

May 3: Final + ≥ 45 exercises from
8AM Ch. 10-19

①

Today: some highlights from the semester:

Ch. 1 units & significance figures

$$12 \text{ m}^3 \neq (12 \text{ m})^3 = 12^3 \text{ m}^3 = 1728 \text{ m}^3$$

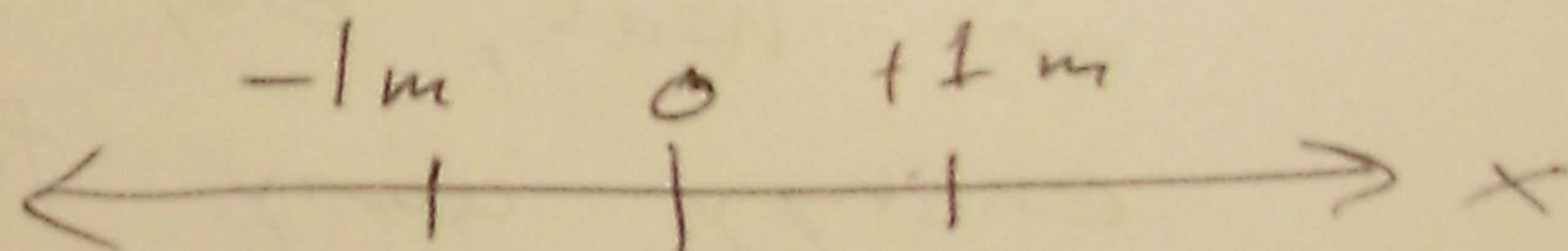
2.5 (2 sig fig) \times 13.47 (4 sig figs) = 34 (2 sig figs) ~~divide~~ & multiply
& squares & square roots
add & subtract

$$13.47 + 2.5 = 16.0$$

\uparrow \uparrow \uparrow
 $\sim \pm 0.01$ $\sim \pm 0.1$ $\sim \pm 0.1$
bigger uncertainty

Ch. 2 1D kinematics (describing motion)

x = position



t = time

$$v = \frac{dx}{dt} = \text{velocity}$$

$$v \approx \frac{\Delta x}{\Delta t} \text{ for small } \Delta t$$

$$a = \frac{dv}{dt} = \text{acceleration}$$

$$\Delta x = x_{\text{final}} - x_{\text{initial}}$$

$$\frac{\Delta x}{\Delta t} = \text{average velocity}$$

\parallel
 v

$$\Delta t = t_{\text{final}} - t_{\text{initial}}$$

$$\Delta v = v_{\text{final}} - v_{\text{initial}}$$

$$\frac{\Delta v}{\Delta t} = \text{avg. accel.} = \bar{a}$$

$$\Delta v = \int_{v_{\text{initial}}}^{v_{\text{final}}} dv = \int_{t_i}^{t_f} a dt = a \Delta t \quad \left[\begin{array}{l} \text{if } a \text{ constant} \\ \text{if } a \text{ constant} \end{array} \right] \quad (2)$$

$$a = \frac{dv}{dt} \quad v = \frac{dx}{dt}$$

$$\Delta x = \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v dt \quad \begin{array}{l} v \Delta t \text{ if } v \text{ const} \\ \frac{1}{2} a (\Delta t)^2 + v_{\text{init}} \Delta t \\ \text{if } a \text{ const} \end{array}$$

Ch. 3: 2D & 3D kinematics

position $\vec{r} = (r_x, r_y, r_z) = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$

$\hat{i} = (1, 0, 0) \quad \hat{j} = (0, 1, 0) \quad \hat{k} = (0, 0, 1)$

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2} \quad |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \left(\frac{r_x}{|\vec{r}|}, \frac{r_y}{|\vec{r}|}, \frac{r_z}{|\vec{r}|} \right)$$

$|\hat{r}| = 1$ (no units) \uparrow direction of \vec{r}

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dr_x}{dt}, \frac{dr_y}{dt}, \frac{dr_z}{dt} \right)$$

$$\vec{a} = \frac{d\vec{v}}{dt}; \quad \text{speed} = |\vec{v}| \quad \text{velocity} = \vec{v}$$

2D: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$$\Delta \vec{v} = \int d\vec{v} = \int \vec{a} dt = (\int a_x dt, \int a_y dt, \int a_z dt)$$

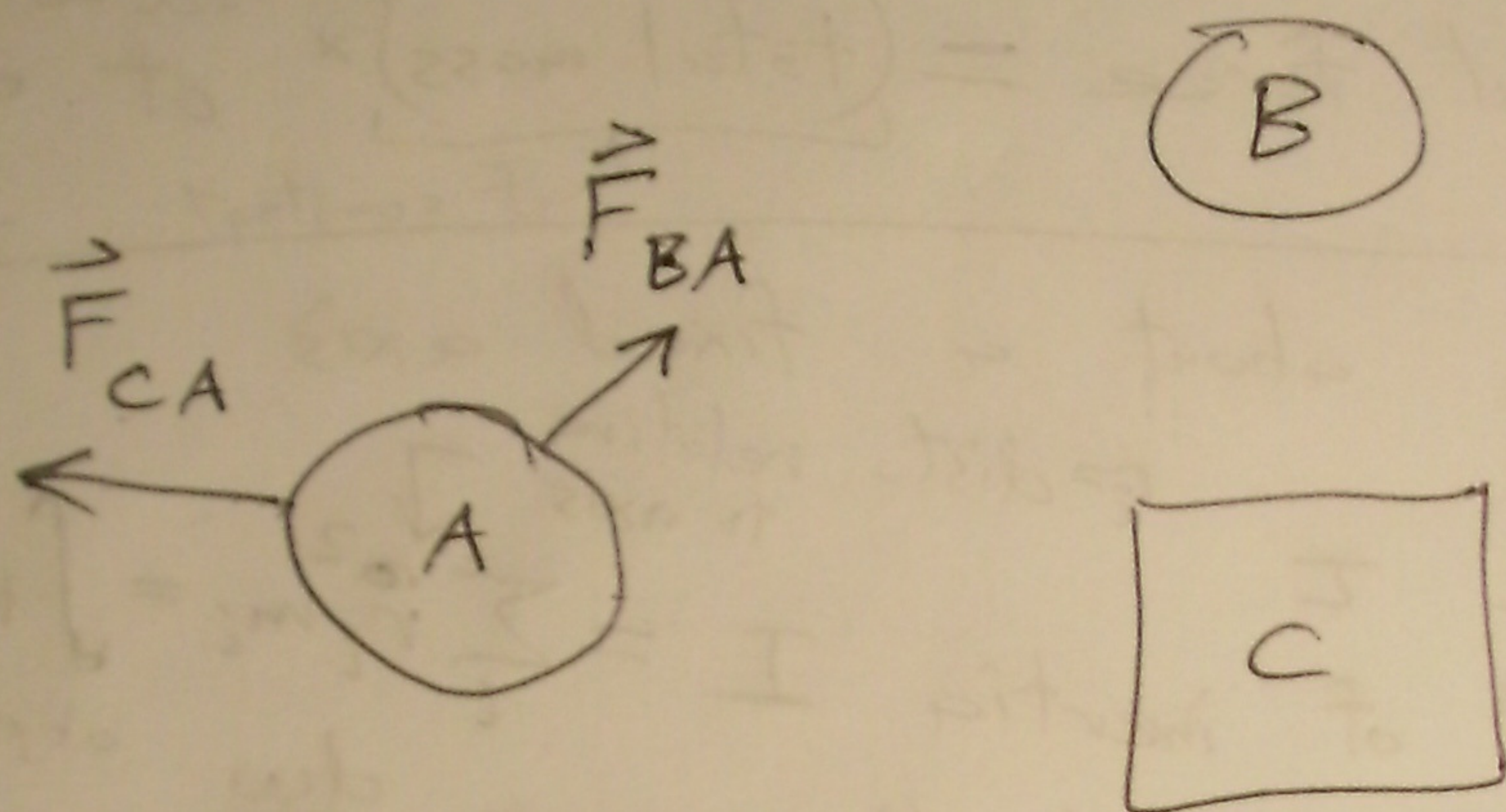
$$\Delta \vec{r} = \int d\vec{r} = \int \vec{v} dt \quad \hat{a} \text{ is "down"}$$

Balistic motion $|\vec{a}| = g = 9.80 \text{ m/s}^2$
 \vec{a} constant in direction too.

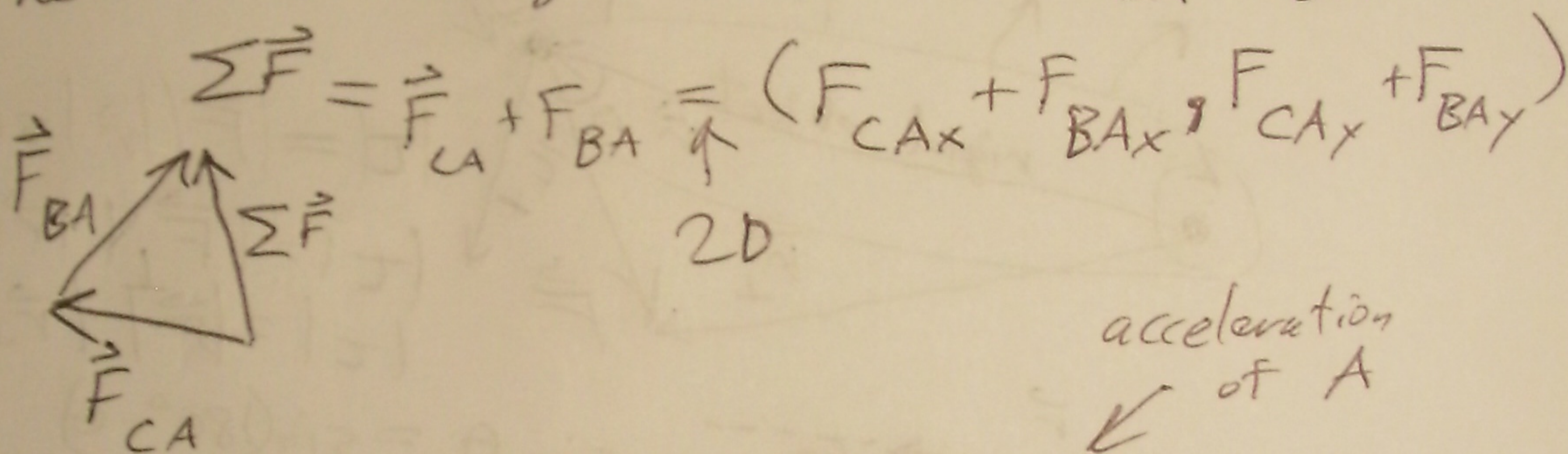
Ch. 4 Forces

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Newton's 1st Law: $\sum \vec{F} = \vec{0} \Rightarrow \vec{v}$ constant



