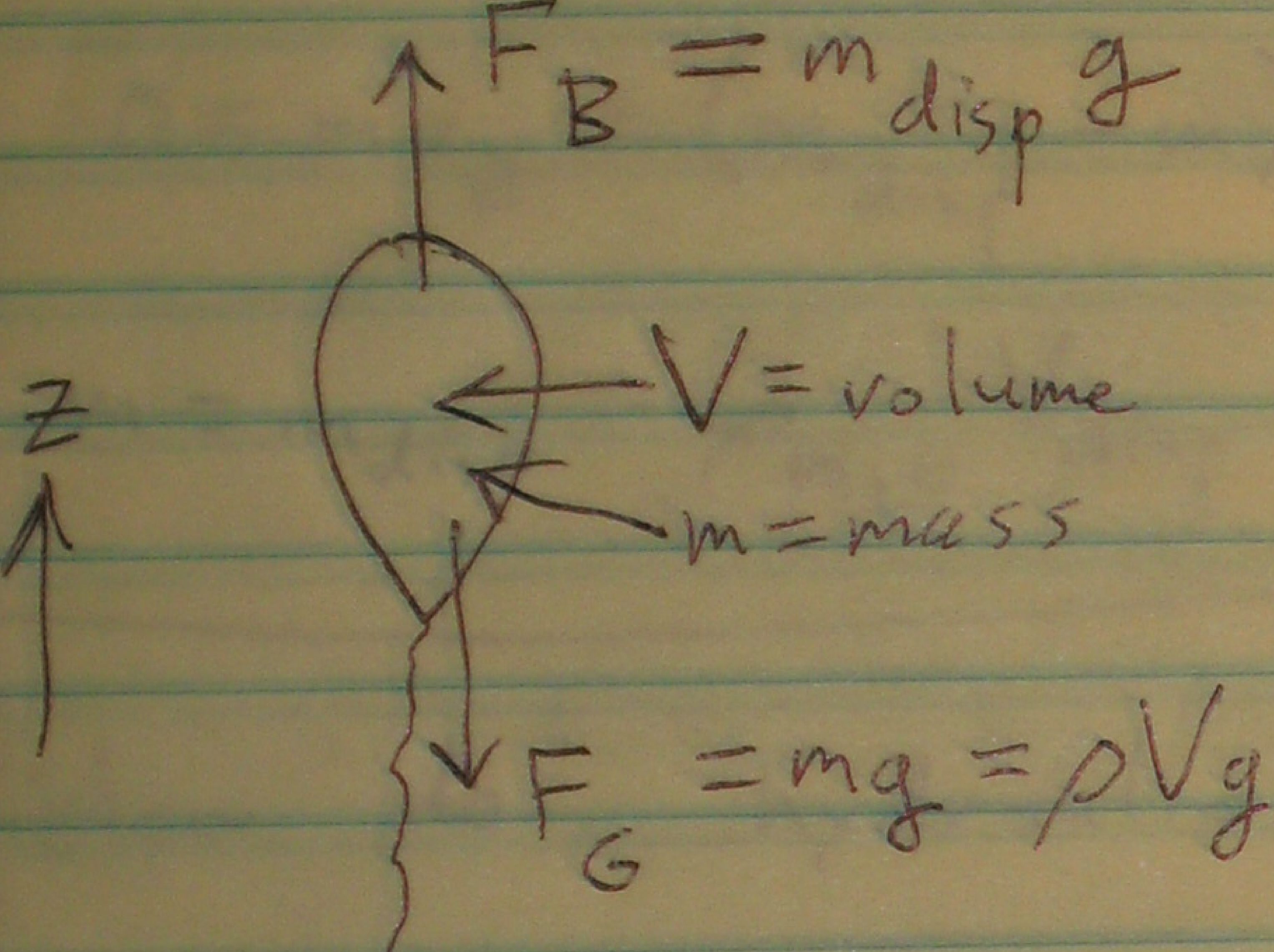
Chapter 13: Fluids

Density = mass/volume $\rho = m/V$

$\rho_{air} = 1.29\ kg/m^3$

Why do rocks fall & helium balloons rise?



F_B = buoyant force

m_{disp} = mass of air displaced

$m_{disp} = \rho_{air} V$

$$ma_z = -mg + m_{disp}\, g = (m_{disp} - m)\, g = (\rho_{air} - \rho) V g$$

If $\rho_{air} \ll \rho$ (rocks), then $ma_z \approx -mg$.

because $\rho_{air} - \rho \approx -\rho$.

