May 3: Final + ≥45 exercises from Ch. 10–19 ①

8AM

Today: some highlights from the semester

Ch. 1 units & significance figures

$$12\,m^3 \neq (12\,m)^3 = 12^3 m^3 = 1728\,m^3$$

$\underbrace{2.5}_{\text{2 sig fig}} \times \underbrace{13.47}_{\text{4 sig figs}} = \underbrace{34}_{\text{2 sig figs}}$ divide & multiply & squares & square roots

add & subtract

$13.47 + 2.5 = 16.0$

$\sim \pm 0.01$ $\quad \underbrace{\sim \pm 0.1}_{\text{bigger uncertainty}}$ $\quad \sim \pm 0.1$

Ch. 2 1D kinematics (describing motion)

−1m 0 +1m x

x = position

t = time

$v = \frac{dx}{dt}$ = velocity

$v \approx \frac{\Delta x}{\Delta t}$ for small Δt

$a = \frac{dv}{dt}$ = acceleration

$\Delta x = x_{final} - x_{initial}$

$\Delta t = t_{final} - t_{initial}$

$\Delta v = v_{final} - v_{initial}$

$\frac{\Delta x}{\Delta t}$ = average velocity = $\bar{v}$

$\frac{\Delta v}{\Delta t}$ = avg. accel. = $\bar{a}$

$$\Delta v = \int_{v_{initial}}^{v_{final}} dv = \int_{t_i}^{t_f} a\, dt = a\Delta t \quad \text{if } a \text{ constant}$$

$$a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

$$\Delta x = \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v\, dt \begin{cases} = v\Delta t & \text{if } v \text{ const} \\ = \frac{1}{2}a(\Delta t)^2 + v_{init}\Delta t & \text{if } a \text{ const} \end{cases}$$

Ch. 3: 2D & 3D kinematics

position $\vec{r} = (r_x, r_y, r_z) = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}$

$\hat{\imath} = (1,0,0) \quad \hat{\jmath} = (0,1,0) \quad \hat{k} = (0,0,1)$

$|\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2} \qquad |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$\hat{\mathbf{r}} = \frac{\vec{r}}{|\vec{r}|} = \left(\frac{r_x}{|\vec{r}|}, \frac{r_y}{|\vec{r}|}, \frac{r_z}{|\vec{r}|}\right)$$

direction of $\vec{r}$

$|\hat{r}| = 1$ (no units)

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dr_x}{dt}, \frac{dr_y}{dt}, \frac{dr_z}{dt}\right)$$

$\vec{a} = \frac{d\vec{v}}{dt}$; speed $= |\vec{v}|$ velocity $= \vec{v}$

$\boxed{\text{2D: } |\vec{A}| = \sqrt{A_x^2 + A_y^2}}$

$$\Delta\vec{v} = \int d\vec{v} = \int \vec{a}\, dt = \left(\int a_x dt, \int a_y dt, \int a_z dt\right)$$

$$\Delta\vec{r} = \int d\vec{r} = \int \vec{v}\, dt$$

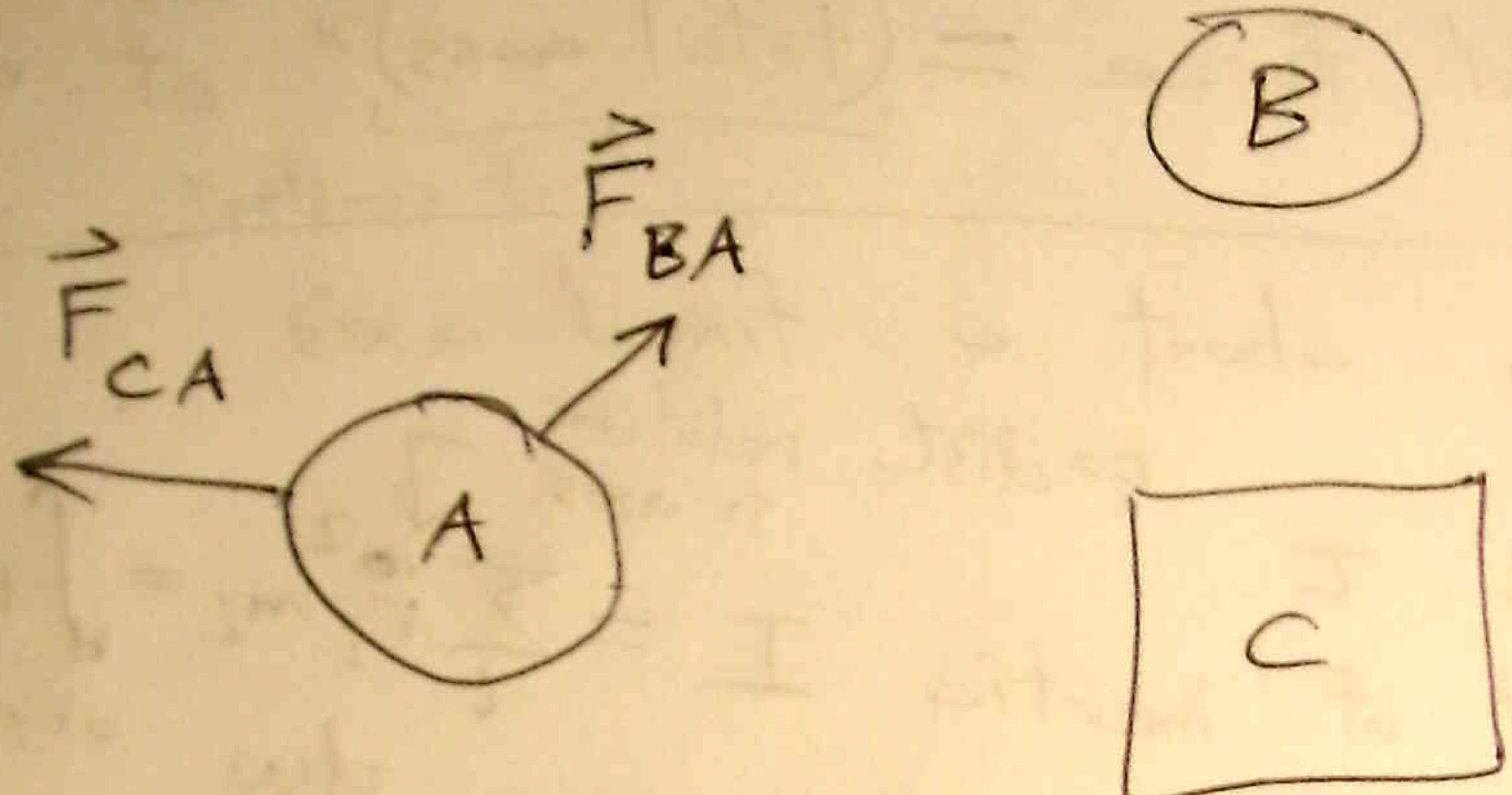
$\hat{a}$ is "down"

Balistic motion $|\vec{a}| = g = 9.80\ \text{m/s}^2$

$\vec{a}$ constant in direction too.

Ch. 4 Forces

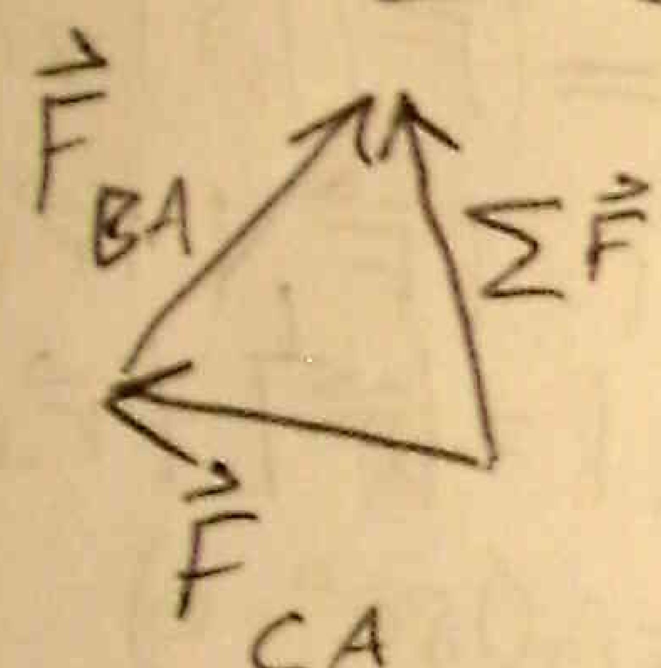
Newton's 1st Law: $\sum \vec{F} = \vec{0} \Rightarrow \vec{v}$ constant



$\sum$ for "sum"

net force acting on A is $\vec{F}_{CA} + \vec{F}_{BA} = \sum \vec{F}$

$$\sum \vec{F} = \vec{F}_{CA} + F_{BA} \underset{\text{2D}}{=} (F_{CAx} + F_{BAx}, F_{CAy} + F_{BAy})$$