net force acting on A is $\vec{F}_{CA} + \vec{F}_{BA} = \sum \vec{F}$



Σ for "sum" \downarrow

acceleration of A

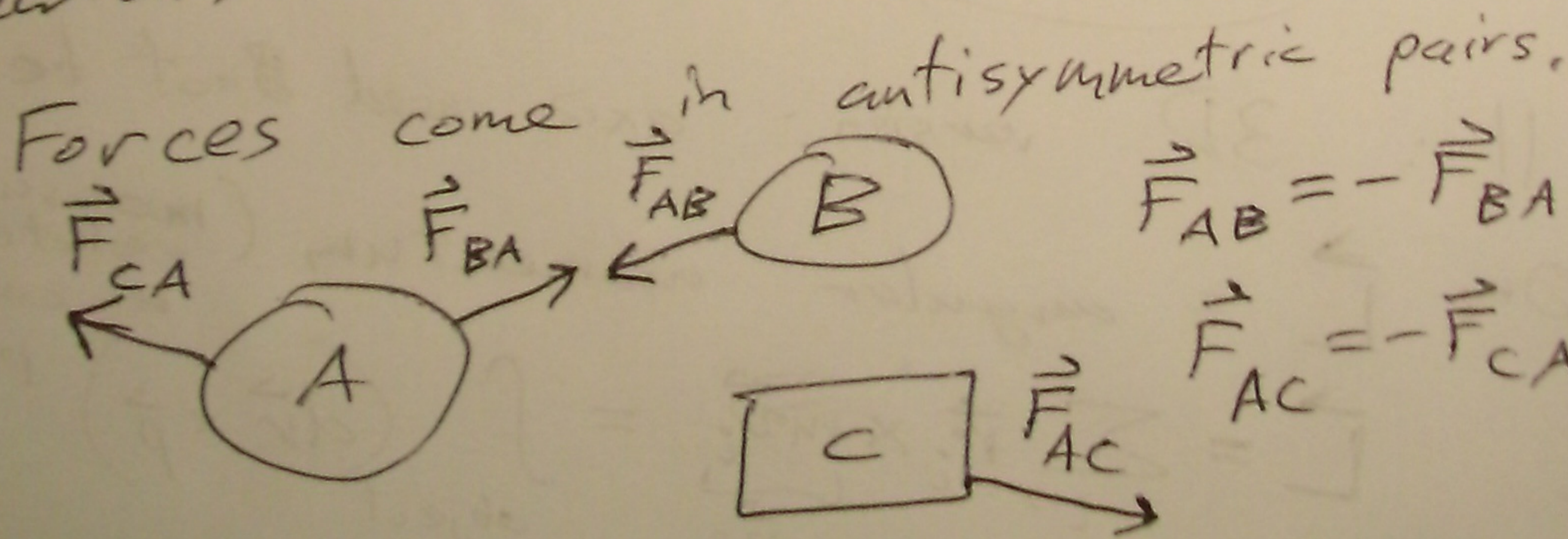
Newton's 2nd Law: $\sum \vec{F} = m \vec{a}$

Equiv: $\frac{1}{m} \sum \vec{F} = \frac{d\vec{v}}{dt}$

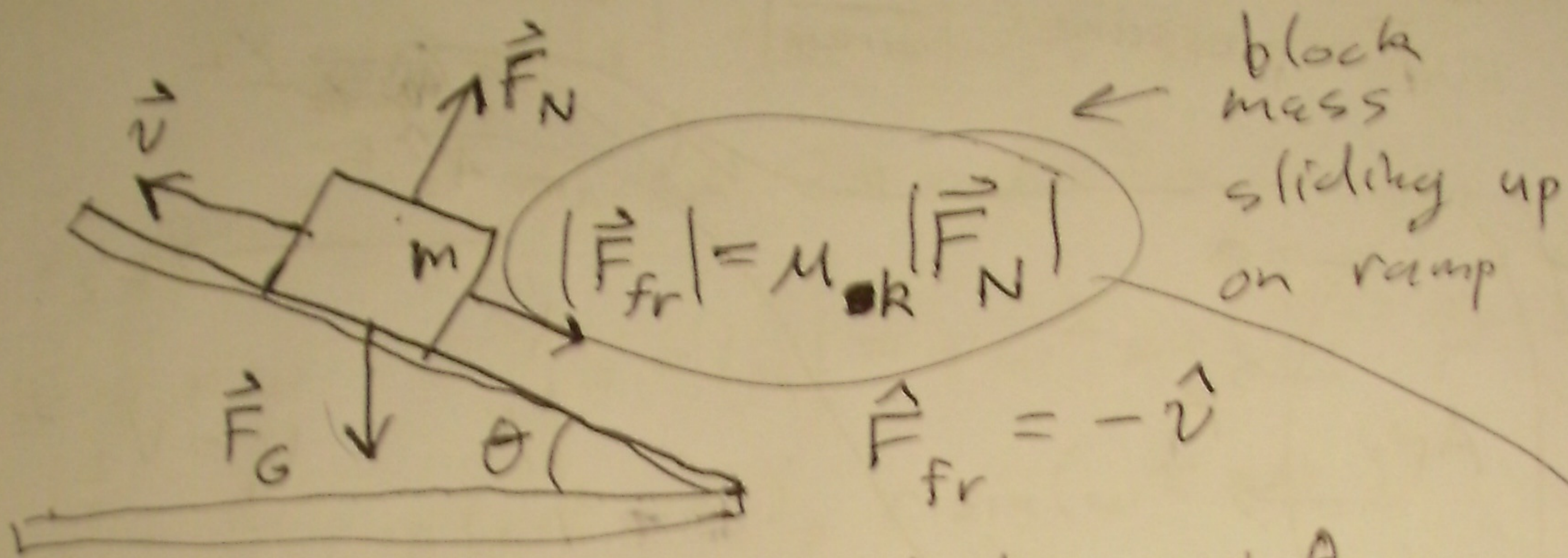
sum of forces acting on A

mass of A

Newton's 3rd Law:



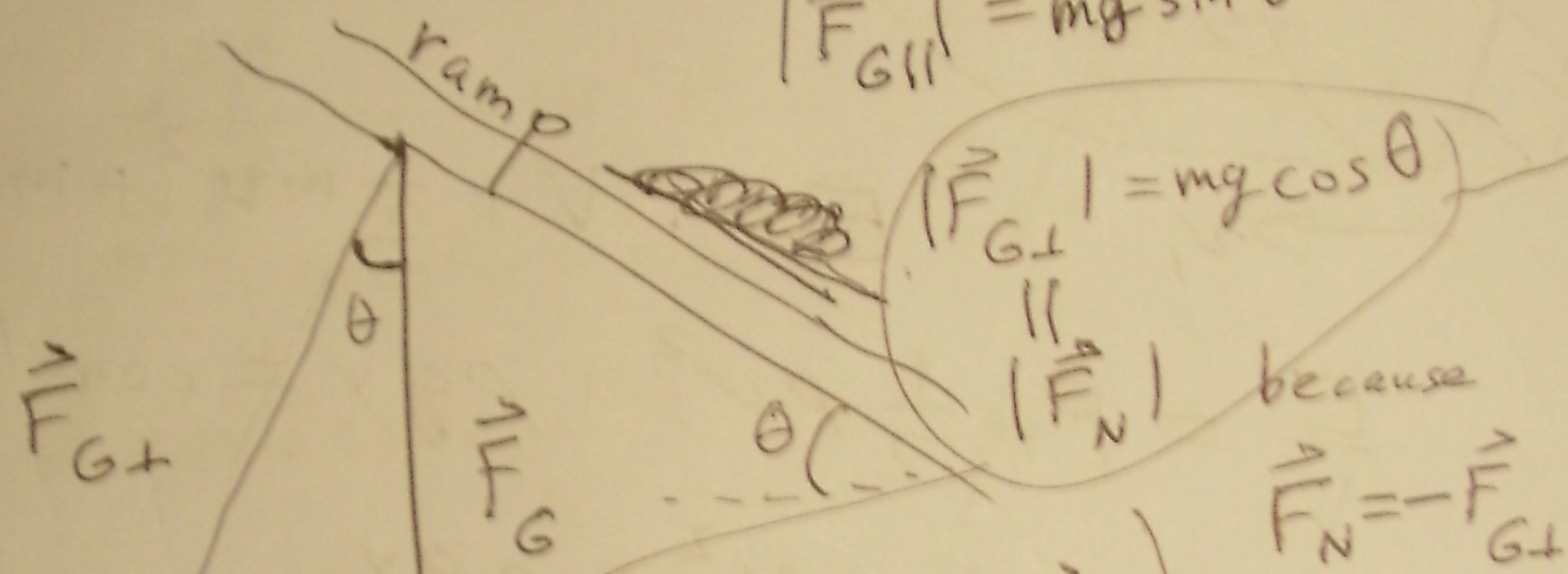
Ch. 5 friction & circular motion



$|\vec{F}_{fr}| = \mu_k |\vec{F}_N|$

$\hat{F}_{fr} = -\hat{v}$

$|\vec{F}_{G||}| = mg \sin \theta$



$|\vec{F}_{G\perp}| = mg \cos \theta$

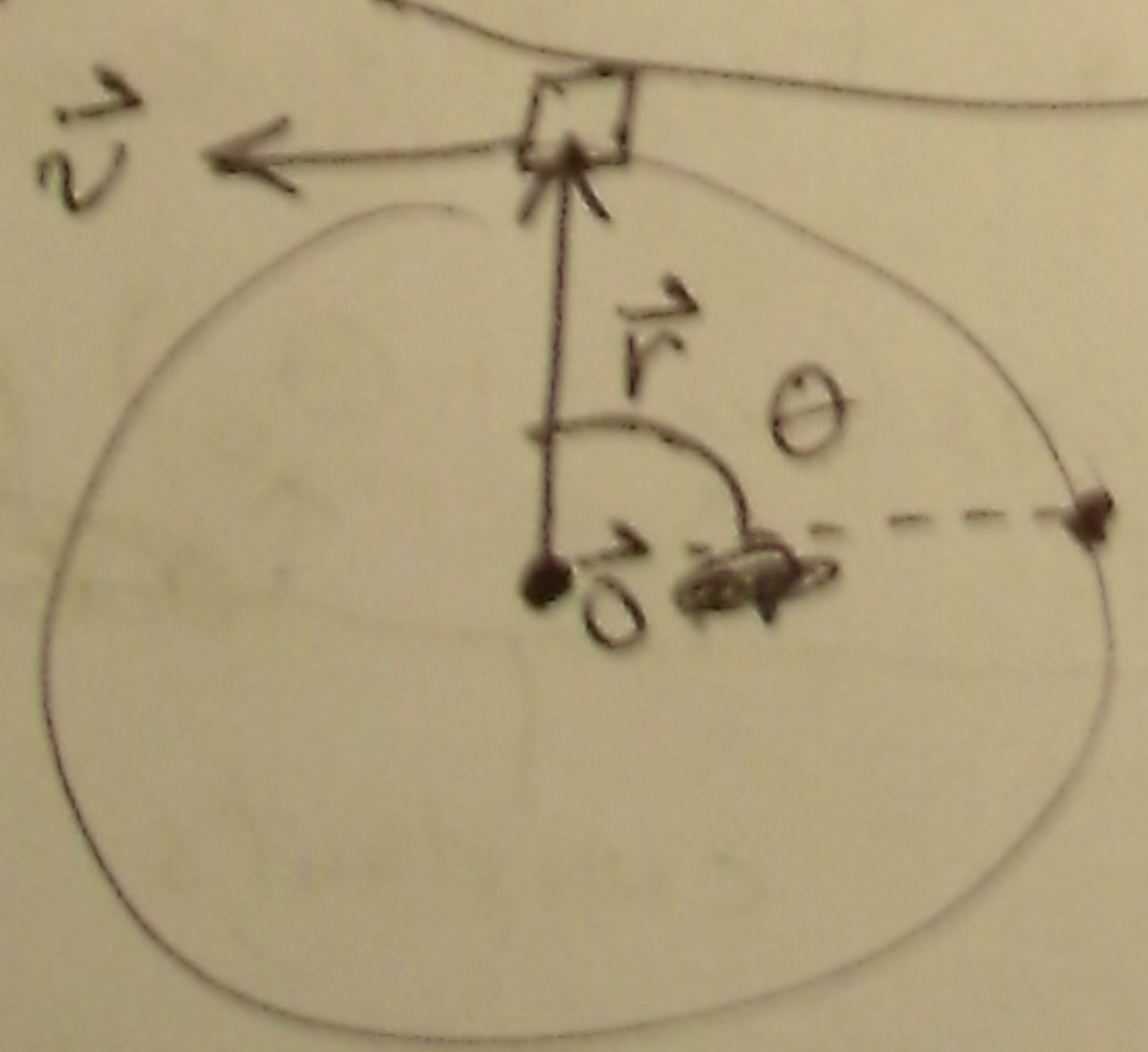
$|\vec{F}_N|$ because $\vec{F}_N = -\vec{F}_{G\perp}$