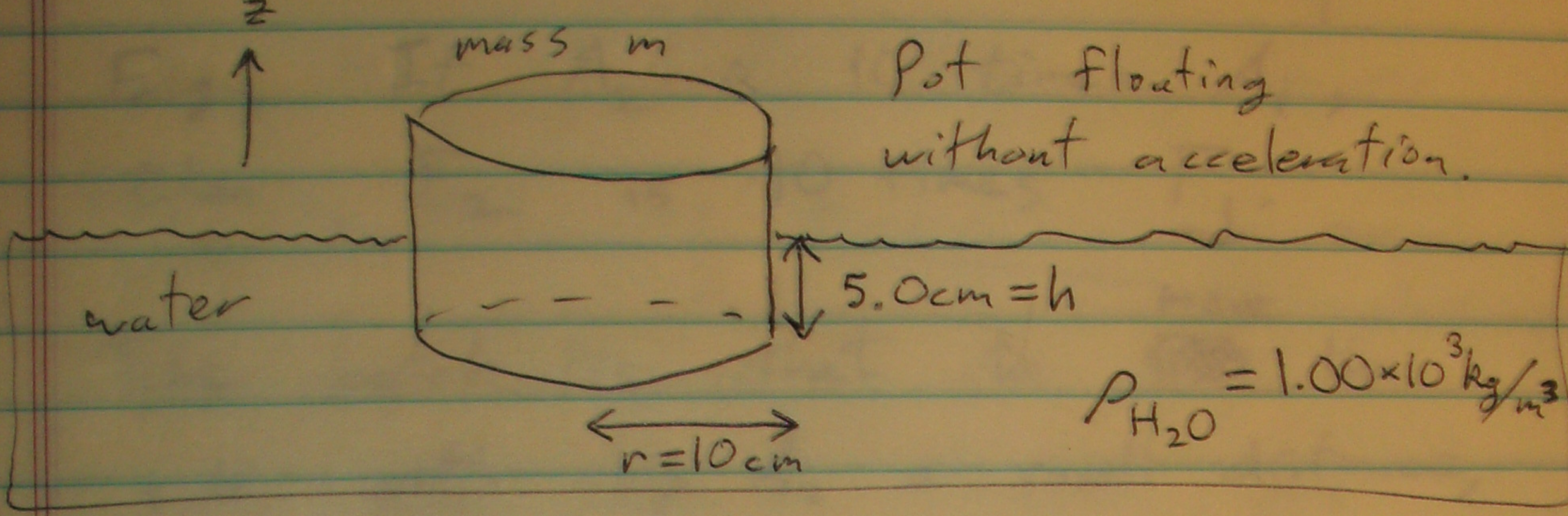
~~If $\rho_{air} \approx \rho$, then ma_z~~

For ρ_{air} closer to ρ, ma_z can significantly differ from $-mg$.

For $\rho_{air} > \rho$, $ma_z > 0$ (balloon rises).

Another example: find mass of pot:

Pot floating without acceleration.



