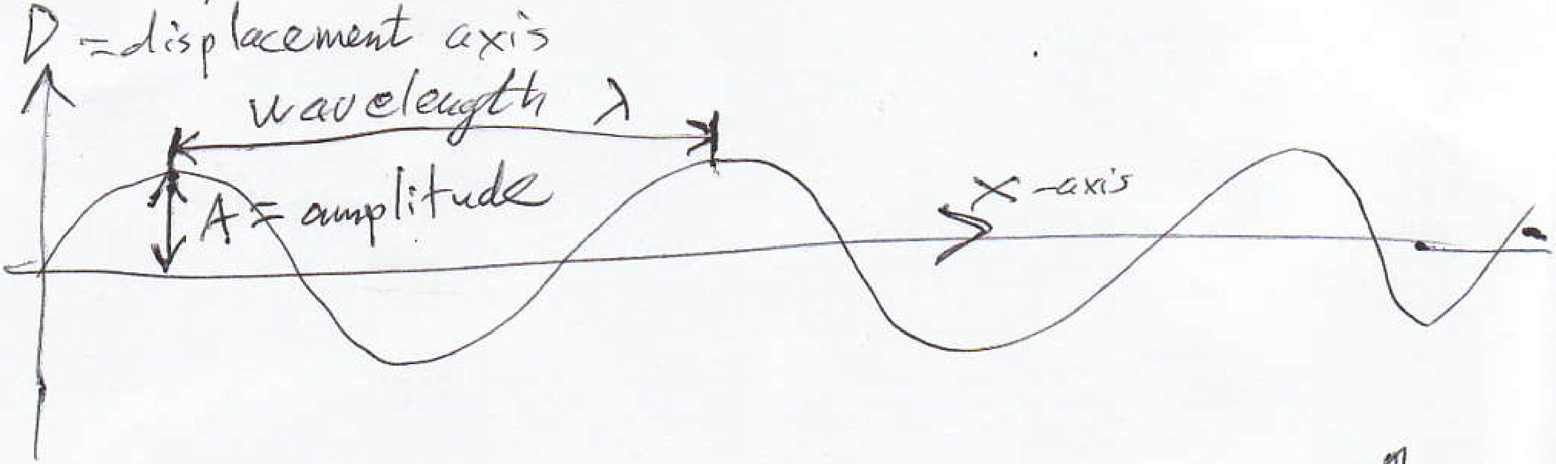
Reminder: simulations due Apr. 19.

Today: Waves & Sound

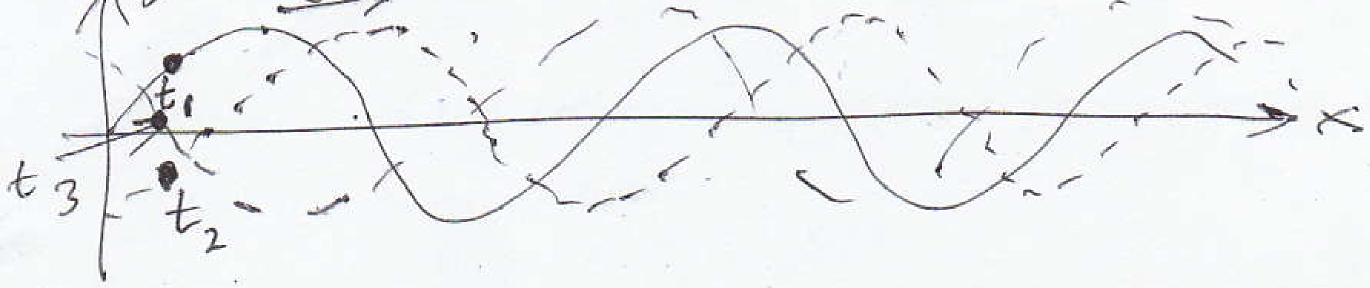
Ch. 15: 1, 2, 3, 4, 6, 9
Ch. 16: 4, 7
Sections

Simplest waves: sine waves




Snapshot at a particular time

waves moving to the right at speed v:
(v = wave speed)



Individual parts of string/air column/etc oscillate with period $T$.

$$\frac{\lambda}{T} = v$$

phase shift

Snapshot

$$D = A\sin\left(\frac{2\pi}{\lambda}x + \phi\right) \quad @\ t=0$$

$$D = A\sin(\phi) \quad @\ t=0,\ x=0$$

Motion: To go to right:

$$x \longmapsto x - vt$$

To go to left

$$x \longmapsto x + vt$$

$$D = A\sin\left(\frac{2\pi}{\lambda}(x - vt) + \phi\right)$$

(moving to right)