$\vec{F}_G = \vec{F}_{G\perp} + \vec{F}_{G||}$ because $\vec{a}_\perp = \vec{0}$
 $|\vec{F}_G| = mg$

$|\vec{F}_{fr}| = \mu_k mg \cos \theta$

$\vec{a}_{||} = \vec{F}_{G||} + \vec{F}_{fr}$

uniform circular motion:



$|\vec{r}| = r = \text{constant}$

$|\vec{v}| = v = \text{constant}$

$\vec{r} \perp \vec{v}$

$\theta = \text{angular position \& displacement}$

$\theta \begin{cases} + = \text{counterclockwise} \\ - = \text{clockwise} \end{cases}$

$$\omega = \frac{d\theta}{dt} = \text{angular velocity} = \text{angular frequency} \quad (5)$$

units like rad/s or rev/min

$$\frac{v}{r} = \omega \text{ const.} \quad 1 \text{ rev} = 2\pi \text{ rad} = 360^\circ = 2\pi$$

$$v = r\omega \quad a = |\vec{a}| = r\omega^2 = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$$

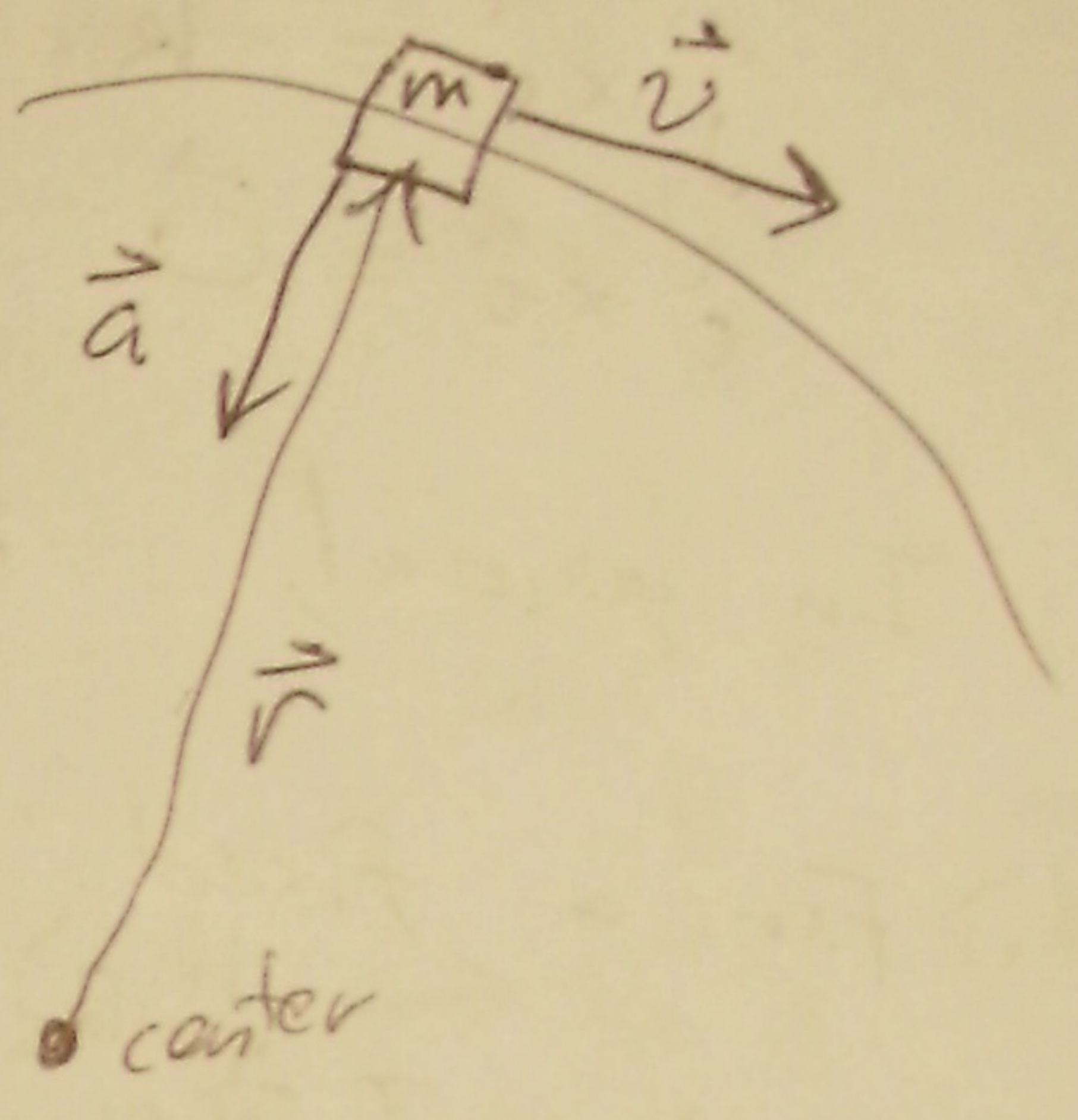
$$\vec{r} = (r \cos(\omega t + \theta_0), r \sin(\omega t + \theta_0))$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -\omega^2 \vec{r}$$

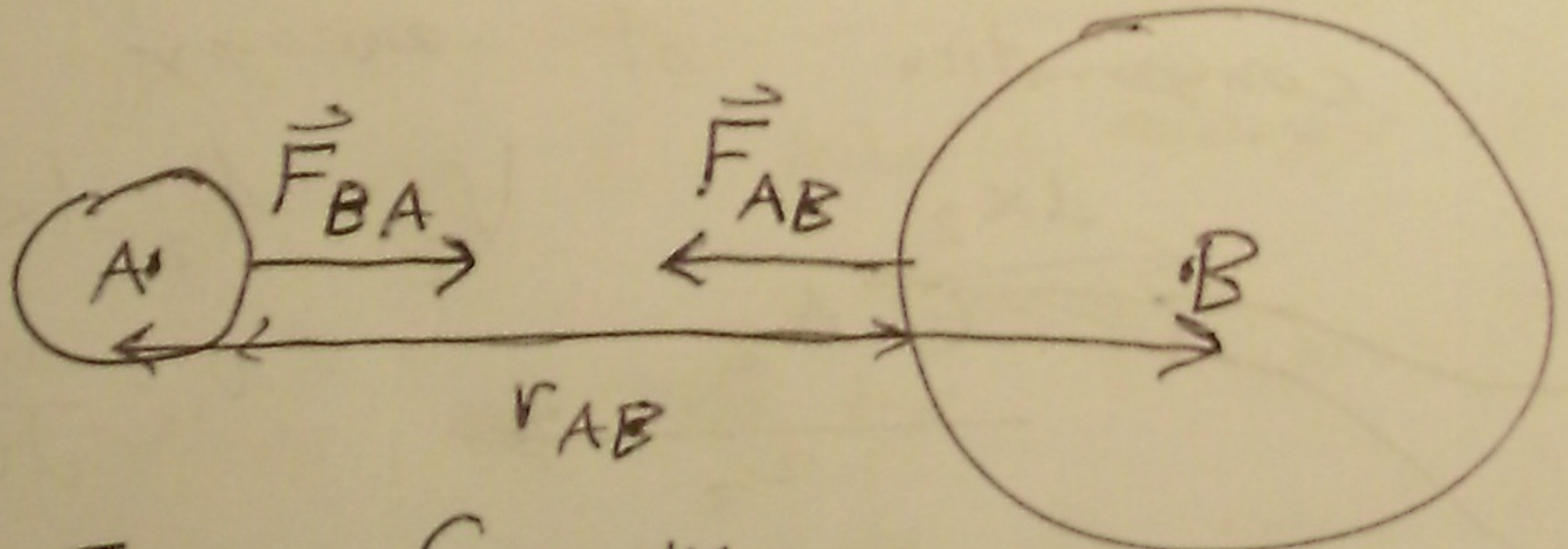
$$|\Sigma \vec{F}| = m\omega^2 r = mv^2/r$$

$$\Sigma \vec{F} = m\vec{a} = -m\omega^2 \vec{r}$$

$$-\frac{mv^2}{r} \hat{r} = -m \frac{v^2 \vec{r}}{r^2}$$



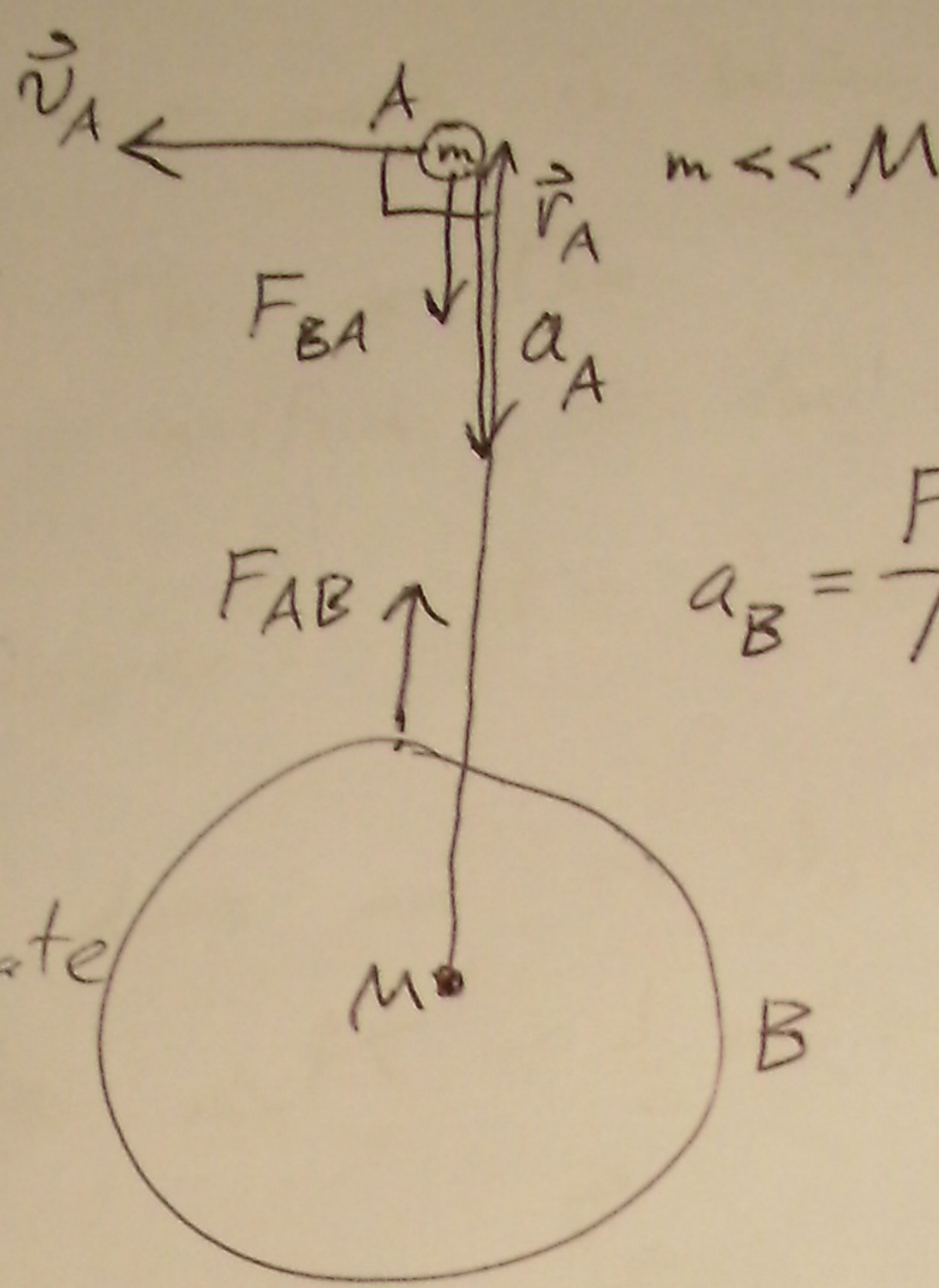
Ch. 6 Gravity $G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$