$$0 = ma_z = (m_{disp} - m)g \Rightarrow m_{disp} = m$$

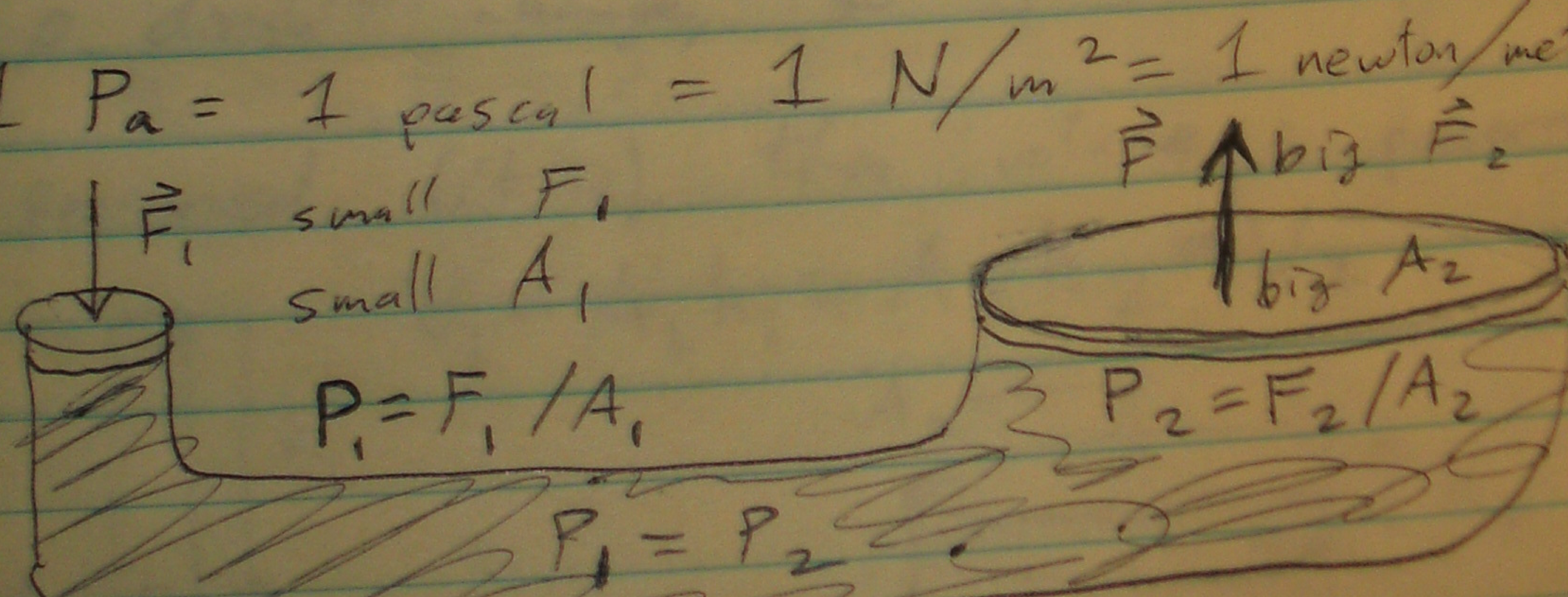
$$m = m_{disp} = \rho_{H_2O} V_{disp} = \rho_{H_2O} \pi r^2 h = \boxed{1.57 \, kg}$$

How do hydraulic lifts work?

Pascal's Principle: pressure is the same everywhere in a confined fluid.

P = Pressure = $|\vec{F}|/A$ = force/area

$1 \, Pa = 1$ pascal $= 1 \, N/m^2 = 1$ newton/meter^2



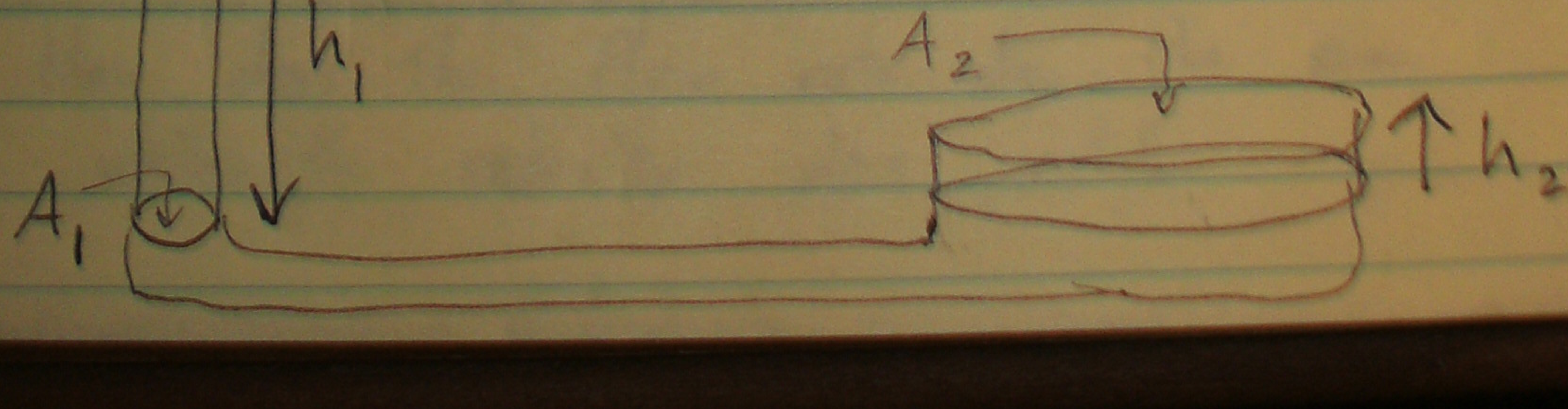
$$\frac{F_1}{A_1} = P_1 = P_2 = \frac{F_2}{A_2} \Rightarrow F_2 = F_1 \frac{A_2}{A_1}$$

E.g., If A_2 is 10 times A_1, then F_2 is 10 times F_1.