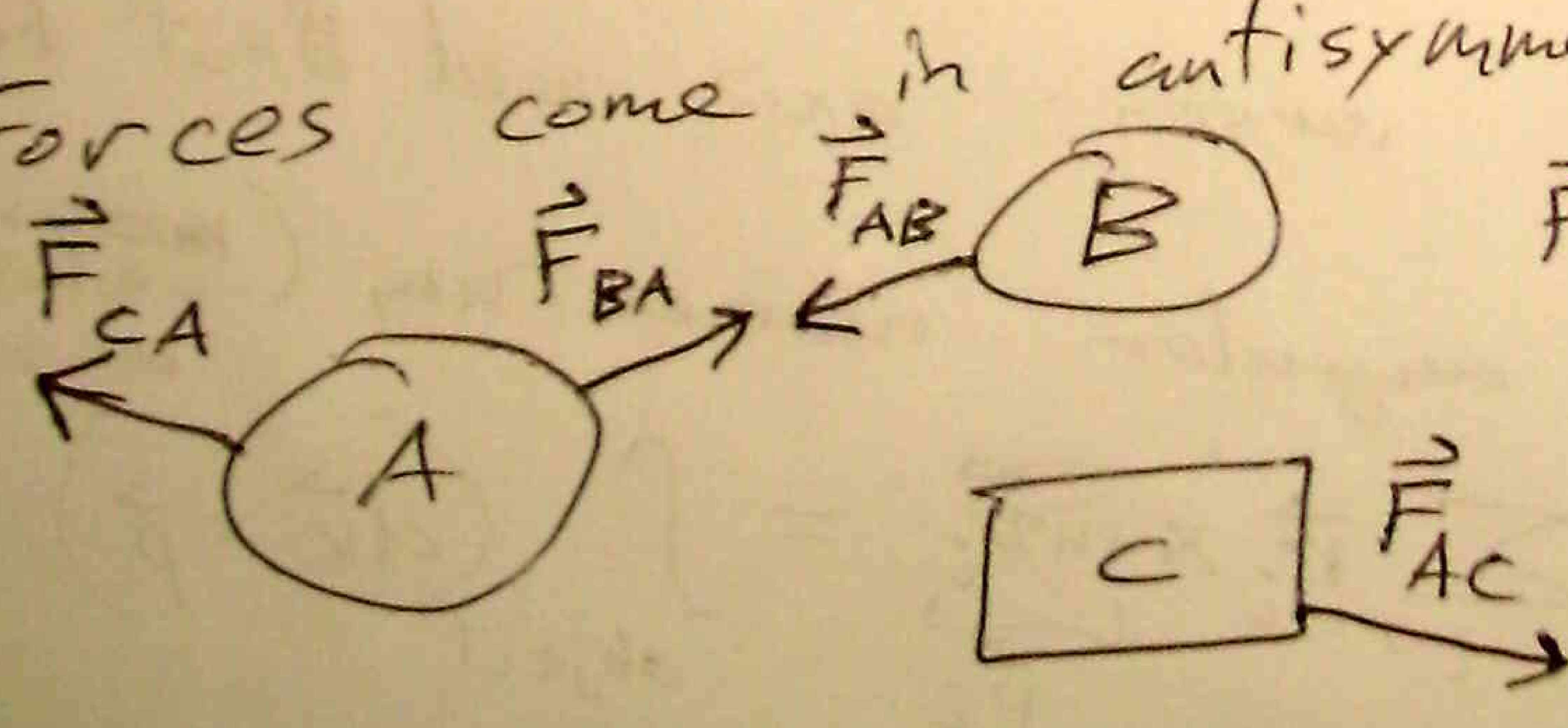


Newton's 2nd Law: $\underbrace{\sum \vec{F}}_{\text{sum of forces acting on A}} = \underset{\text{mass of A}}{m} \overset{\text{acceleration of A}}{\vec{a}}$.

Equiv.: $\frac{1}{m} \sum \vec{F} = \frac{d\vec{v}}{dt}$

Newton's 3rd Law:

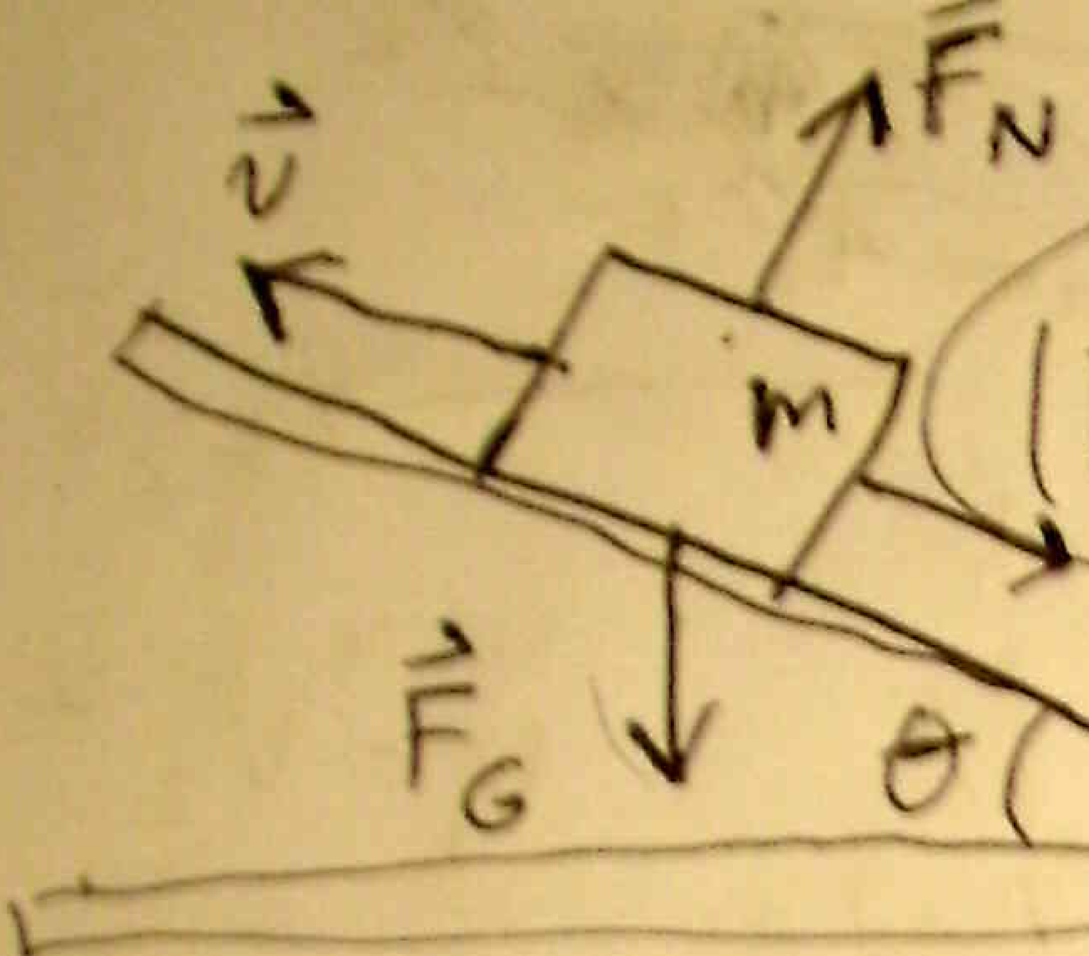
Forces come in antisymmetric pairs.



$\vec{F}_{AB} = -\vec{F}_{BA}$

$\vec{F}_{AC} = -\vec{F}_{CA}$

Ch. 5 friction & circular motion

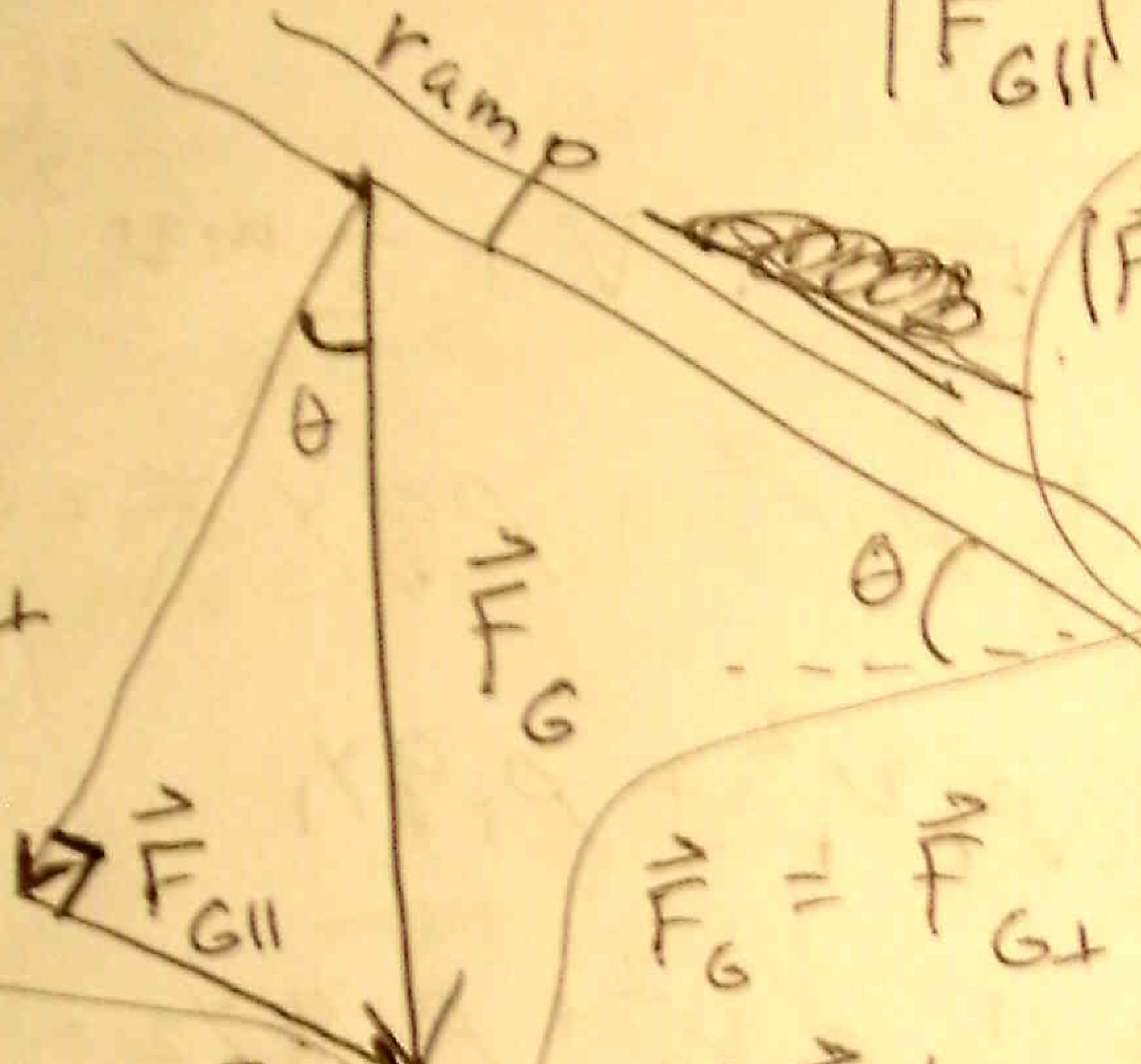


$|\vec{F}_{fr}| = \mu_k |\vec{F}_N|$

block mass sliding up on ramp

$\hat{F}_{fr} = -\hat{v}$

$|\vec{F}_{G\parallel}| = mg \sin\theta$



$|\vec{F}_{G\perp}| = mg\cos\theta$
$\parallel$
$|\vec{F}_N|$ because $\vec{F}_N = -\vec{F}_{G\perp}$ because $\vec{a}_\perp = \vec{0}$