$$F_{AB} = F_{BA} = \frac{G m_A m_B}{r_{AB}^2}$$

$$1N = 1 kg \cdot m/s^2$$

$$\Sigma \vec{F} = m \cdot \vec{a}$$



Simplest case:

$$|\vec{v}_A| \approx \text{constant}$$

$$|\vec{r}_A| \approx \text{constant}$$

$$\vec{a}_B \approx \vec{0}$$

Choose coordinate system such

that \vec{r}_B & \vec{v}_B

are $\approx \vec{0}$

$$a_B = \frac{F_{AB}}{M} \ll \frac{F_{BA}}{m} = a_A$$

$$F_{BA} = m a_A = m v_A^2 / r_A$$

$$\frac{GmM}{r_A^2} \Rightarrow \frac{v_A^2}{r_A} = \frac{GM}{r_A^2} \Leftrightarrow v_A^2 r_A = GM$$

$$v = \omega r$$

$$\omega = \frac{v}{r}$$

$$T = \text{period} = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T} \quad \omega = \frac{2\pi}{T}$$

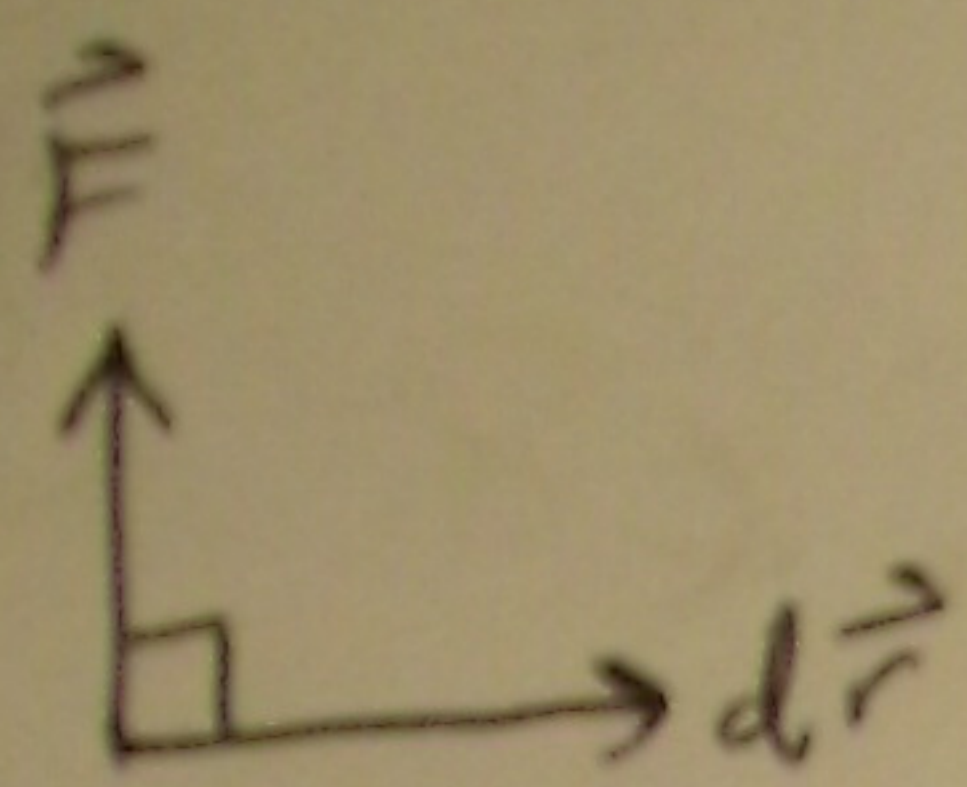
$$\frac{4\pi^2 r^3}{T^2} \parallel \omega^2 r^2 r$$

Ch. 7 Work & kinetic energy & power

Work done ~~by~~ by force F ~~is~~
 from some initial state to some final state

is
$$\int_{\vec{r}_{init}}^{\vec{r}_{final}} \vec{F} \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} |\vec{F}| \cos \theta |d\vec{r}|$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$



\Rightarrow no work is being done.

This is the case for uniform circular motion.

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Kinetic energy: $K = \frac{1}{2} m v^2$

Net work done on an object of mass

m equals $\Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$

$$\int_{\vec{r}_i}^{\vec{r}_f} \sum \vec{F} \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} m \vec{a} \cdot d\vec{r} = m \int_{\vec{r}_i}^{\vec{r}_f} \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

$$= m \int_{t_i}^{t_f} \left(\frac{d\vec{v}}{dt} \cdot \frac{d\vec{r}}{dt} \right) dt = m \int_{t_i}^{t_f} \left(\frac{d\vec{v}}{dt} \cdot \vec{v} \right) dt$$

$$= m \int_{v_i}^{v_f} \underbrace{\vec{v} \cdot d\vec{v}}_{\substack{v_x dv_x \\ + v_y dv_y \\ + v_z dv_z}} = m \int_{v_i^2}^{v_f^2} \frac{1}{2} d(v^2) = \frac{1}{2} m \Delta(v^2)$$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

net work done = ΔK

$$\text{power} = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

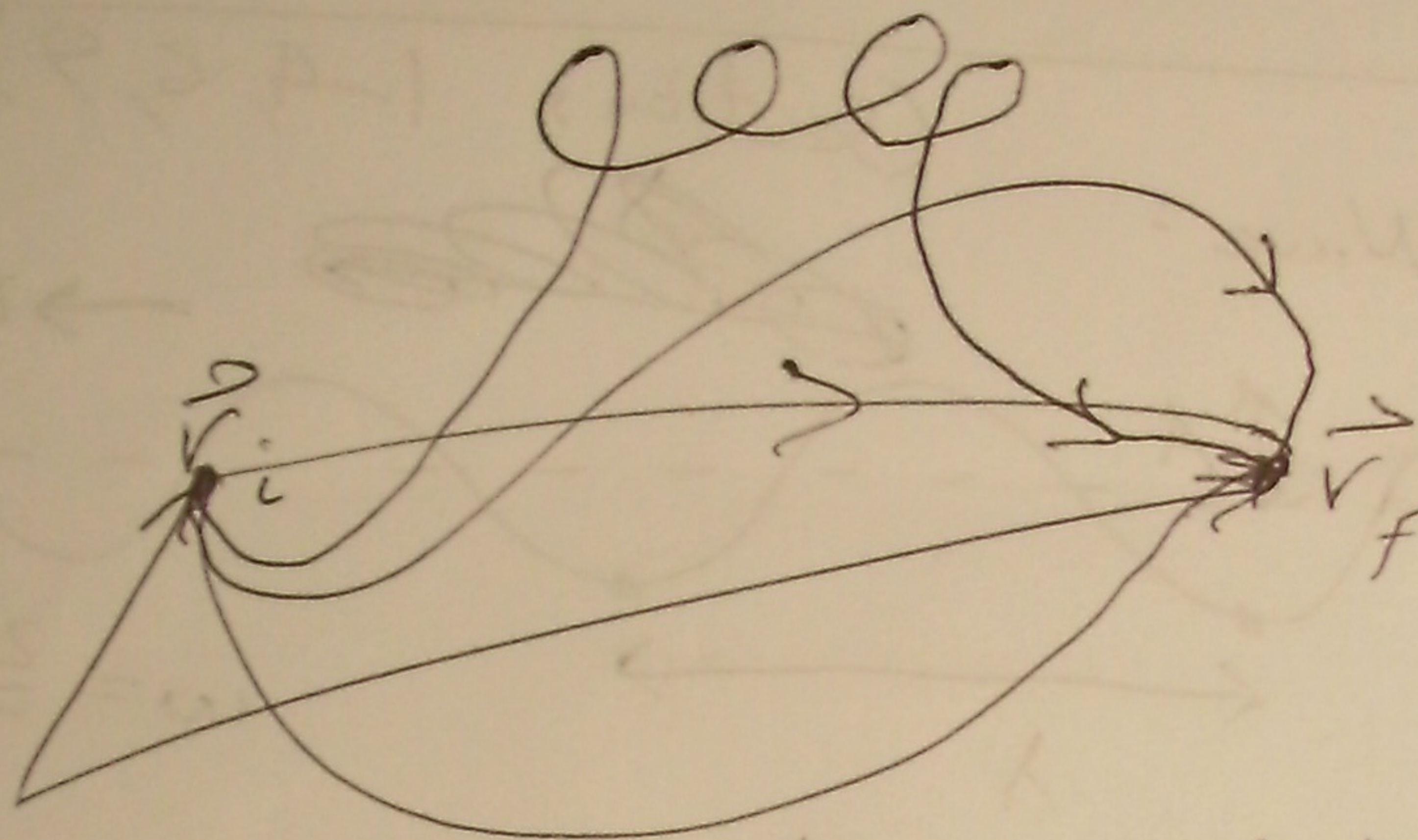
Net work + \Rightarrow speeds up

Net work - \Rightarrow slow down

Ch. 8: Potential energy

8

If W does not depend on the path chosen, but on the endpoints,

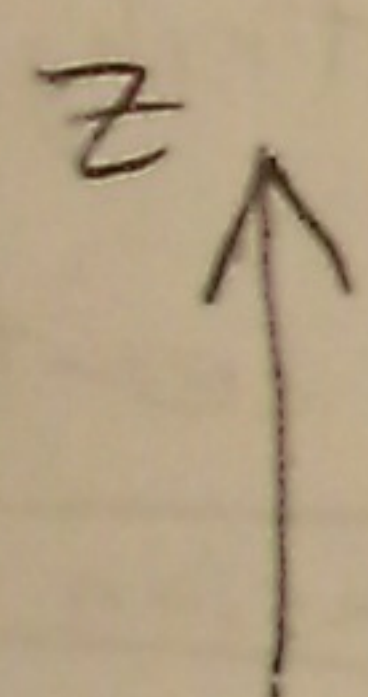


then define $\Delta U = -W = -\Delta K$

ΔU only depends on position.

$$\Delta U_g = - \int_{\vec{r}_i}^{\vec{r}_f} \underbrace{\vec{F}_g}_{\substack{\downarrow \\ F_g = mg}} \cdot d\vec{r} = - \int_{z_i}^{z_f} (-mg) dz$$