The catch is that to move the side with big A_2 a small distance, you have to move the side with small A_1 a large distance. (So, you want a long, maybe coiled, thin tube for the small A_1 end.)

This is conservation of mass. For an incompressible fluid like H_2O (liquid), ρ doesn't change, so if mass is preserved (it is), then volume is preserved:

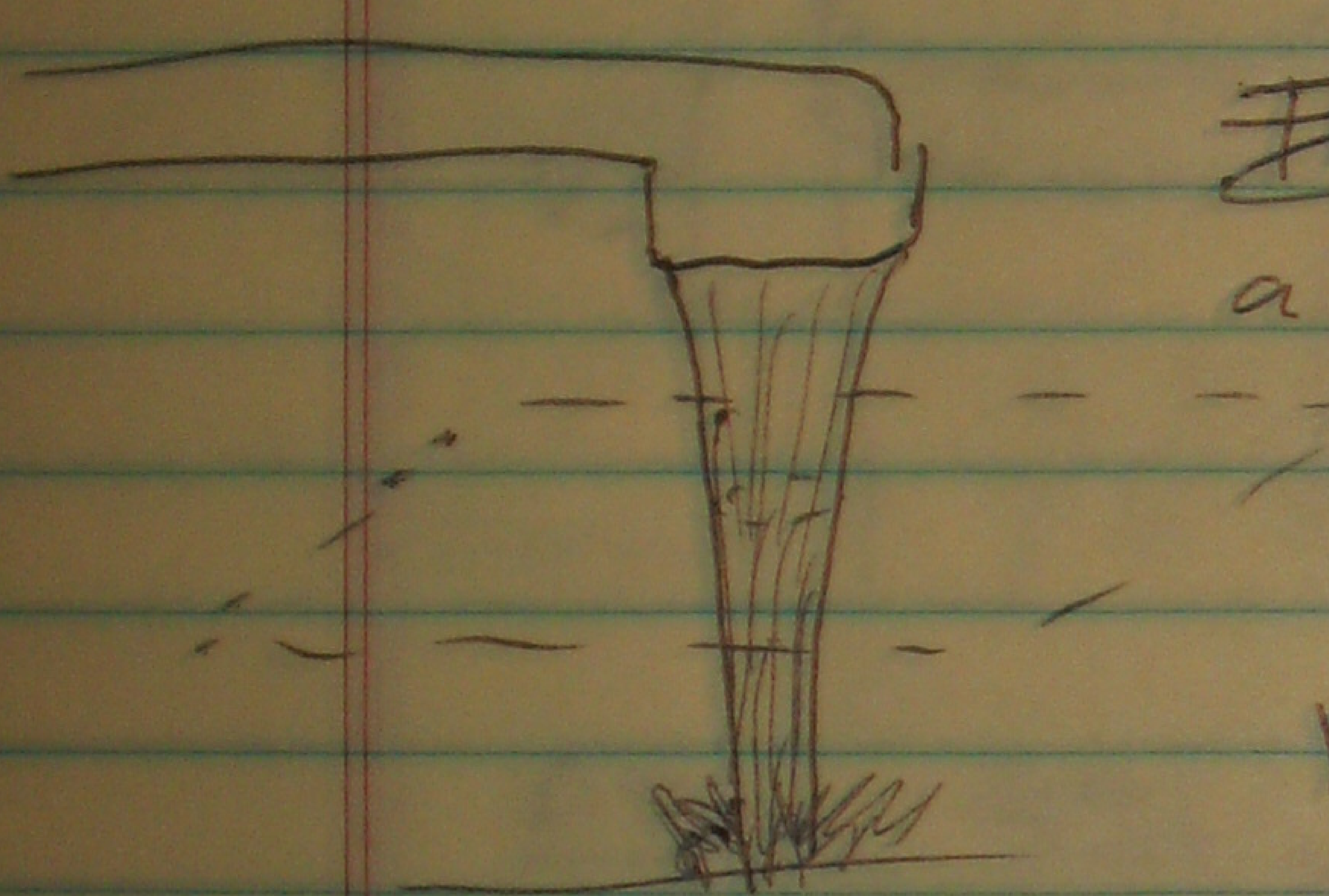
$$V_1 = A_1 h_1 = A_2 h_2 = V_2$$



$h_2 = h_1 A_1 / A_2$

E.g., if A_2 is $10 \times A_1$, then moving the A_1 end 10 m only moves the A_2 end 1.0 m.