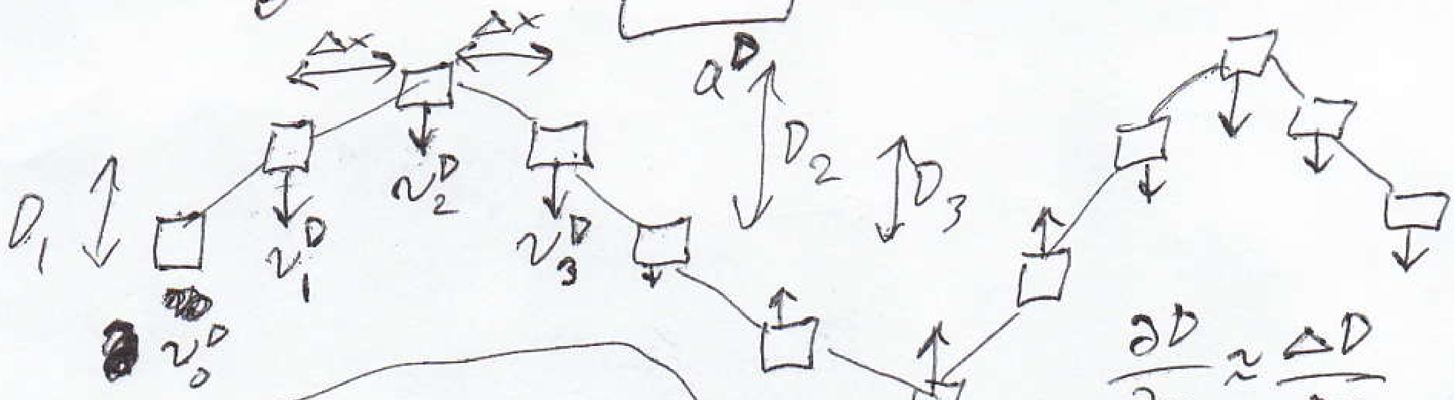
Use $\frac{\lambda}{T} = v$: $D = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \phi\right)$

(moving to right).

All waves are sums of sine waves.

(If you want to simulate waves, go to optional section 15.5:

$$= \frac{\partial^2 D}{\partial x^2} = \underbrace{\frac{\partial^2 D}{\partial t^2}}_{a^D} \cdot \frac{1}{v^2} \iff v^2 \frac{\partial^2 D}{\partial x^2} = \frac{\partial^2 D}{\partial t^2}$$




$$\frac{\partial D}{\partial x} \approx \frac{\Delta D}{\Delta x}$$

$$dv_2^D = a_2^D \, dt$$

$$v_2^D = v_2^D + dv_2^D$$

$$\frac{\partial^2 D}{\partial x^2} \approx \frac{\Delta(\Delta D)}{(\Delta x)^2} \Rightarrow \frac{D_1 - 2D_2 + D_3}{(\Delta x)^2}$$

$$\boxed{dv_2^D = \frac{v^2}{(\Delta x)^2}(D_1 - 2D_2 + D_3)\, dt}$$

Because $\Delta(\Delta D) = (D_3 - D_2) - (D_2 - D_1)$

$$dD_2 = v_2\,dt$$
$$D_2 = D_2 + dD_2$$

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Adding left + right to get up&down.

$$D = A\sin\Bigl(\overbrace{\tfrac{2\pi}{\lambda}\underbrace{(x - vt)}_{\text{right}\rightarrow}}^{\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}}\Bigr) + A\sin\Bigl(\overbrace{\tfrac{2\pi}{\lambda}\underbrace{(x + vt)}_{\leftarrow\text{left}}}^{\frac{2\pi x}{\lambda} + \frac{2\pi t}{T}}\Bigr)$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cdot\cos\beta \pm \sin\beta\cos\alpha$$

$$D = A\Bigl[\Bigl(\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T} - \sin\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}\Bigr) + \Bigl(\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T} + \sin\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}\Bigr)\Bigr]$$