$\vec{F}_G = \vec{F}_{G\perp} + \vec{F}_{G\parallel}$

$|\vec{F}_G| = mg$

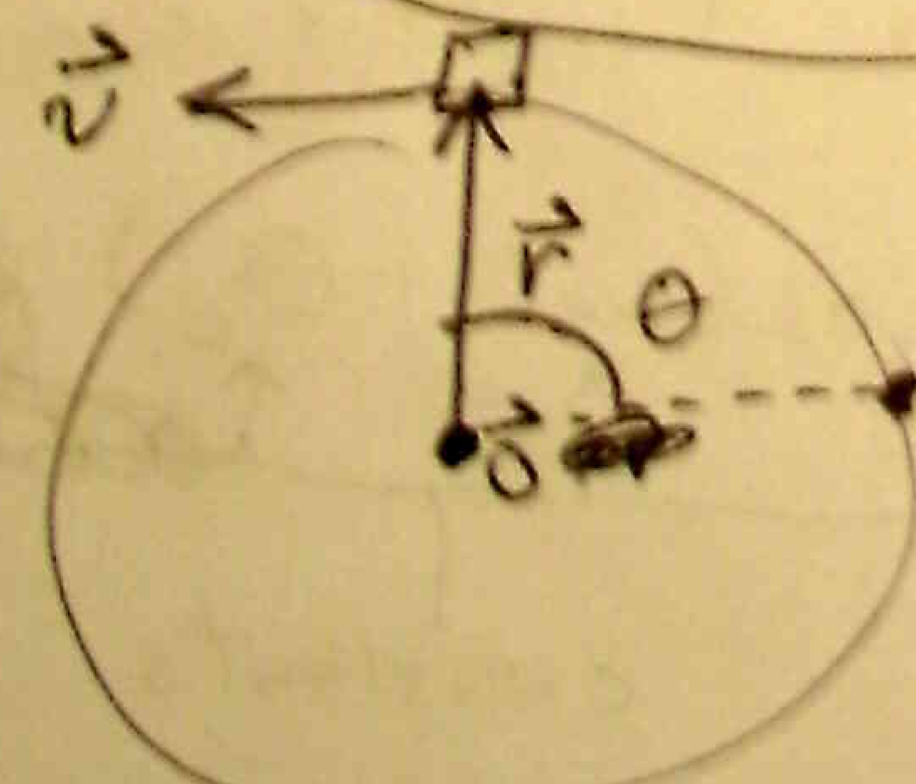
$|\vec{F}_{Fr}| = \mu_k mg\cos\theta$

$\vec{a}_\parallel = \vec{F}_{G\parallel} + \vec{F}_{fr}$

<u>uniform</u> circular motion:

$|\vec{r}| = r = \text{constant}$

$|\vec{v}| = v = \text{constant}$



$\vec{r} \perp \vec{v}$

θ = angular position & displacement

$\theta \begin{cases} + = \text{counterclockwise} \\ - = \text{clockwise} \end{cases}$

(5)

$$\omega = \frac{d\theta}{dt} = \text{angular velocity} = \text{angular frequency}$$

units like rad/s or rev/min

1 rev = 2π rad = $360° = 2\pi$

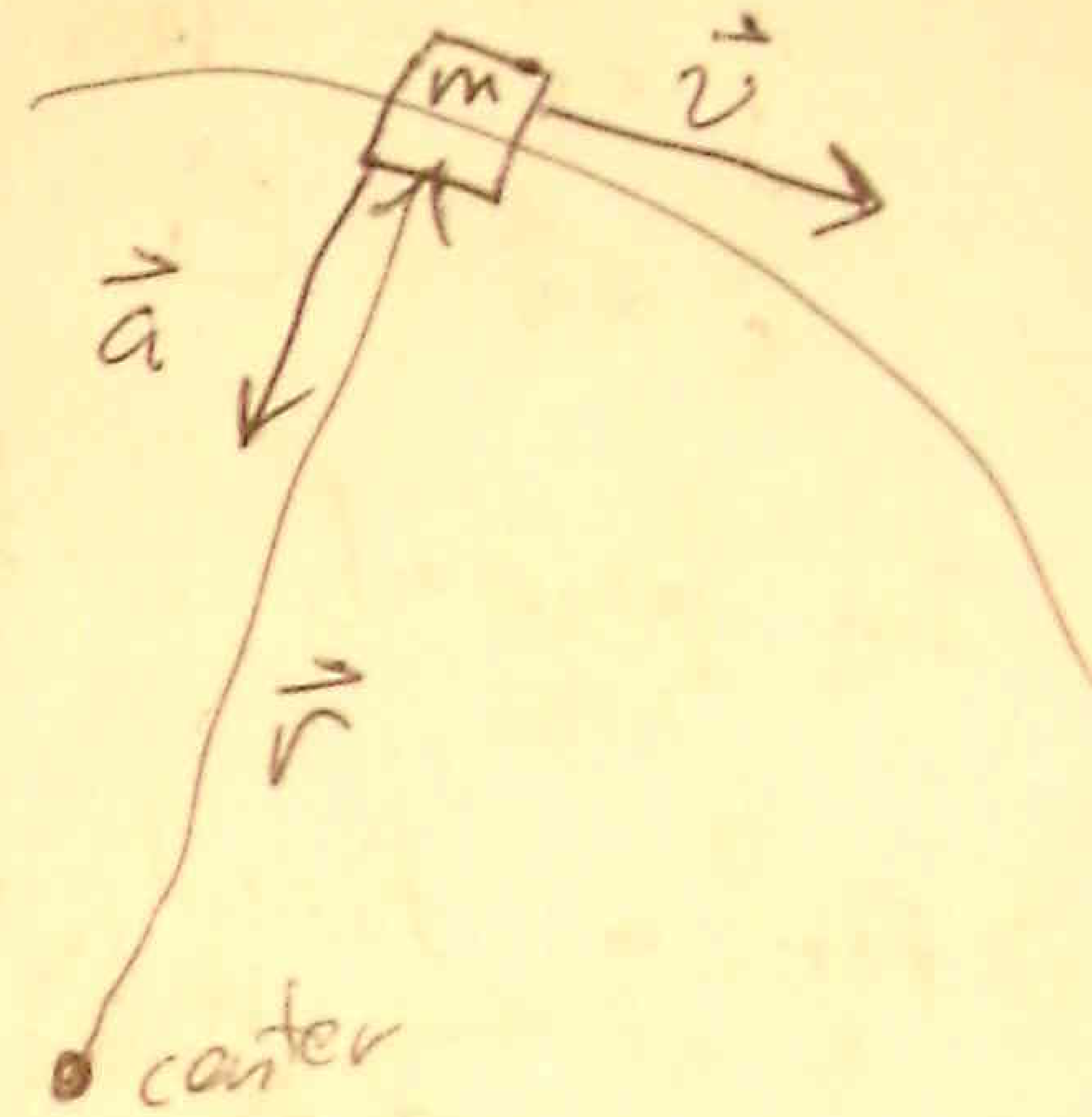
$\frac{v}{r} = \omega$ const.

$\Updownarrow$

$v = r\omega$ $\qquad$ $a = |\vec{a}| = r\omega^2 = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$

$\vec{r} = (r\cos(\omega t + \theta_0), r\sin(\omega t + \theta_0))$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$$