Conservation of mass also explains why water from a faucet gets thinner as it falls:

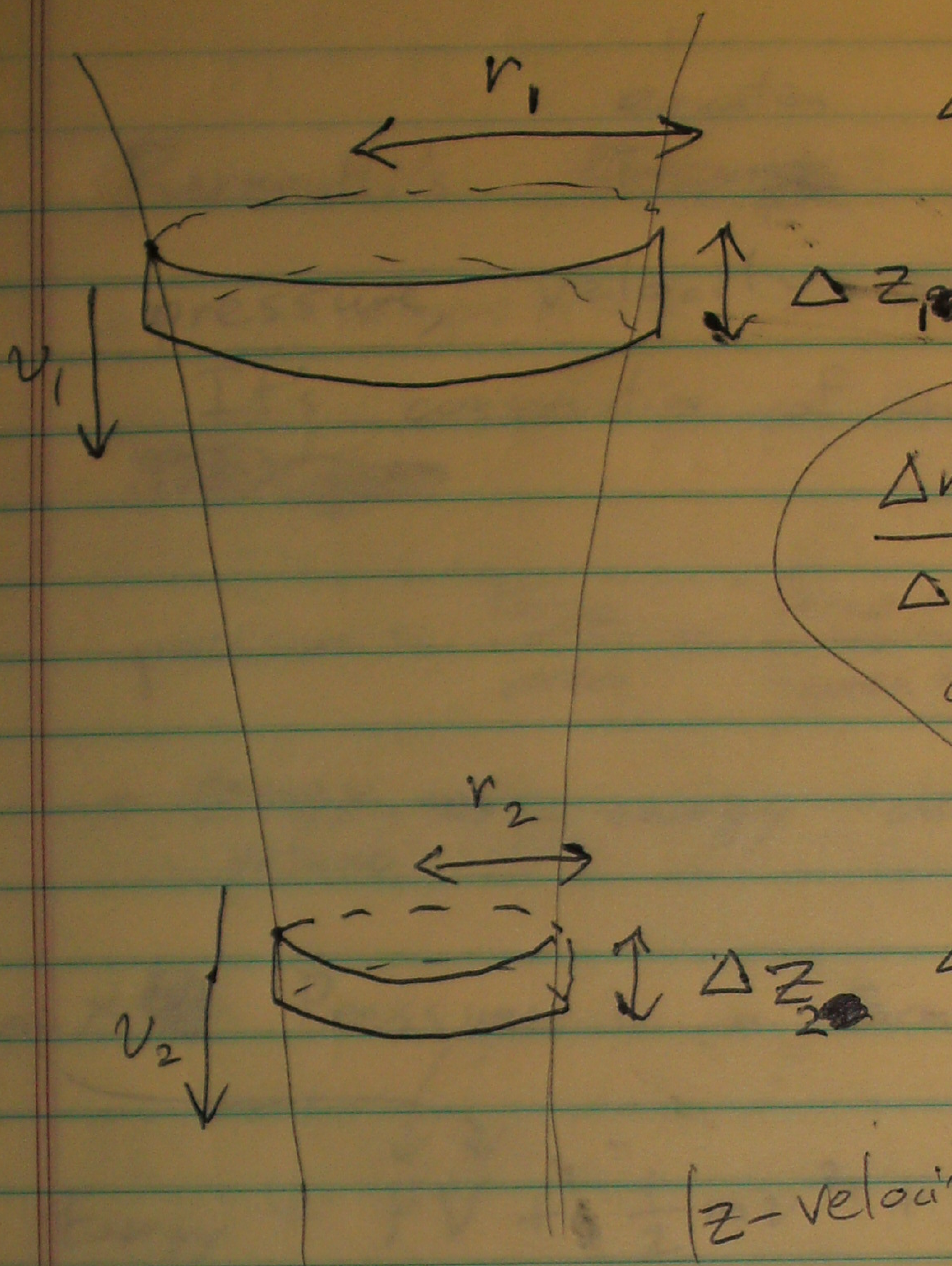


~~The flow rate~~ Imagine a horizontal plane slicing through the stream of water. How many kilograms of H_2O pass ~~flo~~ through the plane each second?