$$D = 2A\Bigl(\sin\frac{2\pi x}{\lambda}\Bigr)\Bigl(\cos\frac{2\pi t}{T}\Bigr)$$

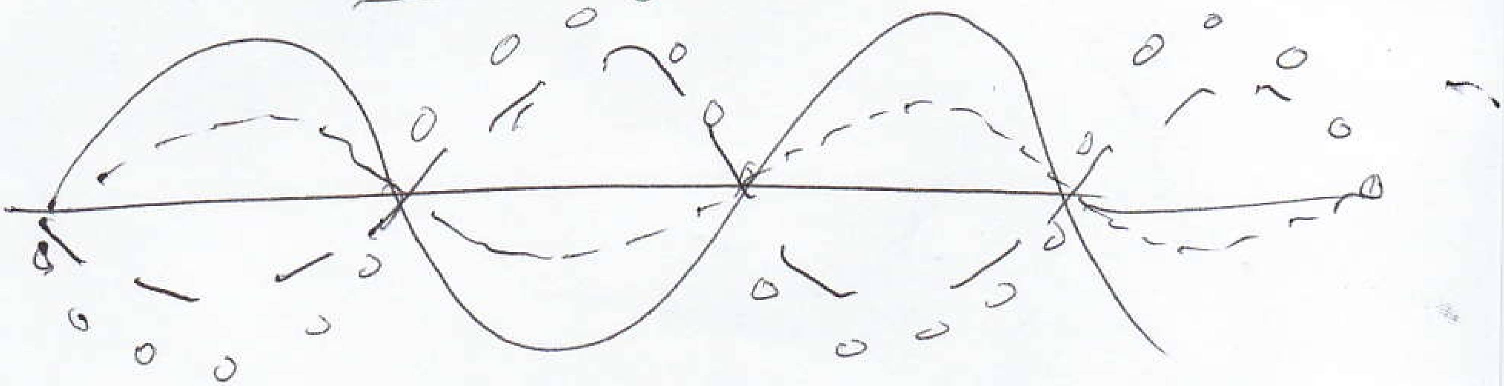
At $x = \frac{\lambda}{4}$, $t = 0$: $D = 2A\sin\frac{\pi}{2}\cos 0$

$$D = 2A$$

Snapshot @ time t:

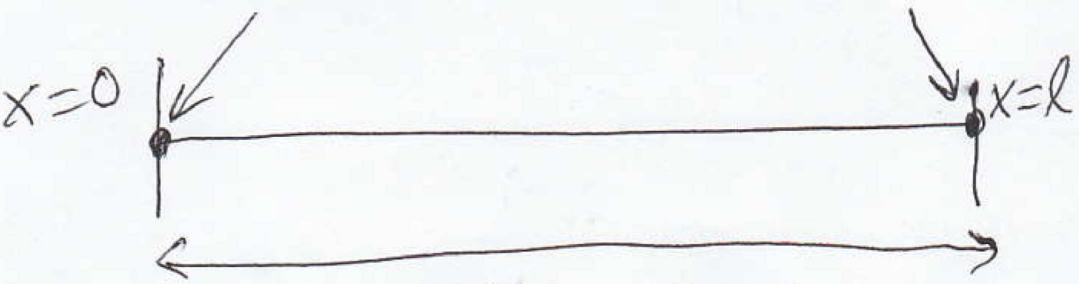
$$D = \left(2A\cos\frac{2\pi t}{T}\right)\sin\frac{2\pi x}{\lambda}$$

(standing waves)



Sound:

D=0 ← at all times → D=0




string with 2 fixed ends

length l

~~Need sin~~ ~~To have a standing wave of a single freque~~

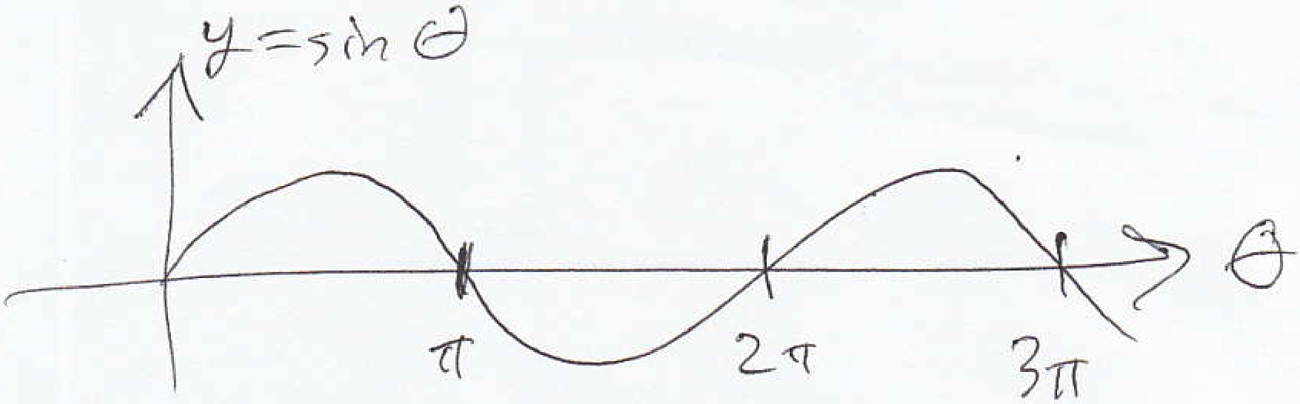
If $D = 2A\cos\frac{2\pi t}{T}\sin\frac{2\pi x}{\lambda}$

is a standing wave on this string,

then $\underbrace{\sin\left(\frac{2\pi(0)}{\lambda}\right)}_{\sin 0 = 0} = 0 = \sin\left(\frac{2\pi \ell}{\lambda}\right)$

real constraint

no information




$$\frac{2\pi \ell}{\lambda} = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$$

$$\lambda = \frac{2\ell}{1}, \frac{2\ell}{2}, \frac{2\ell}{3}, \frac{2\ell}{4}, \ldots$$

$$\lambda_1 = \frac{2\ell}{1}, \lambda_2 = \frac{2\ell}{2}, \lambda_3 = \frac{2\ell}{3}, \ldots$$


$$D = A_1 \cos\left(\frac{2\pi t}{T_1} + \phi\right) \sin\frac{2\pi x}{\lambda_1}$$

$$+ A_2 \cos\left(\frac{2\pi t}{T_2} + \phi\right) \sin\frac{2\pi x}{\lambda_2}$$

$$+ \ldots$$

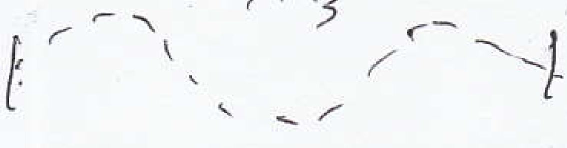
$$D = \cos\left(\frac{2\pi t}{T} + \phi\right)\left(A_1 \sin\frac{2\pi x}{\lambda_1} + A_2 \sin\frac{2\pi x}{\lambda_2} + A_3 \sin\frac{2\pi x}{\lambda_3} + A_4 \sin\frac{2\pi x}{\lambda_4} + \ldots\right)$$

$A_1$ Almost always biggest/loudest

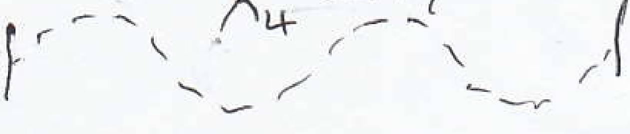
$$\lambda_1 = 2l = 2l/1$$

$$\lambda_2 = l = 2l/2$$

$$\lambda_3 = 2l/3$$



$$\lambda_4 = l/2 = 2l/4$$



$$\frac{\lambda}{T} = v$$

only depends on density & tension of string

$$f = \frac{1}{T}$$

Frequency

$$T_1 = \frac{\lambda_1}{v} = \frac{2l}{1v}$$

$$f_1 = \frac{1v}{2l}$$ Fundamental (pitch you hear most)