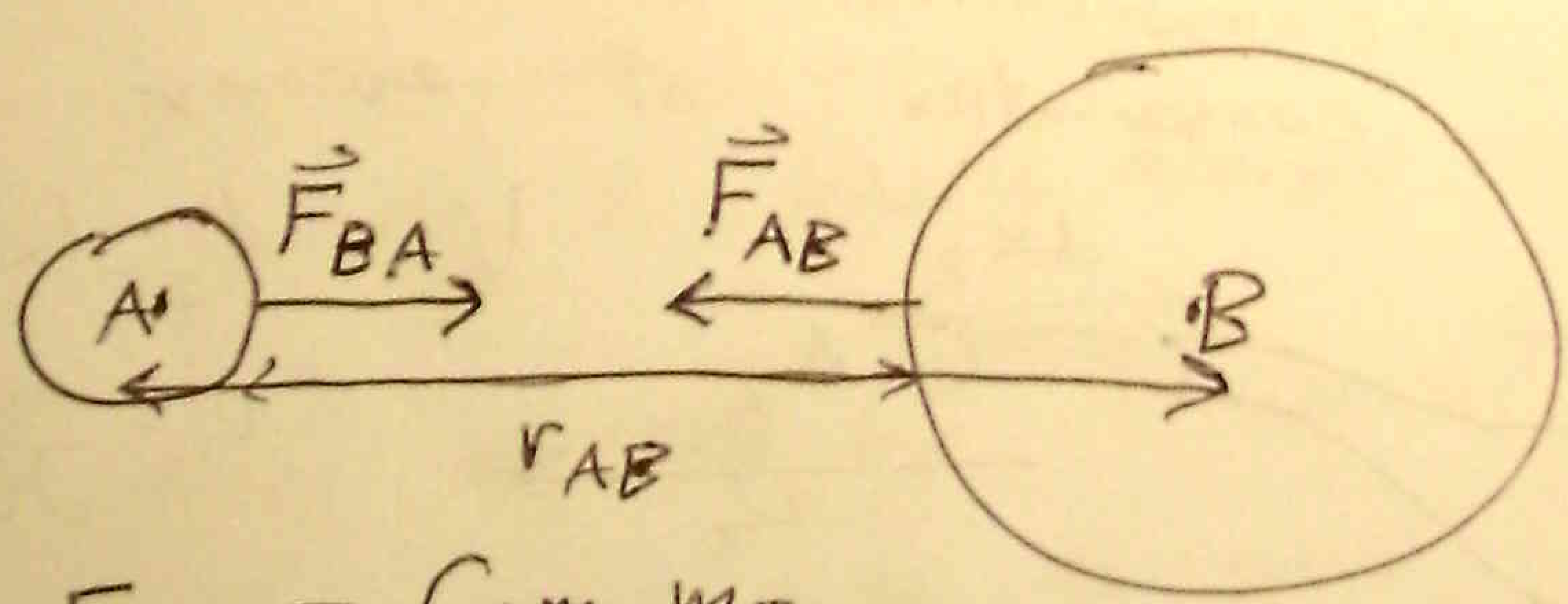
$|\sum\vec{F}| = m\omega^2 r = mv^2/r$

$$\sum\vec{F} = m\vec{a} = -m\omega^2\vec{r}$$

$$\parallel$$

$$-\frac{mv^2}{r}\hat{r} = -m\frac{v^2\vec{r}}{r^2}$$

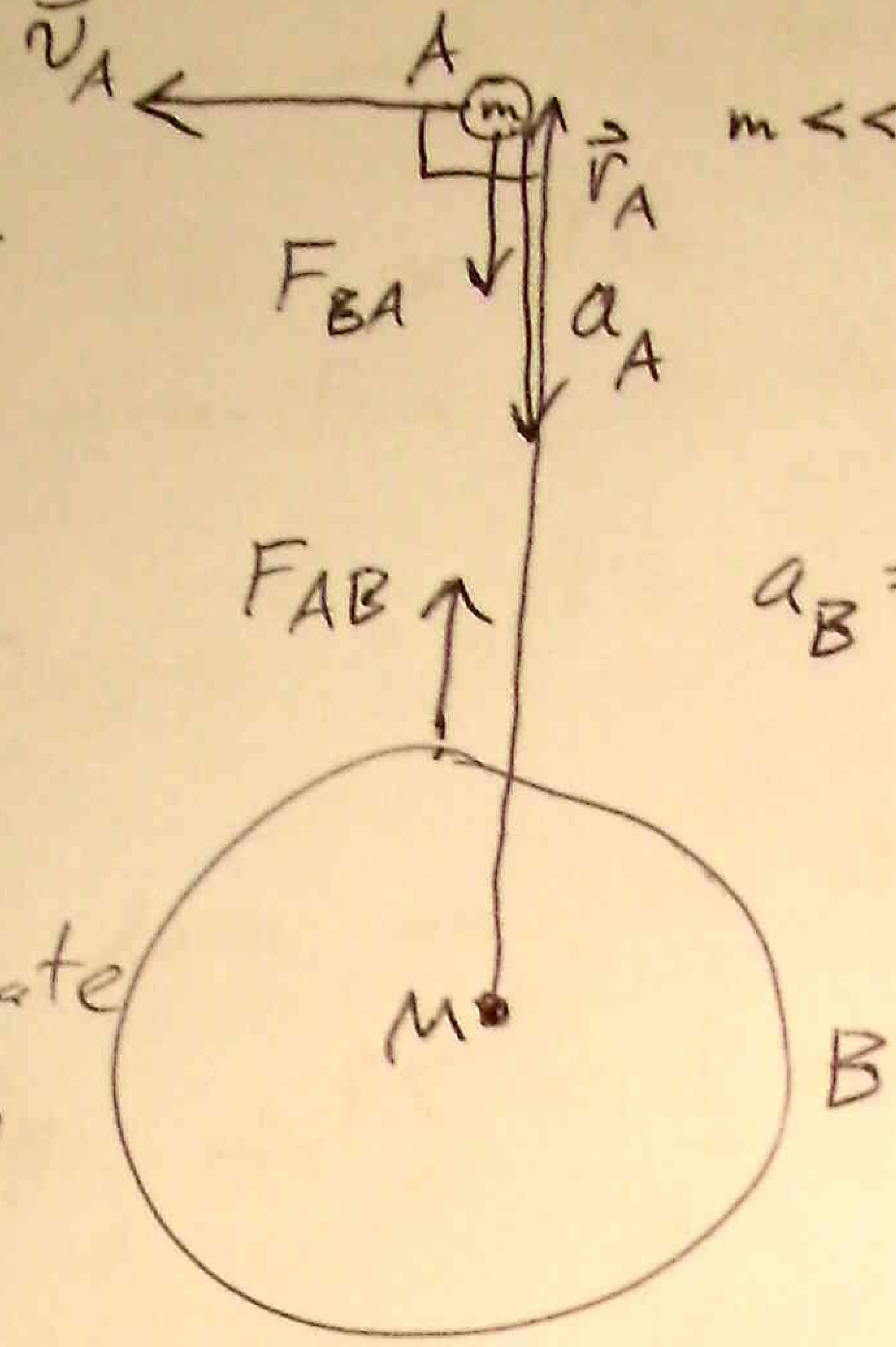
Ch. 6 Gravity $\qquad$ $G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$



$$F_{AB} = F_{BA} = \frac{G m_A m_B}{r_{AB}^2}$$

$1N = 1\ kg \cdot m/s^2$

$\sum\vec{F} = m \cdot \vec{a}$



$m << M$

Simplest case:

$|\vec{v}_A| \approx$ constant

$|\vec{r}_A| \approx$ constant

$\vec{a}_B \approx \vec{0}$

$\downarrow$

Choose coordinate system such that $\vec{r}_B$ & $\vec{v}_B$ are $\approx \vec{0}$

$$a_B = \frac{F_{AB}}{M} \ll \frac{F_{BA}}{m} = a_A$$

$$F_{BA} = m a_A = m v_A^2 / r_A$$

$$\Downarrow$$

$$\frac{GmM}{r_A^2} \Rightarrow \frac{v_A^2}{r_A} = \frac{GM}{r_A^2} \Longleftrightarrow v_A^2 r_A = GM$$

$$\frac{4\pi^2 r^3}{T^2} = \omega^2 r^2 r = v_A^2 r_A = GM$$

$$T = \text{period} = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

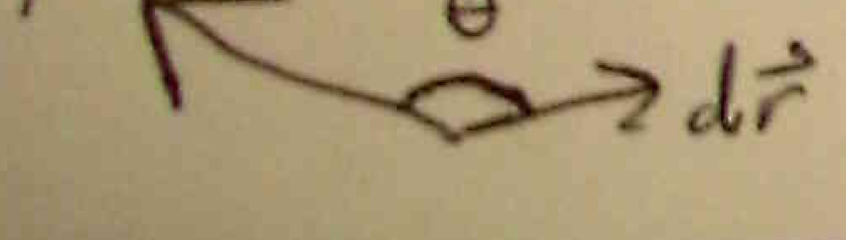
$$v = \omega r \Longleftrightarrow \omega = \frac{v}{r}$$

$$v = \frac{2\pi r}{T} \Longleftrightarrow \omega = \frac{2\pi}{T}$$

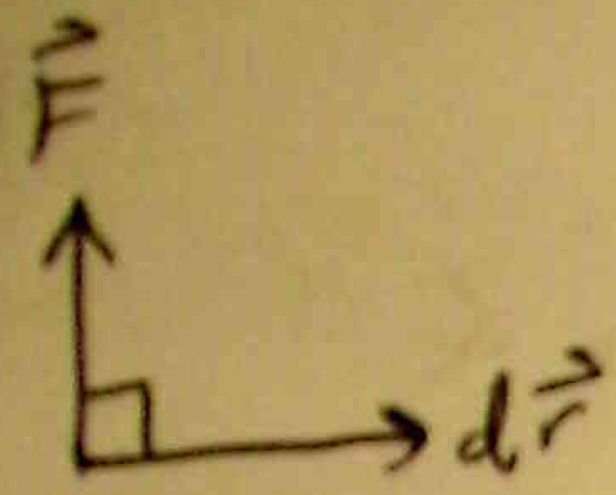
Ch. 7 Work & kinetic energy & power

Work done by force F from some initial state to some final state is

$$\int_{\vec{r}_{init}}^{\vec{r}_{Final}} \underbrace{\vec{F} \cdot d\vec{r}} = \int_{\vec{r}_i}^{\vec{r}_f} |\vec{F}| \cos\theta \, |d\vec{r}|$$



$$\hookrightarrow \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$



$\Rightarrow$ no work is being done.

This is the case for uniform circular motion.