This flow rate (mass/time) is the same everywhere along the stream. Yet, the velocity of the H_2O is faster at the lower parts of the stream than the upper parts because the water is accelerated by gravity as it falls.

How does the flow rate stay the same while the velocity does not?

$\rho = 1.00 \times 10^3\ kg/m^3$



$$\Delta m_1 = \rho \underbrace{\pi r_1^2 \Delta z_1}_{V_1}$$

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

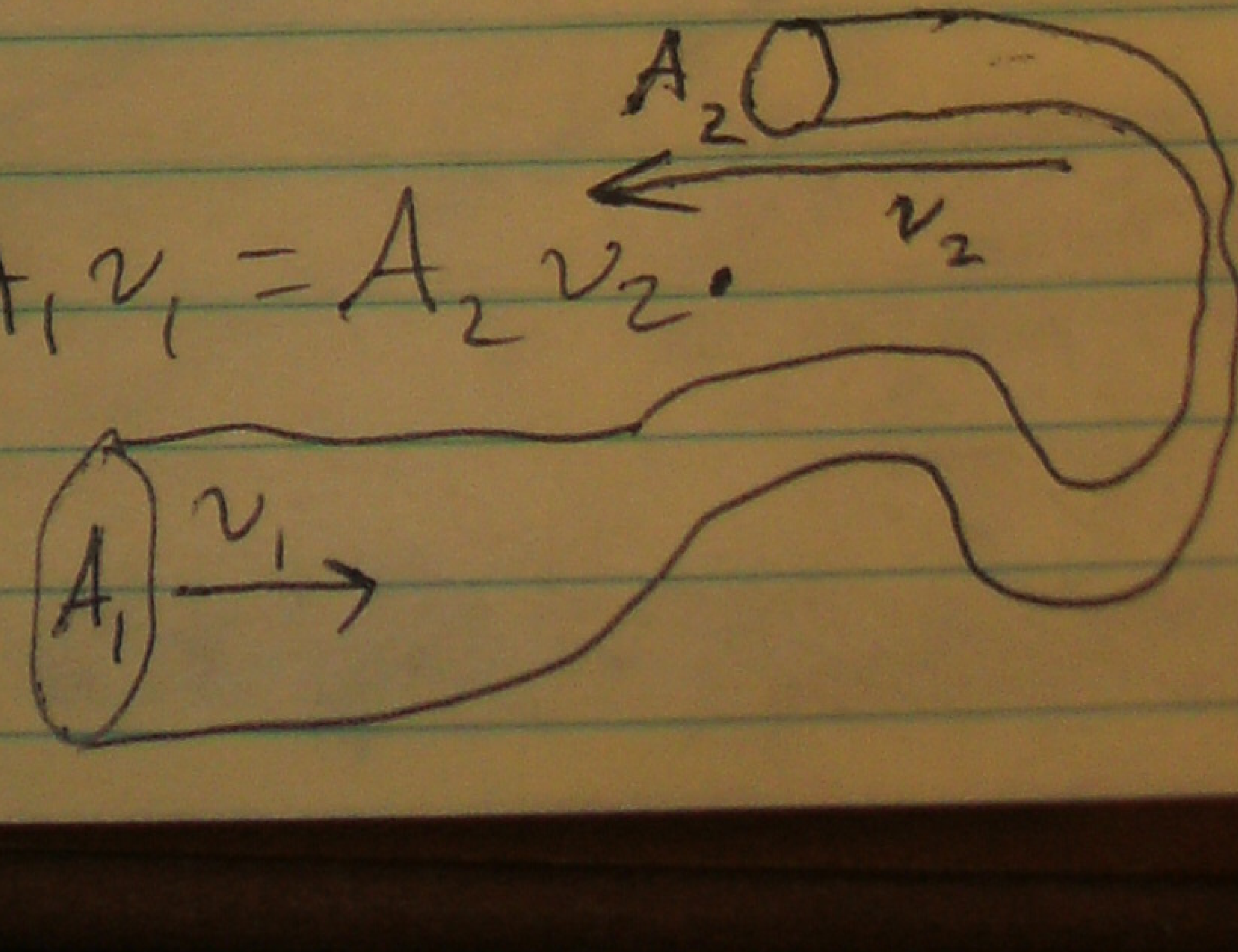
conservation of mass

$$\Delta m_2 = \rho \underbrace{\pi r_2^2 \Delta z_2}_{V_2}$$

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \pi r_1^2 \overbrace{\frac{\Delta z_1}{\Delta t}}^{|z\text{-velocity}|} = \pi r_2^2 \overbrace{\frac{\Delta z_2}{\Delta t}}^{|z\text{-velocity}|}$$

$$\Rightarrow \underbrace{\pi r_1^2}_{\text{bigger area}} \underbrace{v_1}_{\text{smaller speed}} = \underbrace{\pi r_2^2}_{\text{smaller area}} \underbrace{v_2}_{\text{bigger speed}}$$

In general, $A_1 v_1 = A_2 v_2$.