$\rightarrow 0 dx + 0 dy + (-mg) dz$

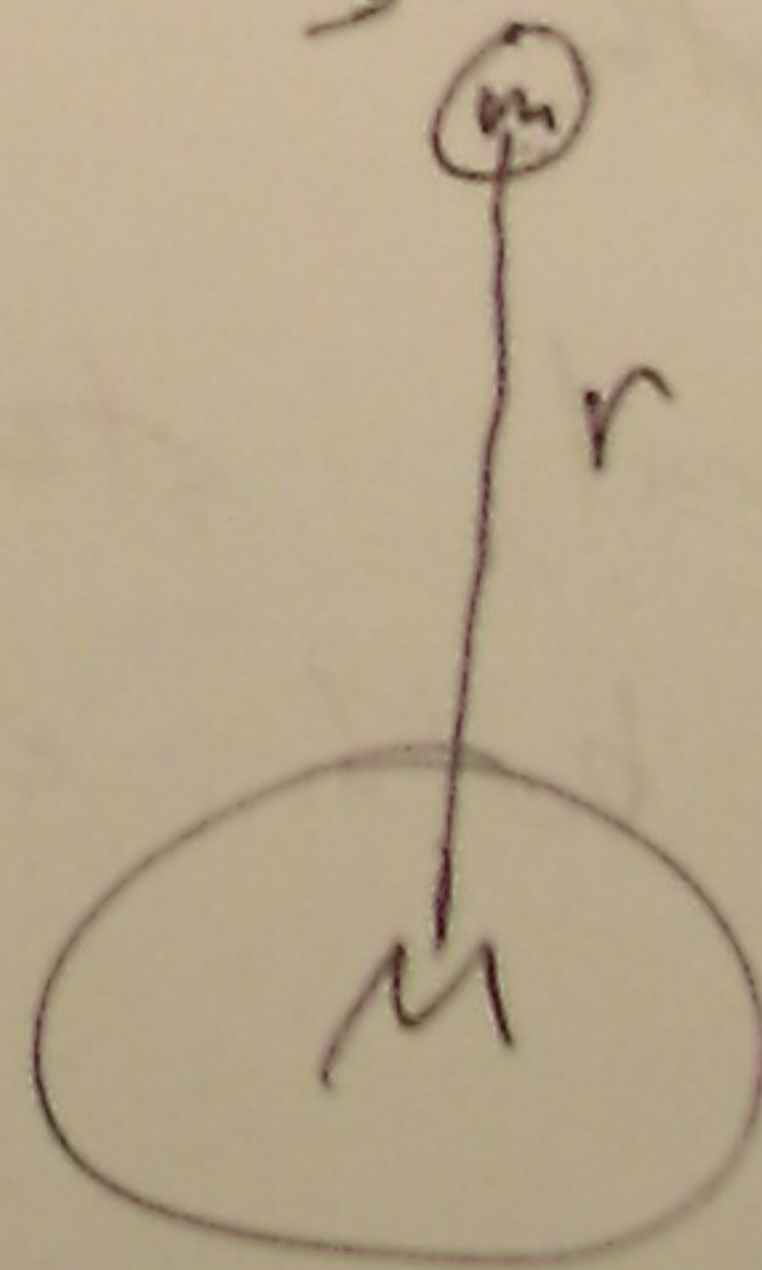


$$\Delta U_g = mg \Delta z$$

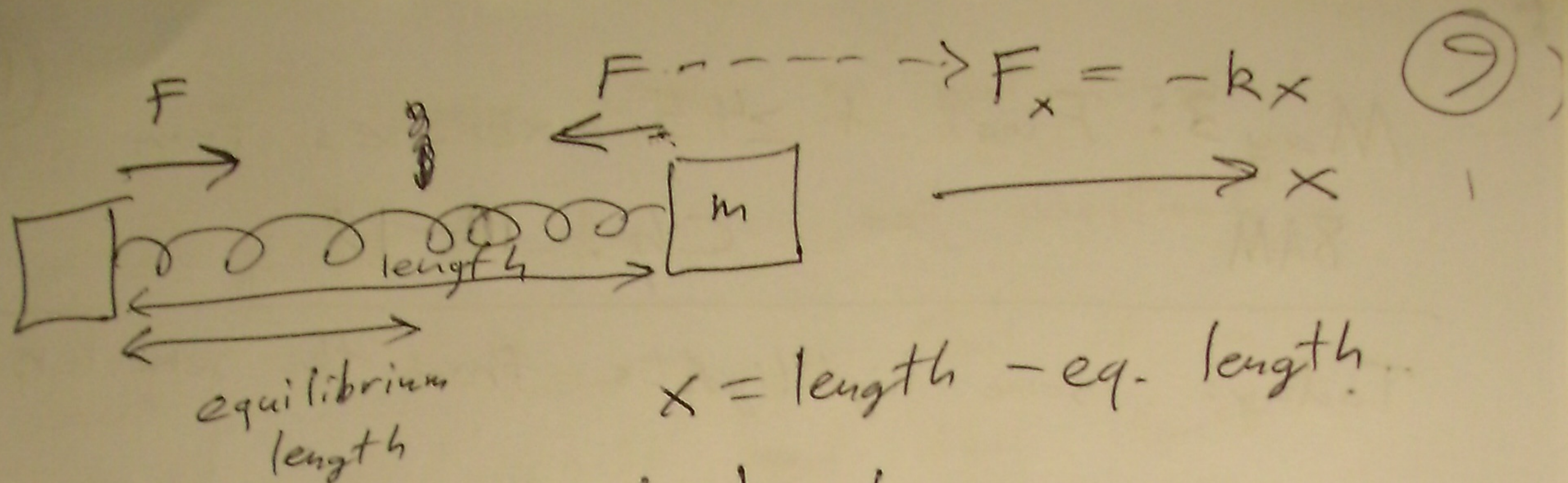
Near Earth's surface

$$\Delta U_G = - \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_G \cdot d\vec{r} = -GMm \Delta \left(\frac{1}{r} \right)$$

universal gravitation



$$g = \frac{GM_E}{R_E^2}$$



$$x = \text{length} - \text{eq. length}$$

$$|F| = kx$$

↑
k = spring constant

$$\Delta U_{sp} = -W = -\int_{x_i}^{x_f} \underbrace{-kx}_{\vec{F} \cdot d\vec{x}} dx = \frac{1}{2} k \Delta(x^2)$$

||
 $\frac{1}{2} k (x_f^2 - x_i^2)$

total energy $E = K + U$ is

constant b/c $-\Delta K = -W = \Delta U$,

except that ~~if~~ if there's friction or air resistance,

or spring damping, or sound, or heat, etc, then these take

energy away from $K + U$ b/c

these resistive forces depend on path.
or dissipative

Ch. 9 (Linear) momentum $\vec{p} = m\vec{v}$

Newton's 2nd Law: $\sum \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

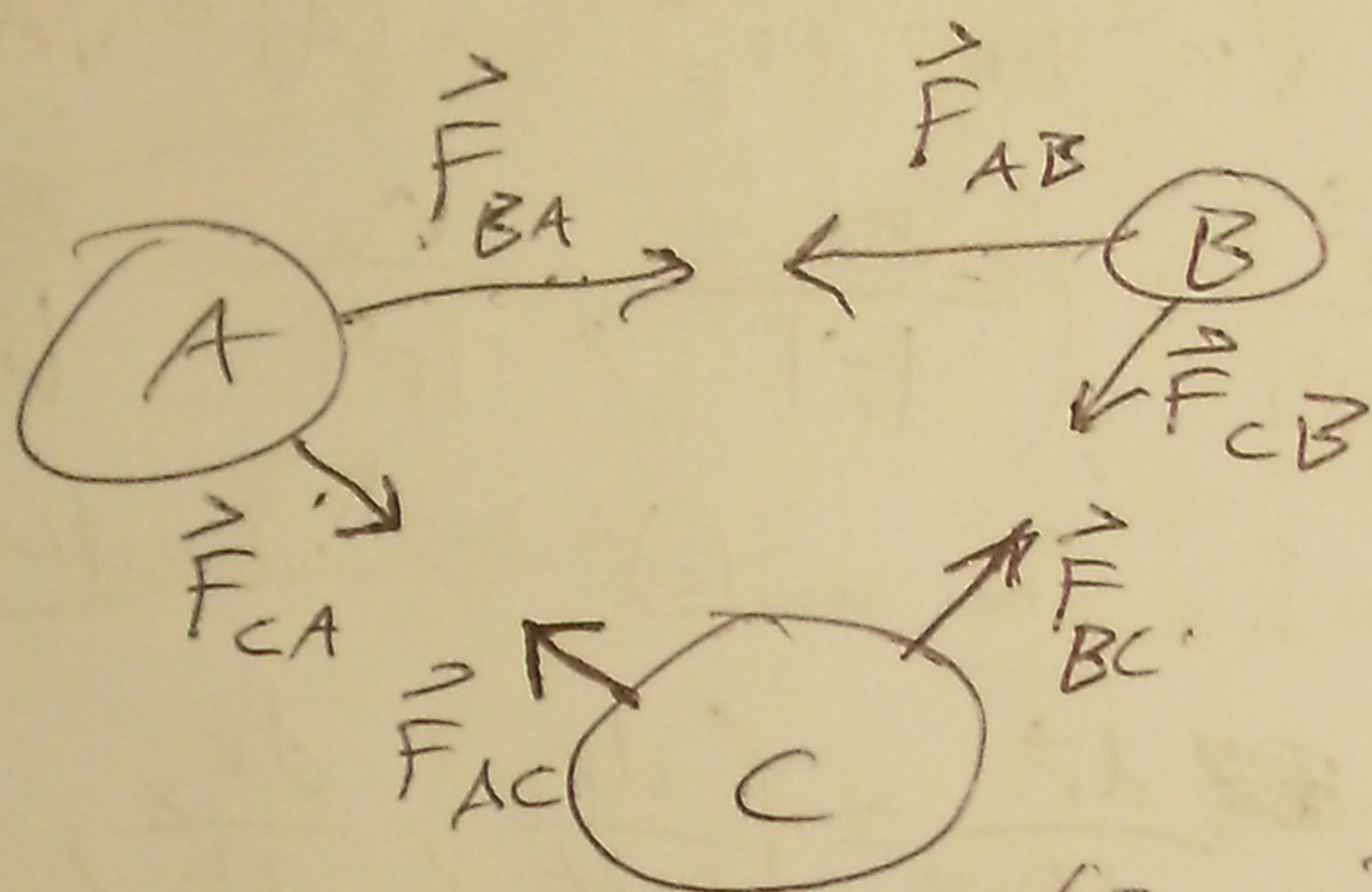
b/c $m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$ if m constant

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \text{ even when } m \text{ not constant.}$$

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$$\sum \vec{F} \neq m\vec{a} \text{ when } m \text{ not constant}$$

Newton's 3rd Law says that if a system of objects and they don't interact with any objects outside the system, or, if all forces on the system from outside the system add up to $\vec{0}$, then the total momentum of the system is constant.



$$\begin{aligned} \frac{d}{dt} (\vec{p}_A + \vec{p}_B + \vec{p}_C) &= (\vec{F}_{BA} + \vec{F}_{CA}) \\ &+ (\vec{F}_{AB} + \vec{F}_{CB}) \\ &+ (\vec{F}_{AC} + \vec{F}_{BC}) \\ &= \vec{0} \end{aligned}$$

$$\text{center of mass} = \frac{\vec{r}_A m_A + \vec{r}_B m_B + \vec{r}_C m_C}{m_A + m_B + m_C}$$

\downarrow
 \vec{r}_{cm} (c.m.)

$$\vec{p}_A + \vec{p}_B + \vec{p}_C = (m_A + m_B + m_C) \vec{v}_{cm}$$

(11)