Kinetic energy: $K = \frac{1}{2} m v^2$

Net work done on an object of mass m equals $\Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$

$$\int_{\vec{r}_i}^{\vec{r}_f} \sum \vec{F} \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} m\vec{a} \cdot d\vec{r} = m \int_{\vec{r}_i}^{\vec{r}_f} \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

$$= m \int_{t_i}^{t_f} \left(\frac{d\vec{v}}{dt} \cdot \frac{d\vec{r}}{dt} \right) dt = m \int_{t_i}^{t_f} \left(\frac{d\vec{v}}{dt} \cdot \vec{v} \right) dt$$

$$= m \int_{v_i}^{v_f} \underbrace{\vec{v} \cdot d\vec{v}}_{v_x dv_x + v_y dv_y + v_z dv_z} = m \int_{v_i^2}^{v_f^2} \frac{1}{2} \, d(v^2) = \frac{1}{2} m \Delta(v^2)$$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

net work done $= \Delta K$

$$\text{power} = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

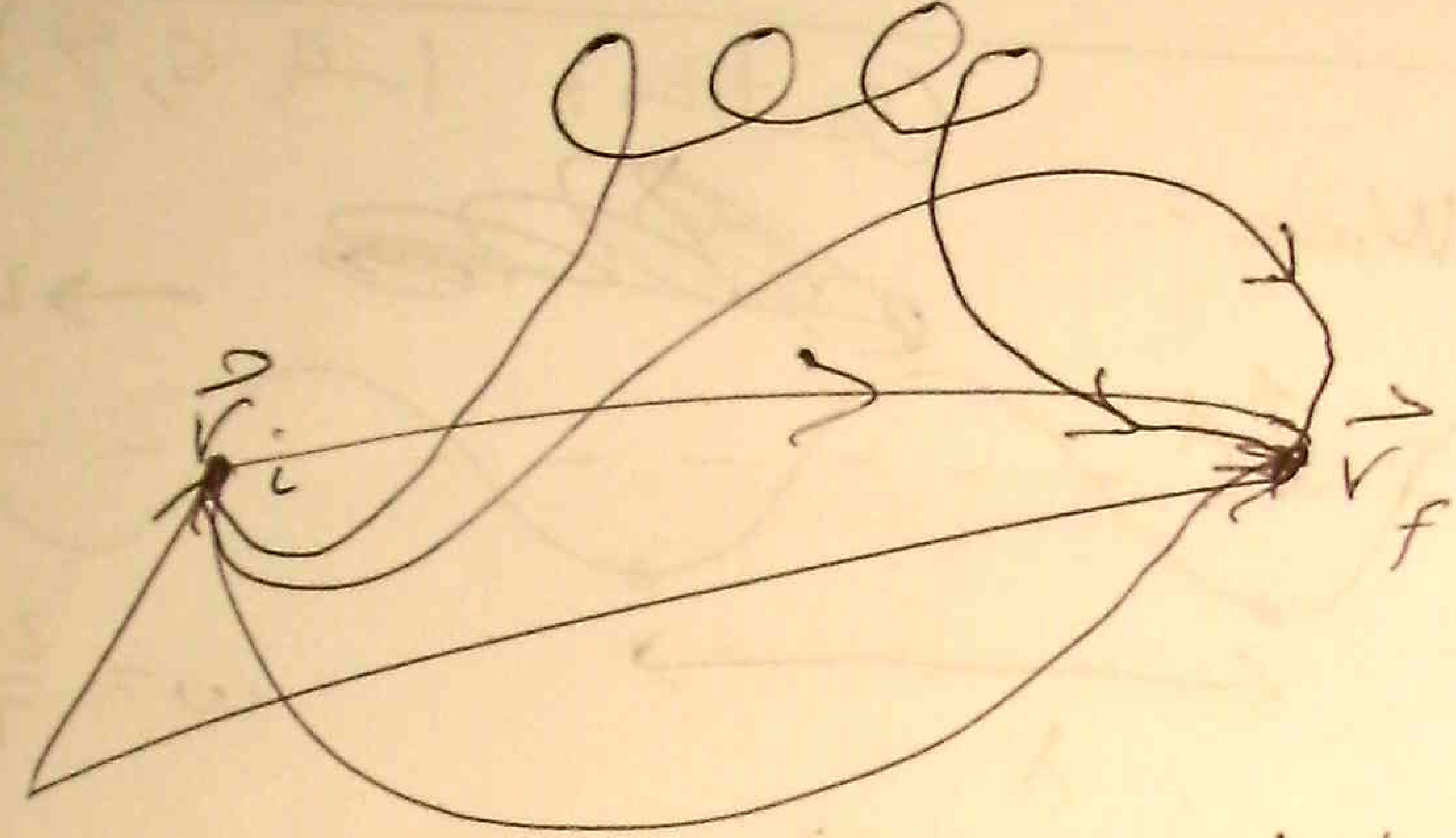
Net work $+ \Rightarrow$ speeds up

Net work $- \Rightarrow$ slow down

Net work

Ch. 8: Potential energy

If W does not depend on the path chosen, but on the endpoints,



then define $\Delta U = -W = -\Delta K$

ΔU only depends on position.

$$\Delta U_g = -\int_{r_i}^{\vec{r}_f} \underbrace{\vec{F}_g \cdot d\vec{r}}_{0\,dx + 0\,dy + (-mg)\,dz} = -\int_{z_i}^{z_f} -mg\,dz$$

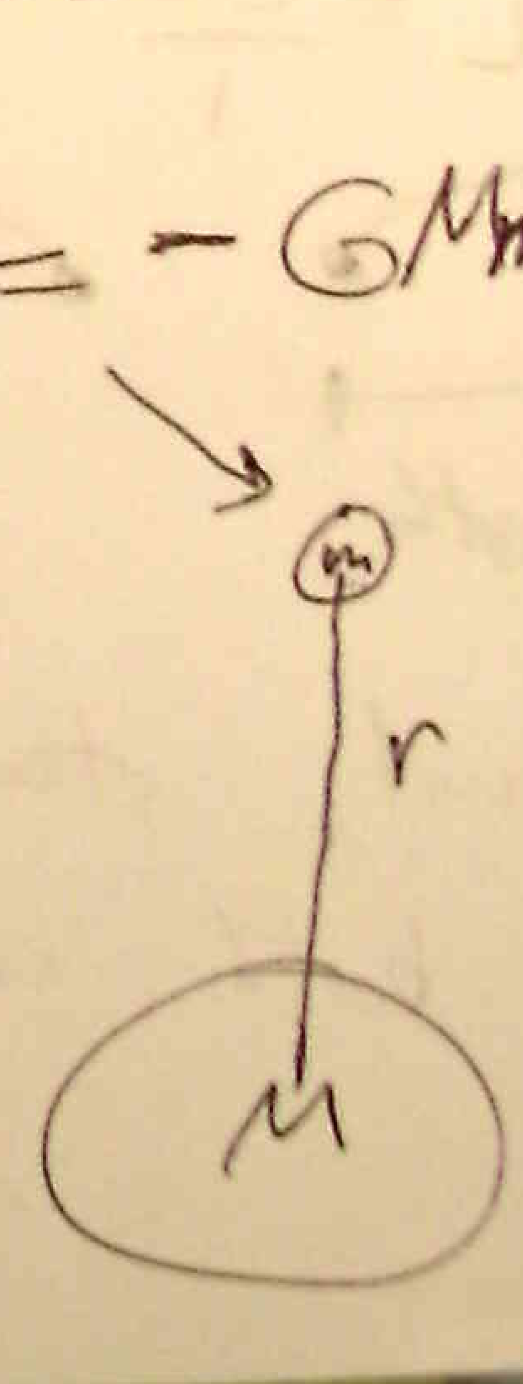
$z \uparrow$ $\quad \downarrow F_g = mg$

$$\Delta U_g = mg\,\Delta z$$

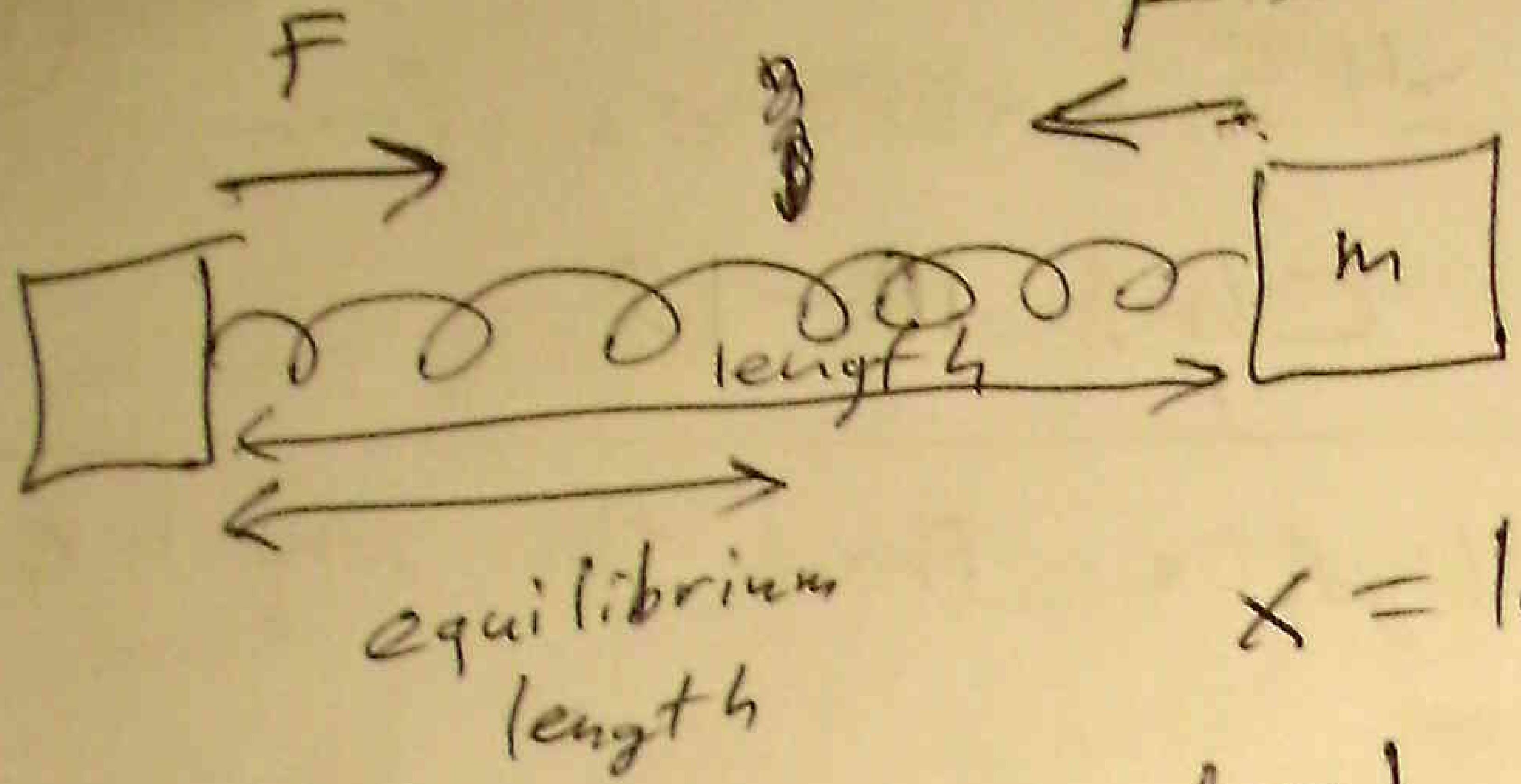
Near Earth's surface

$$\Delta U_G = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_G \cdot d\vec{r} = -GMm\,\Delta\left(\frac{1}{r}\right)$$