Bernoulli's equation connects pressure, velocity, and height.

It's conservation of energy for fluids.

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{\text{force} \times \text{distance}}{\text{area} \times \text{length}} = \frac{\text{work}}{\text{volume}}$$

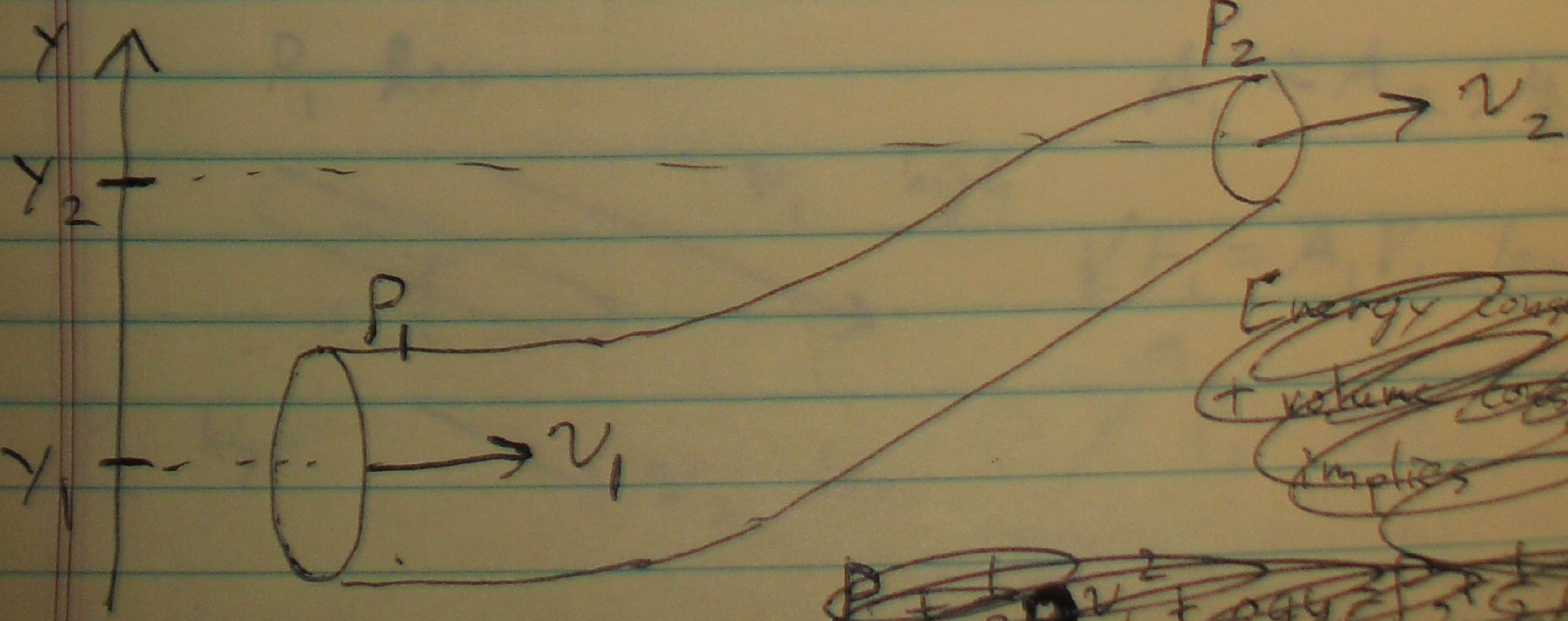
$$= \frac{\text{energy}}{\text{volume}} = \text{energy density.}$$

Volume × Pressure is a form of stored energy!