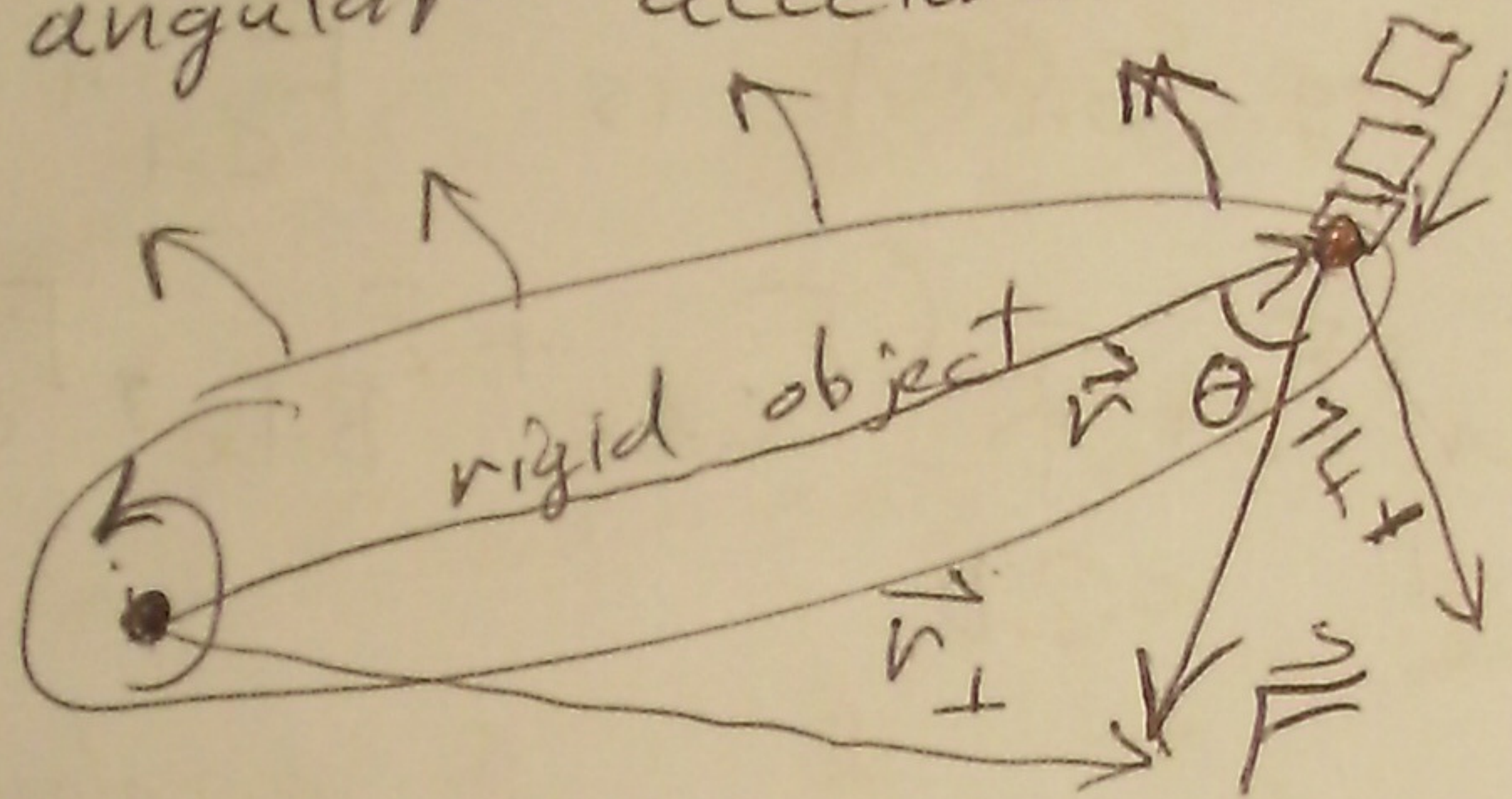
$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} \quad \vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt}$$

Net external force = (total mass) × acceleration of c.m.
↑ if constant

10 rotation about a fixed axis

torque τ
 moment of inertia $I = \sum_i r_i^2 m_i = \int_{\text{object}} r^2 dm$
 $r_i = \text{dist. relative to axis}$

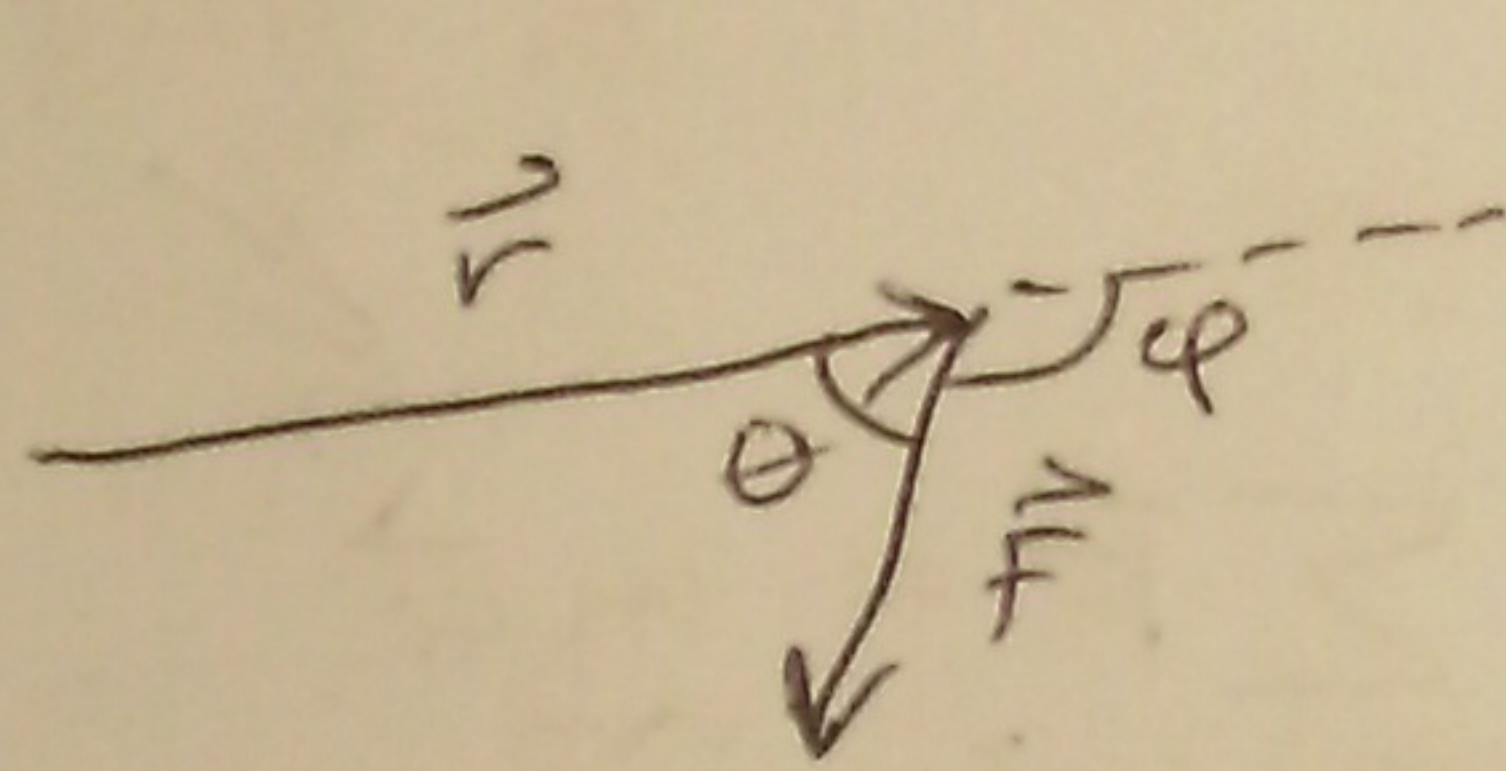
angular acceleration $\alpha = \frac{d\omega}{dt}$



$$|\tau| = |\vec{F}| |\vec{r}| \sin \theta$$

$$|\tau| = |\vec{F}| |\vec{r}|$$

$$|\tau| = |\vec{F}| |\vec{r}_{\perp}|$$



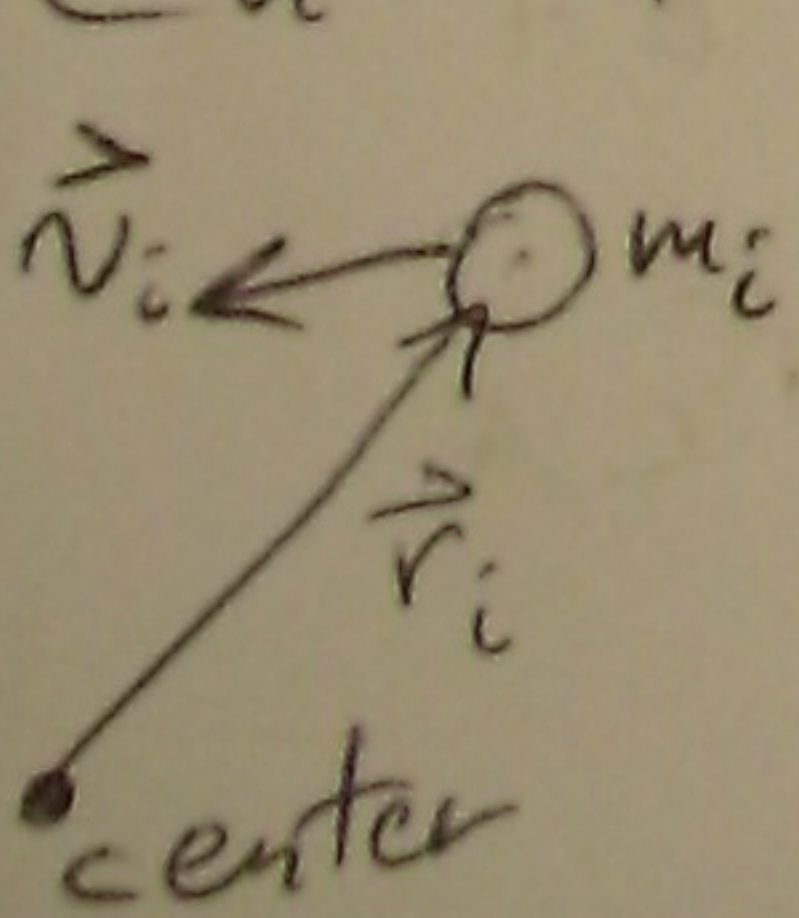
$$\sin \theta = \sin(180^\circ - \theta)$$

$$\sin \theta = \sin \phi$$

Table of I 's for simple shapes
 Ch. 10.

$$\tau = I \alpha \quad (\text{like } \vec{F} = m \vec{a})$$

Ch. 11: 3D version; axes need not be fixed



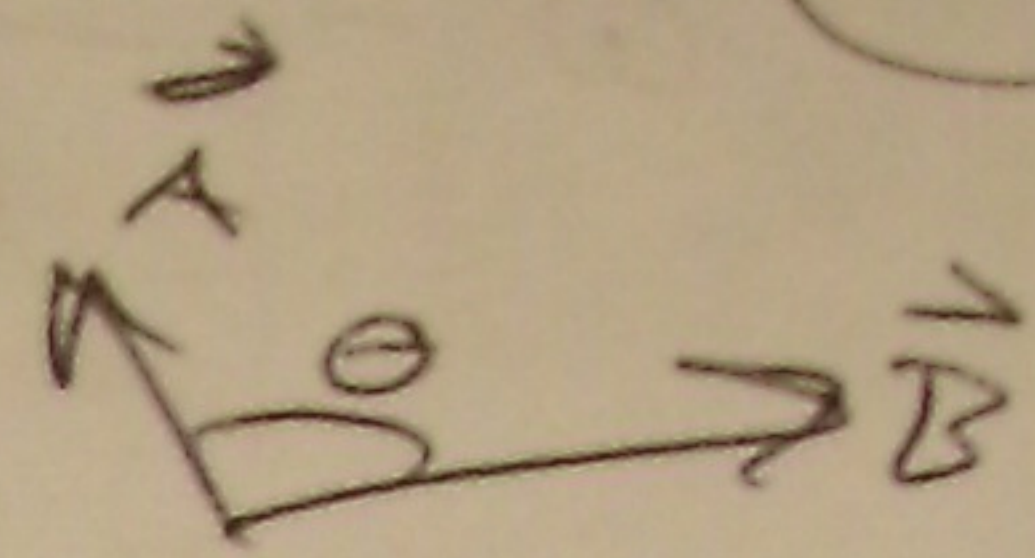
$$\vec{L} = \text{angular momentum (measured relative to a center point)}$$

$$\vec{L} = \sum_i \vec{r}_i \times m_i \vec{v}_i = \int_{\text{object}} (d\vec{r} \times \vec{p})$$

$$(\vec{A} \times \vec{B}) \perp \vec{A} \quad (\vec{A} \times \vec{B}) \perp \vec{B}$$