universal gravitation



$$g = \frac{GM_E}{R_E^2}$$



$F_x = -kx$

$\longrightarrow x$

x = length − eq. length

$$|F| = kx$$

k = spring constant

$$\Delta U_{sp} = -W = -\int_{x_i}^{x_f} \underbrace{-kx\,dx}_{\vec{F}\cdot d\vec{x}} = \frac{1}{2}k\Delta(x^2) = \frac{1}{2}k(x_f^2 - x_i^2)$$

total energy $E = K + U$ is constant b/c $-\Delta K = -W = \Delta U$, except that if there's friction or air resistance, or spring damping, or sound, or heat, etc, then these take energy away from $K+U$ b/c these resistive forces depend on path.
or dissipative

Ch. 9 (Linear) momentum $\vec{p} = m\vec{v}$

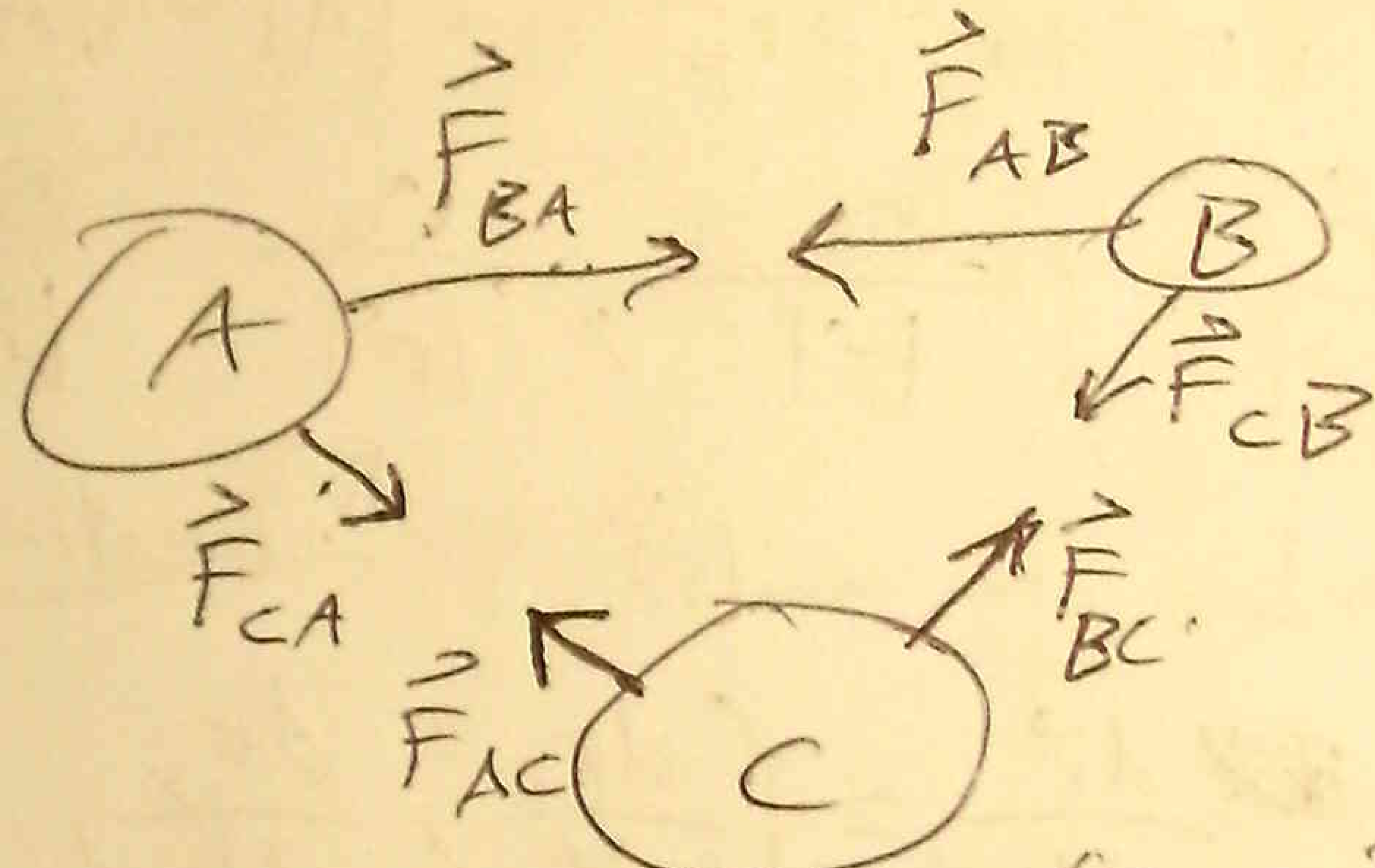
Newton's 2nd Law: $\sum \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

b/c $m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$ if m constant

$\sum \vec{F} = \frac{d\vec{p}}{dt}$ even when m not constant.

$\sum \vec{F} \neq m\vec{a}$ when m not constant

Newton's 3rd Law says that if a system of objects and they don't interact with any objects outside the system, or, if all forces on the system from outside the system add up to $\vec{0}$, then the total momentum of the system is constant.



$$\frac{d}{dt}(\vec{p}_A + \vec{p}_B + \vec{p}_C) = (\vec{F}_{BA} + \vec{F}_{CA}) + (\vec{F}_{AB} + \vec{F}_{CB}) + (\vec{F}_{AC} + \vec{F}_{BC}) = \vec{0}$$

center of mass $= \dfrac{\vec{r}_A m_A + \vec{r}_B m_B + \vec{r}_C m_C}{m_A + m_B + m_C}$

$\vec{r}_{cm}$ (c.m.)

$\vec{p}_A + \vec{p}_B + \vec{p}_C = (m_A + m_B + m_C)\vec{v}_{cm}$

$\vec{a}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} \qquad \vec{v}_{cm} = \dfrac{d\vec{r}_{cm}}{dt}$

Net external force $=$ $\underbrace{(\text{total mass})}_{\text{if constant}}$ $\times$ acceleration of c.m.

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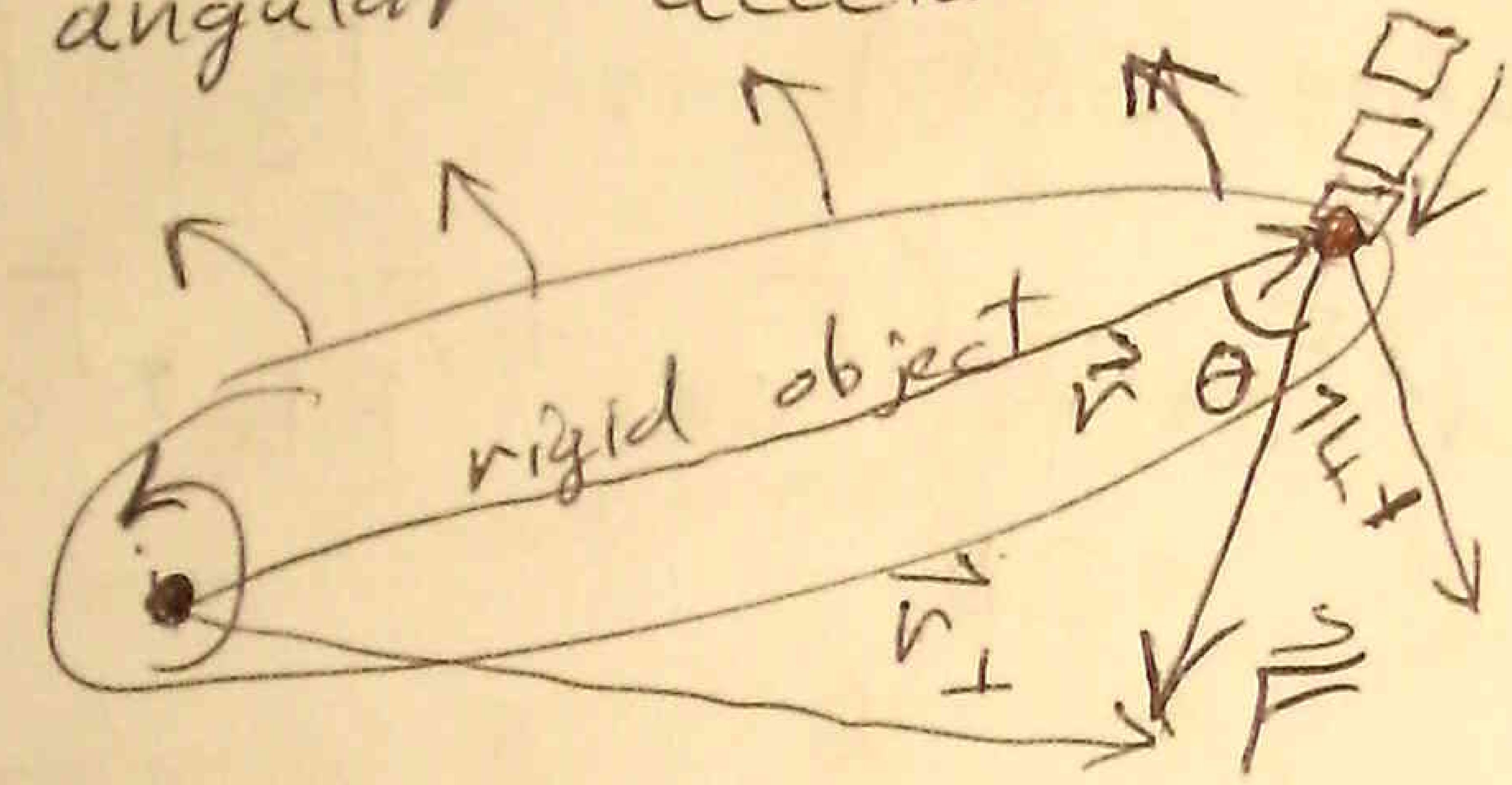
10 rotation about a fixed axis