$$\text{Energy} = PV + \frac{1}{2}mv^2 + mgy$$

$$E = PV + \frac{1}{2}\rho V v^2 + \rho V g y$$

$$E/V = P + \frac{1}{2}\rho v^2 + \rho g y$$



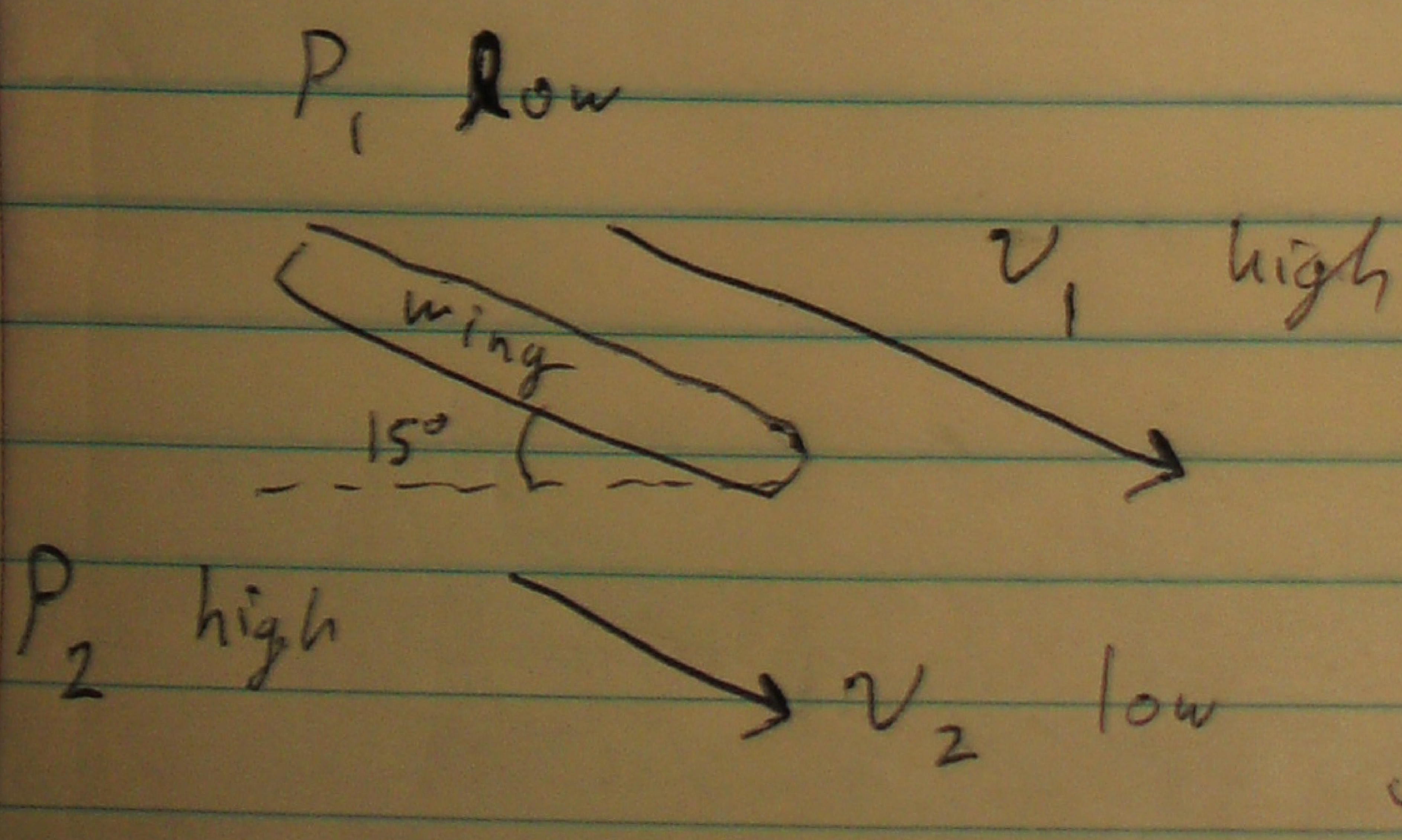
~~Energy conservation + volume conservation implies~~

$\cancel{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1}$

Conservation of mass & energy for an incompressible fluid implies conservation of mass, density, volume, & energy, which implies $\underbrace{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$, Bernoulli's equation.

Example: How do planes fly?

With a good angle of attack, like 15°, friction causes the speed of air above the wing (speed measured relative to the wing) to be higher than the speed of air below the wing.



$A_1 \approx A_2$; $y_1 \approx y_2$.

$F_1 = A_1 P_1$ low

$\underbrace{F_2 = A_2 P_2 \text{ high}}_{\text{force on wing}}$