(12)

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$



$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Use right-hand rule to visually/graphically find direction

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{i} \times \hat{i} = \vec{0}$$

$$\hat{j} \times \hat{j} = \vec{0}$$

$$\hat{k} \times \hat{k} = \vec{0}$$

like $\Sigma \vec{F} = \frac{d\vec{p}}{dt}$

In general,

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

For a fixed axis of symmetry,

$$\vec{\tau} = I \vec{\alpha}$$

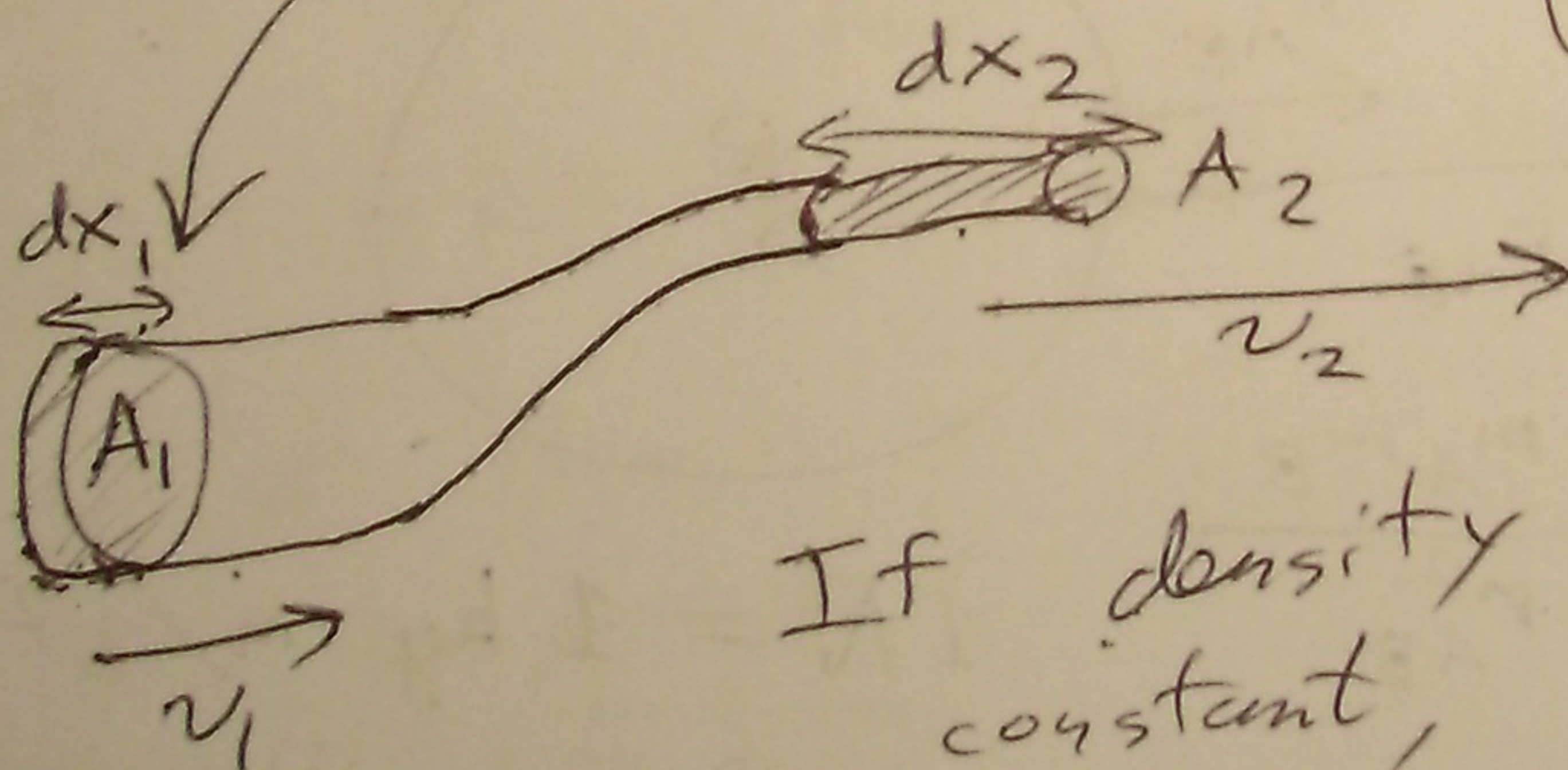
(much simpler).

$$\vec{L} = I \vec{\omega}$$

Ch. 13
(Fluids)

2 big ideas:

conservation of mass
conservation of energy



$$V_2 = A_2 dx_2$$

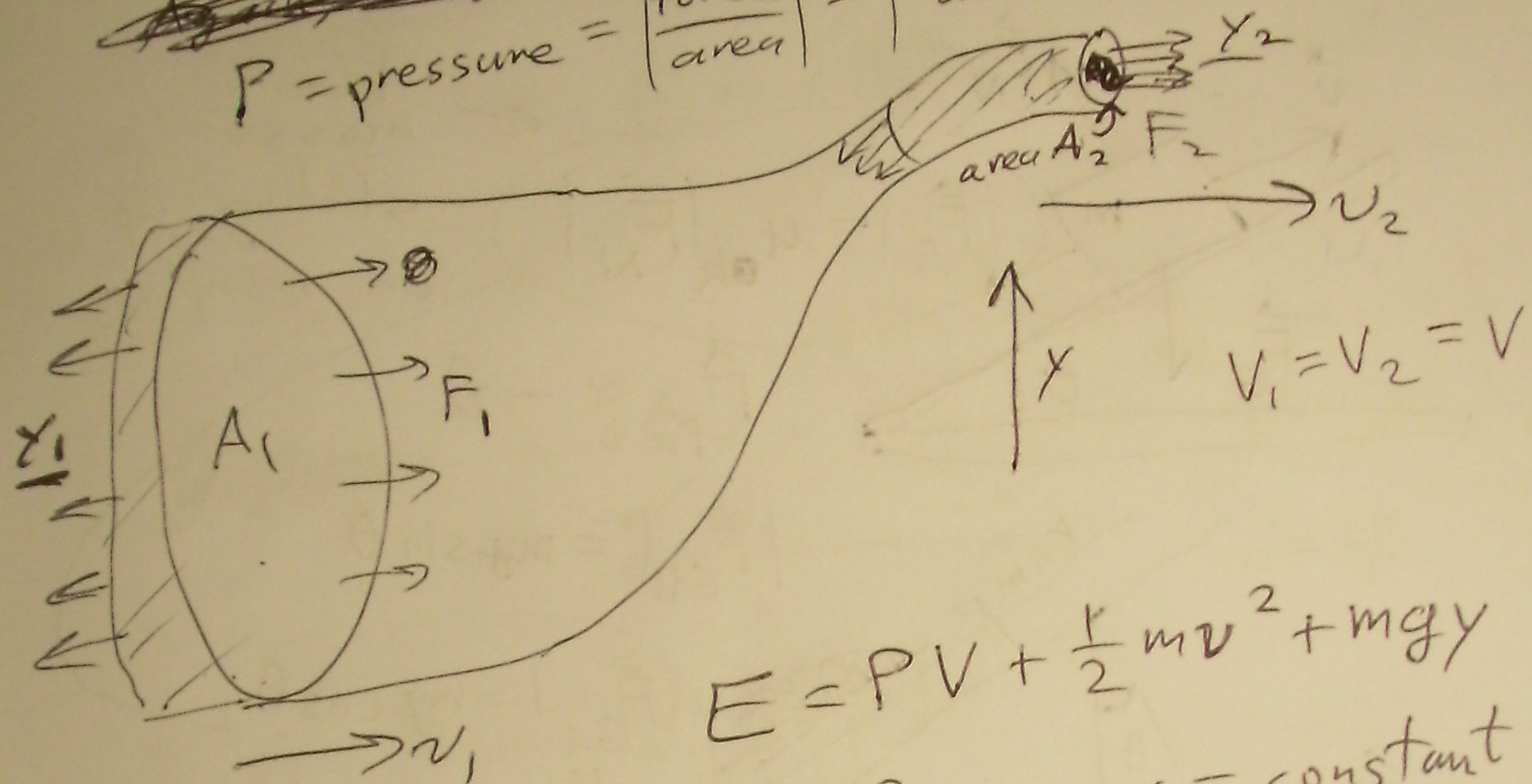
$$v_1 = \frac{dx_1}{dt_1}, \quad v_2 = \frac{dx_2}{dt_2}$$

If density $\rho = \frac{\text{mass}}{\text{volume}}$ is constant, then volume is conserved too, implying

$$V_1 = A_1 dx_1, \quad \left\{ \begin{array}{l} v_1 = v_2 \\ dt_1 = dt_2 \end{array} \right\} \text{ so } A_1 v_1 = A_2 v_2$$

Conservation of energy:

~~Pressure = force / area~~
 $P = \text{pressure} = \frac{\text{force}}{\text{area}} = \frac{\text{force} \cdot \text{distance}}{\text{area} \cdot \text{distance}} = \frac{\text{work}}{\text{volume}}$



$$E = PV + \frac{1}{2}mv^2 + mgy$$

$$\frac{E}{V} = P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

$$P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 g y_1 = P_2 + \frac{1}{2}\rho_2 v_2^2 + \rho_2 g y_2$$

Ch. 14 Simple Harmonic oscillation:

1D; $F = -kx$ (e.g. spring)
 k constant

$$m \frac{d^2x}{dt^2} = -kx \Rightarrow \text{solutions to this equation}$$

are all of form $x = A \cos(\omega t + \theta_0)$
 constants

where $\omega = \sqrt{\frac{k}{m}}$ = ang. freq.
 A = amplitude; θ_0 = phase shift;

$$E = \sqrt{\frac{1}{2}mv^2 + \frac{1}{2}kx^2} = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \theta_0) + \frac{1}{2}kA^2 \cos^2(\omega t + \theta_0) \quad (14)$$

$$v = \frac{dx}{dt}$$

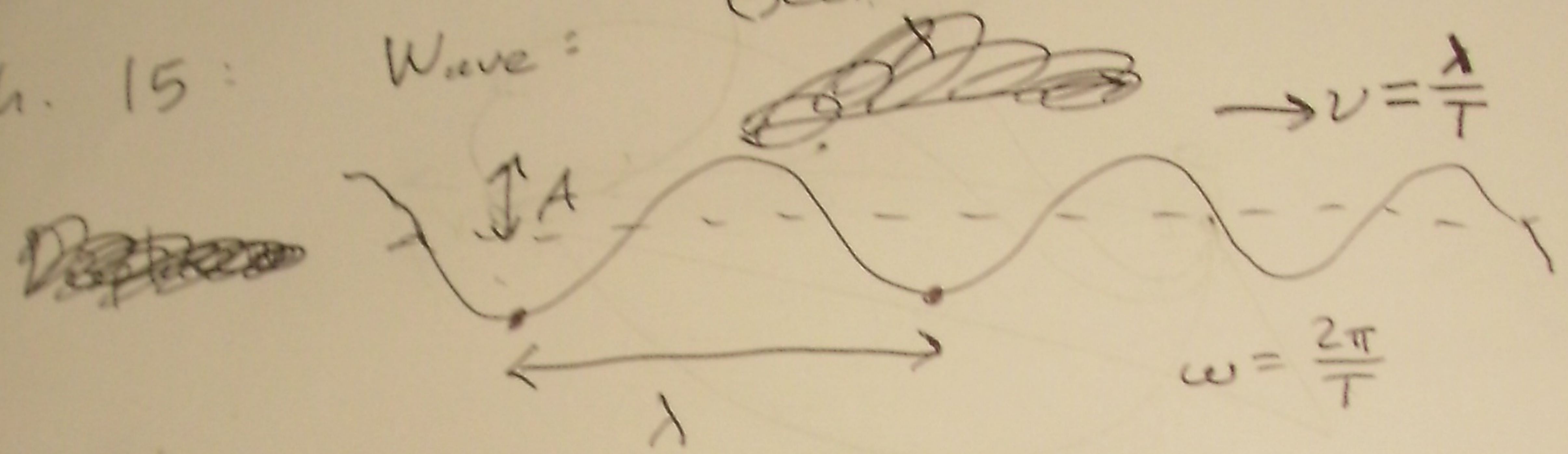
$$m\omega^2 = k$$

$$E = \frac{1}{2}kA^2 = \text{constant}$$

Ch. 15:

Wave:

(Sections 1-4, 6, 9)



$$D(x,t) = A \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$$

↑
sinusoidal wave travelling to right at speed v

Ch. 16 Sound:

(Sections 4, 7)

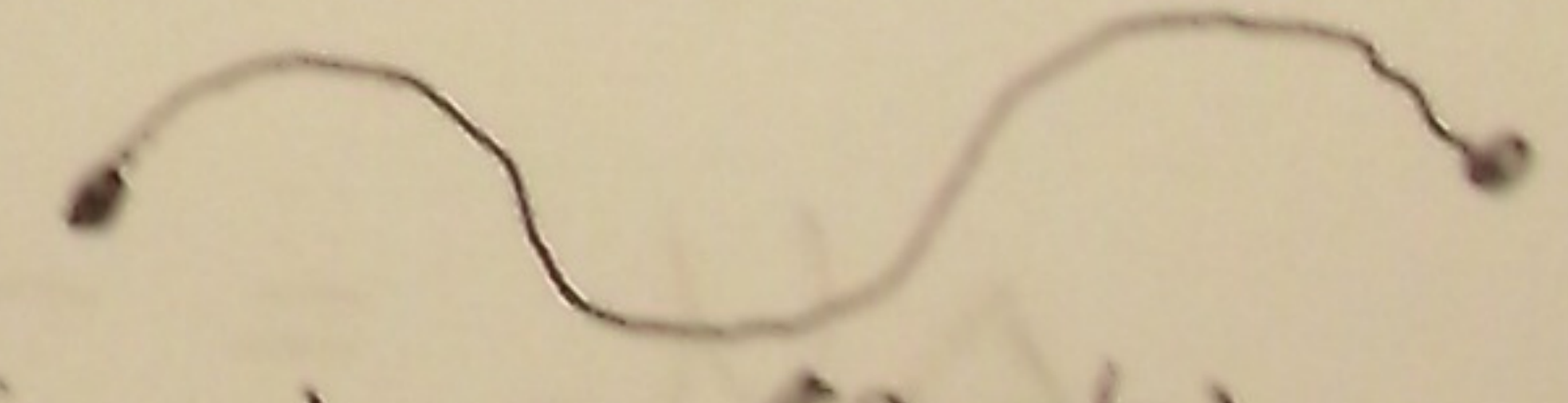
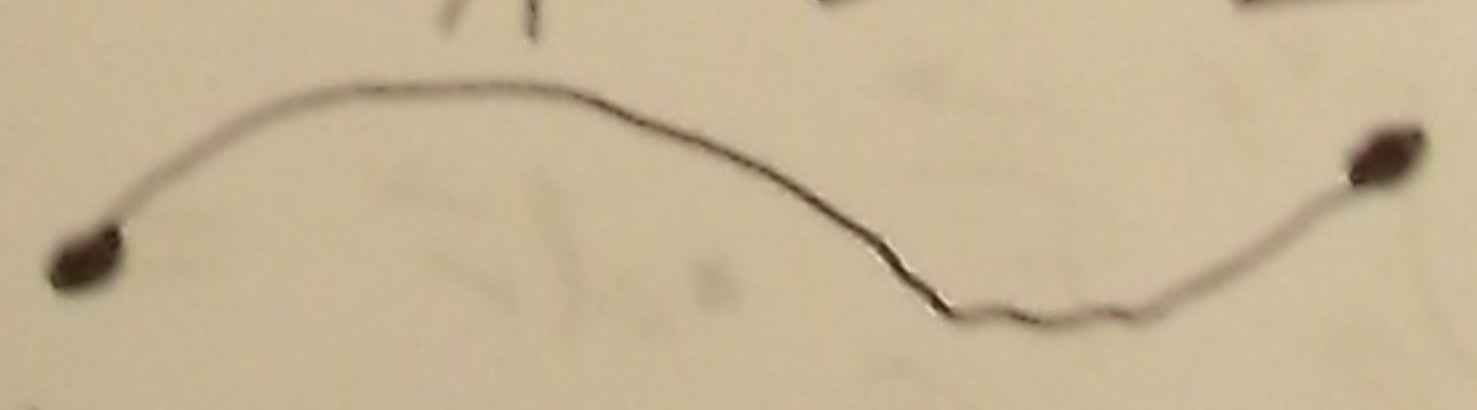
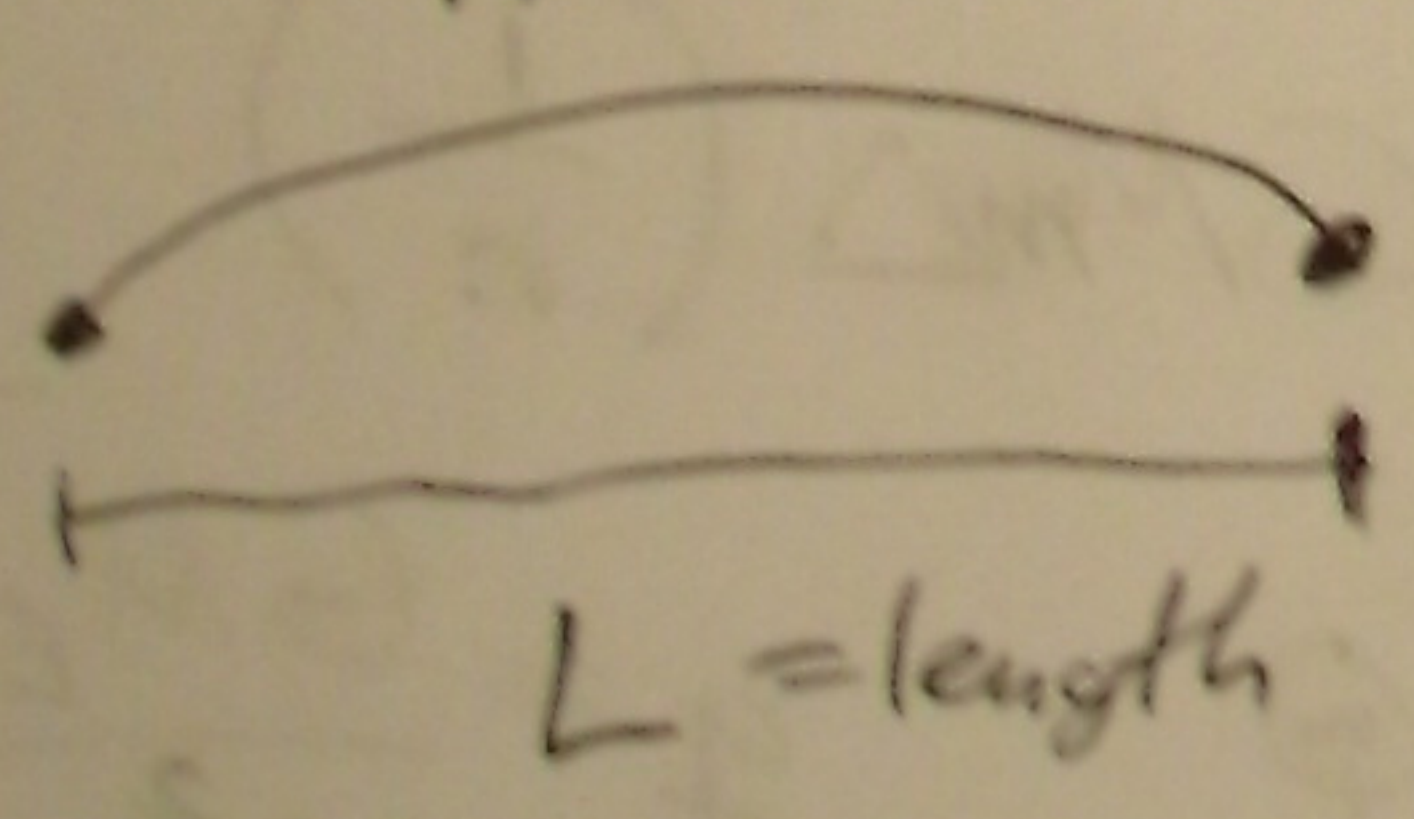
~~Standing~~ Standing waves of vibrating strings

& columns of air happen only at certain wavelengths. $v = \sqrt{\frac{\text{tension} \cdot \text{length}}{\text{mass}}}$

$$\lambda = 2L = \frac{2L}{1}$$

$$\lambda = L = \frac{2L}{2}$$

$$\lambda = \frac{2L}{3}$$



$$D(x,t) = A \cos\left(\frac{2\pi x}{\lambda}\right) \cos\left(\frac{2\pi t}{T}\right)$$

In general, standing waves @ $\frac{2L}{n} = \lambda$ when both ends are fixed.

Just section 1

Ch. 17, 18, 19 - Ideal gasses &

(15)

thermodynamics.

An ideal gas is a "box of bullets" bouncing around. For a monatomic

ideal gas, temperature $T = \frac{2}{3k} \overline{K}$ (or $\frac{3}{2} \overline{K}$) where \overline{K} is the average kinetic energy of a gas particle: $\overline{K} = \frac{1}{N} (K_1 + K_2 + \dots + K_N)$.

T is measured in Kelvins ($^{\circ}K$) (too many K's!)

Room temperature is $\sim 300^{\circ}K$

$$0^{\circ}C = 273.15^{\circ}K = 32^{\circ}F$$

$$x^{\circ}C = (273.15 + x)^{\circ}K$$

Ideal gas law: $PV = NkT = nRT$
 ↑ pressure ↑ volume ↑ # particles ↑ Boltzmann constant ↑ temperature

$$n = \frac{N}{N_A}; \quad R = kN_A \quad N_A = 6.022 \times 10^{23}$$

 ↑ # moles ↑ gas constant $R = 8.314 \frac{J}{mol \cdot ^{\circ}K}$

$$\text{Work done by gas} = \int_{V_i}^{V_f} P dV$$

$$V_i = \frac{F}{A} \quad \frac{F}{A} dx$$