torque τ

r_i = dist. relative to axis

moment of inertia $I = \sum_i r_i^2 m_i = \int_{\text{object}} r^2\, dm$

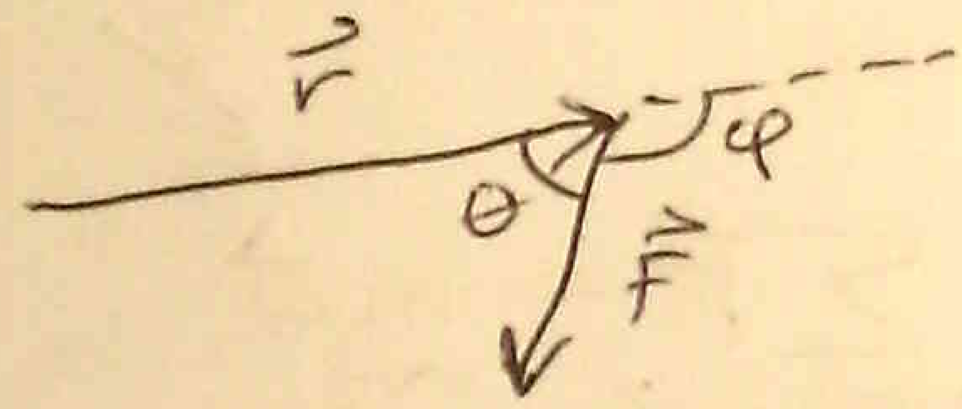
angular acceleration $\alpha = \dfrac{d\omega}{dt}$



$|\tau| = |\vec{F}||\vec{r}|\sin\theta$

$|\tau| = |\vec{F}_\perp||\vec{r}|$

$|\tau| = |\vec{F}|\,|\vec{r}_\perp|$



$\sin\theta = \sin(180° - \theta)$

$\sin\theta = \sin\varphi$

Table of I's for simple shapes in Ch. 10.

$\tau = I\alpha$ (like $\vec{F} = m\vec{a}$).

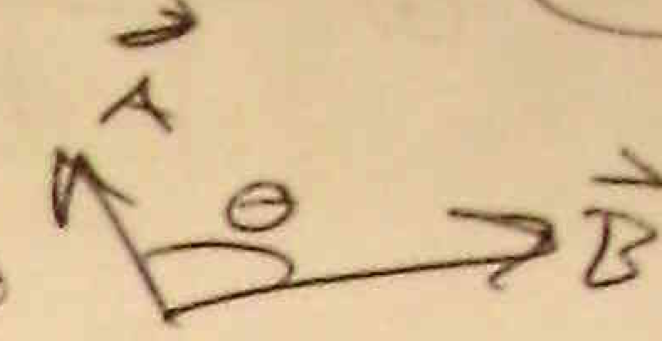
Ch. 11: 3D version; axis need not be fixed

$\vec{L}$ = angular momentum (measured relative to a center point)

$\vec{L} = \sum_i \vec{r}_i \times \underbrace{m\vec{v}_i}_{\vec{p}_i} = \int_{\text{object}} (d\vec{r} \times \vec{p})$

$(\vec{A} \times \vec{B}) \perp \vec{A}$ $\qquad$ $(\vec{A} \times \vec{B}) \perp \vec{B}$ $\qquad$ 12

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$



$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

↳ Use right-hand rule to visually/graphically find direction

$\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\quad \hat{\jmath} \times \hat{k} = \hat{\imath}$, $\quad \hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

$\hat{\imath} \times \hat{\imath} = \vec{0}$ $\quad \hat{\jmath} \times \hat{\jmath} = \vec{0}$ $\quad \hat{k} \times \hat{k} = \vec{0}$

In general, $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$ ← like $\sum \vec{F} = \frac{d\vec{p}}{dt}$

For a fixed axis of symmetry,

$\vec{\tau} = I\vec{\alpha}$ & $\vec{L} = I\vec{\omega}$

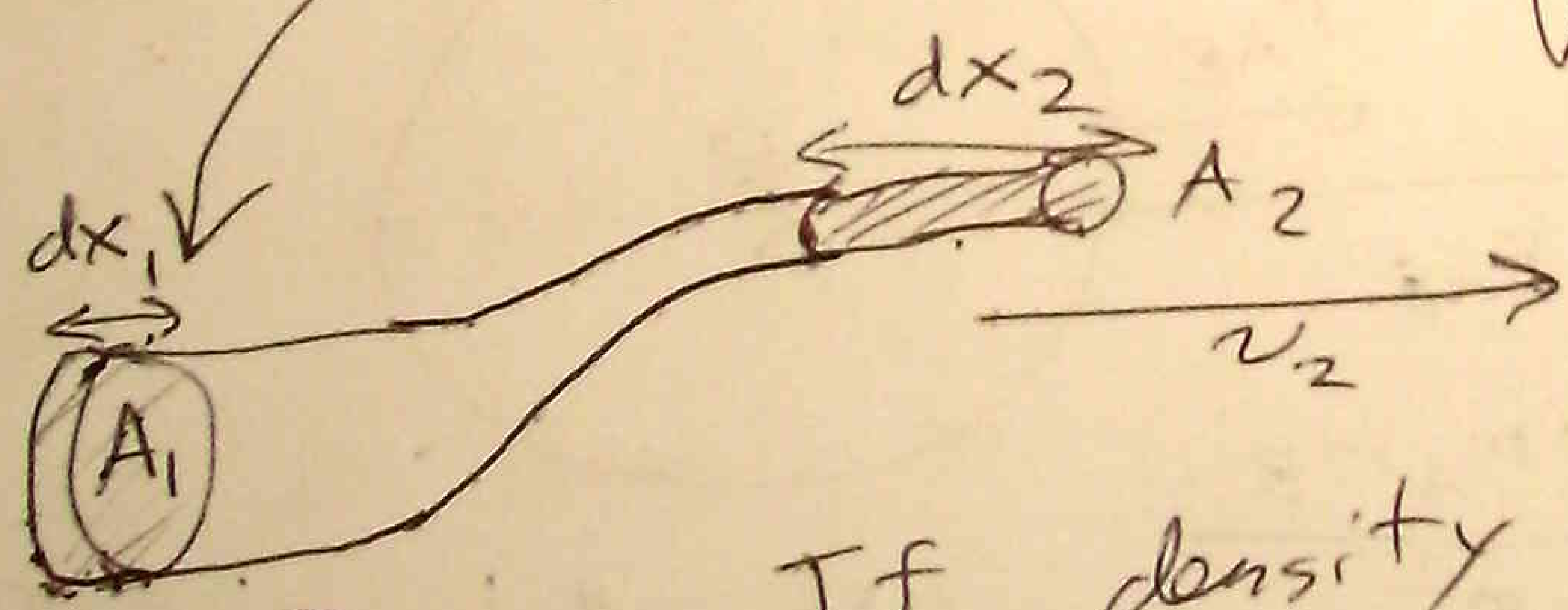
(much simpler).

Ch. 13 (Fluids)