$$T_2 = \frac{\lambda_2}{v} = \frac{2l}{2v}$$

$$f_2 = \frac{2v}{2l}$$ ← 1st ~~Fundamental~~ Harmonic

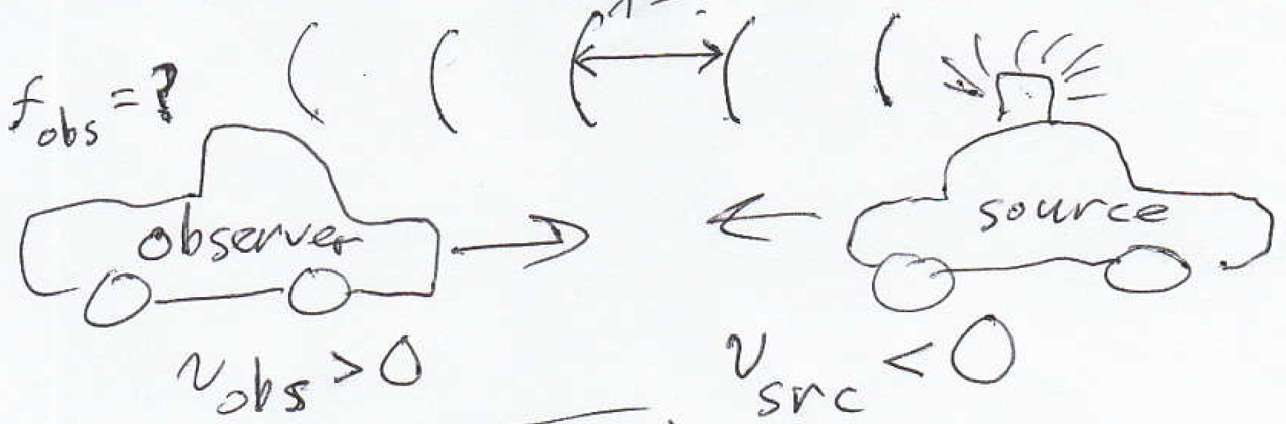
$$T_3 = \frac{\lambda_3}{v} = \frac{2l}{3v}$$

$$\vdots$$

$$f_3 = \frac{3v}{2l}$$ ← 2nd Harmonic

The sequence $A_1, A_2, A_3, \ldots$ of amplitudes distinguishes 2 instruments playing same pitch.

(And for more ways 2 instruments playing the same pitch can sound different, read about "timbre." Start at Wikipedia if necessary.)




For obs. left of src.

$\longrightarrow x$

$$f_{obs} = f_{src}\left(\frac{1 + \frac{v_{obs}}{v_{snd}}}{1 + \frac{v_{src}}{v_{snd}}}\right)$$

~~approaching~~

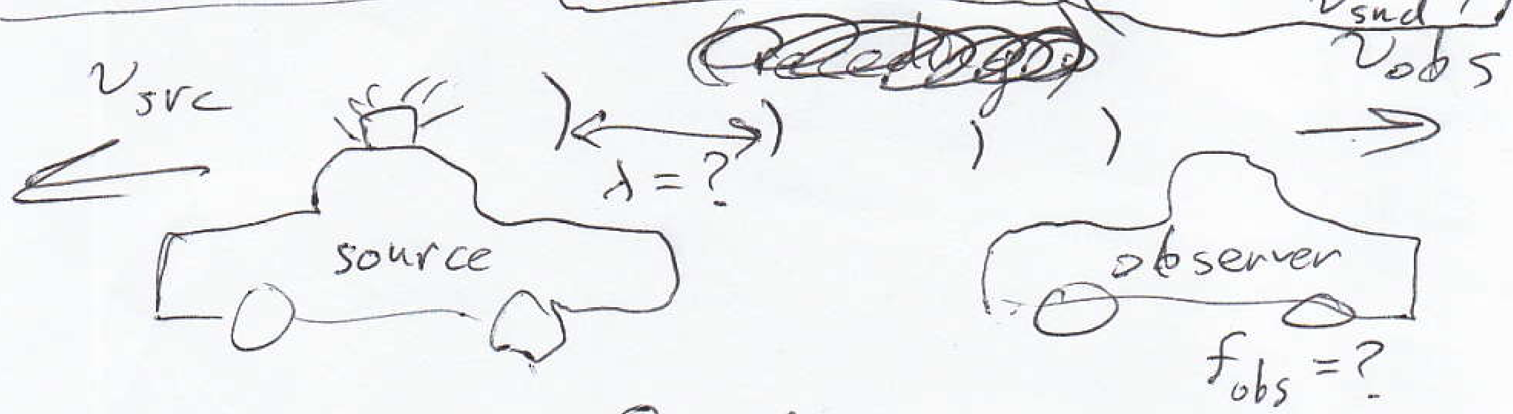
$v_{snd} = 343$ m/s @ 20°C (1 atm pressure)

speed of sound in air

For obs. right of src.

$$\boxed{f_{obs} = f_{src}\left(\frac{1 - \frac{v_{obs}}{v_{snd}}}{1 - \frac{v_{src}}{v_{snd}}}\right)}$$

~~receding~~




Doppler Effect