2 big ideas:
conservation of mass
conservation of energy

$V_2 = A_2 \, dx_2$

$v_1 = \frac{dx_1}{dt_1}$, $v_2 = \frac{dx_2}{dt_2}$



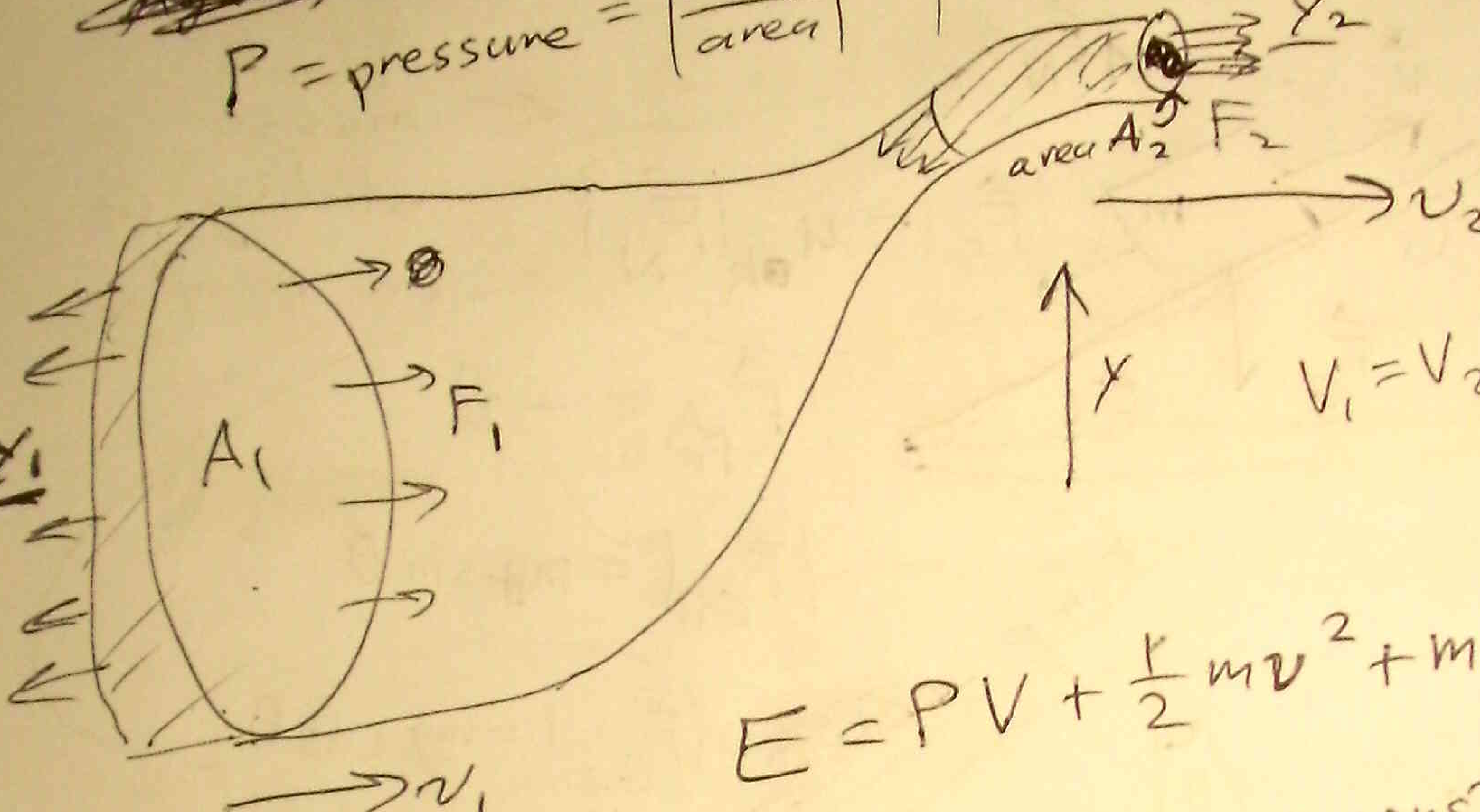
If density $\rho = \frac{\text{mass}}{\text{volume}}$ is constant, then volume is conserved too, implying

$V_1 = A_1 \, dx_1$ $\left\{ \begin{matrix} V_1 = V_2 \\ dt_1 = dt_2 \end{matrix} \right\}$, so $A_1 v_1 = A_2 v_2$.

" "

conservation of energy:

P = pressure = $\left|\dfrac{\text{force}}{\text{area}}\right| = \left|\dfrac{\text{force}\cdot\text{distance}}{\text{area}\cdot\text{distance}}\right| = \left|\dfrac{\text{work}}{\text{volume}}\right|$



$V_1 = V_2 = V$

$$E = PV + \frac{1}{2}mv^2 + mgy$$

$$\frac{E}{V} = P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 g y_1$$

$$P_2 + \frac{1}{2}\rho_2 v_2^2 + \rho_2 g y_2 .$$

Ch. 14 Simple Harmonic oscillation:

1D; $F = -kx$ (e.g. spring)

k constant

$m\dfrac{d^2x}{dt^2} = -kx \Rightarrow$ solutions to this equation

are all of form $x = A\cos(\omega t + \theta_0)$

constants

where $\omega = \sqrt{\dfrac{k}{m}}$ = ang. freq.

A = amplitude; θ_0 = phase shift;

Just section 1

15

Ch. 17, 18, 19: Ideal gasses & thermodynamics.

An ideal gas is a "box of bullets" bouncing around. For a monoatomic ideal gas, temperature $T = \frac{2}{3k}\overline{K}$ ~~(or $\frac{3}{2}\overline{K}$?)~~ where $\overline{K}$ is the average kinetic energy of a gas particle: $\overline{K} = \frac{1}{N}(K_1 + K_2 + \cdots + K_N)$.

T is measured in Kelvins (°K) (too many K's!)

Room temperature is ~300 °K

$0°C = 273.15°K = 32°F$

$x°C = (273.15 + x)°K$

Ideal gas law: $\underbrace{P}_{\text{pressure}}\underbrace{V}_{\text{volume}} = \underbrace{N}_{\#\text{particles}}\overbrace{k}^{\text{Boltzmann constant}}\underbrace{T}_{\text{temperature}} = nRT$

$\underbrace{n}_{\#\text{moles}} = \frac{N}{N_A}$; $\underbrace{R}_{\text{gas constant}} = kN_A$ $\quad N_A = 6.022 \times 10^{23}$

$R = 8.314 \frac{J}{mol \cdot °K}$

$$\text{Work done by gas} = \int_{V_i}^{V_f} \underbrace{P}_{\frac{F}{A}}\, \underbrace{dV}_{A\,dx}$$

$$E = \overbrace{\frac{1}{2}mv^2}^{K} + \overbrace{\frac{1}{2}kx^2}^{U} = \frac{1}{2}mA^2\omega^2\sin^2(\omega t+\theta_0) + \frac{1}{2}kA^2\cos^2(\omega t+\theta_0)$$ (14)

$$v = \frac{dx}{dt}$$

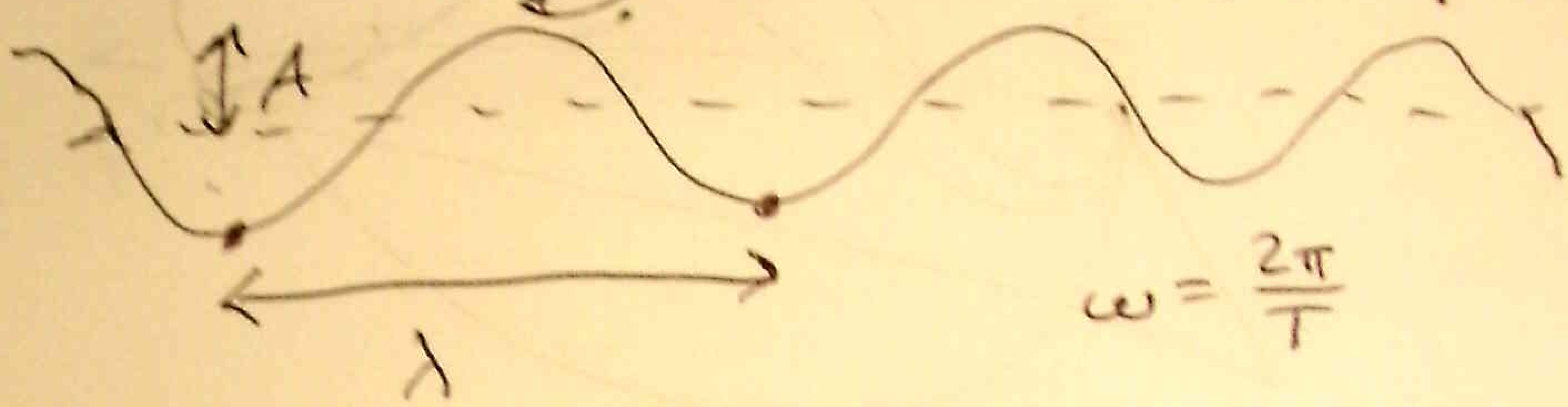
$$E = \frac{1}{2}kA^2 = \text{constant}$$

$$m\omega^2 = k$$

(Sections 1–4, 6, 9)

Ch. 15: Wave:

$\rightarrow v = \frac{\lambda}{T}$



$$\omega = \frac{2\pi}{T}$$

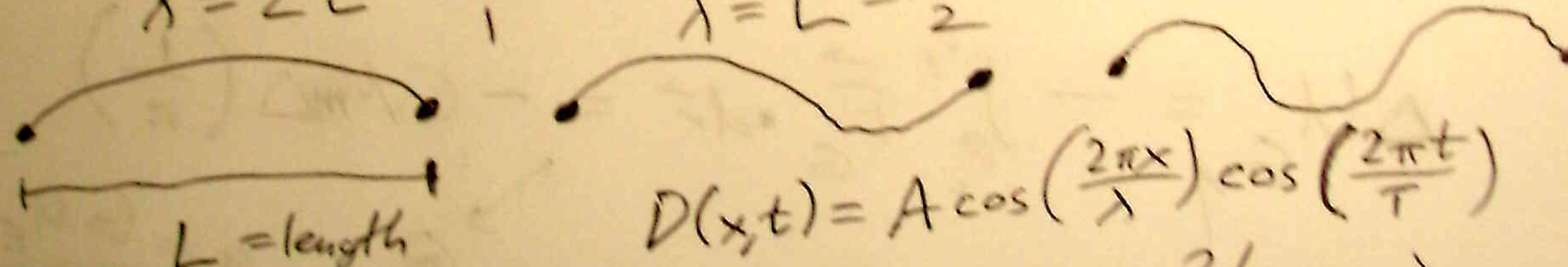
$$D(x,t) = A\cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$$

↑ sinusoidal wave travelling to right at speed v

Ch. 16 Sound: (Sections 4, 7)

Standing waves of vibrating strings & columns of air happen only at certain wavelengths. E.g. string $\left(v = \sqrt{\frac{\text{tension} \cdot \text{length}}{\text{mass}}}\right)$:

$\lambda = 2L = \frac{2L}{1}$ $\qquad \lambda = L = \frac{2L}{2}$ $\qquad \lambda = \frac{2L}{3}$



$$D(x,t) = A\cos\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$$

In general, standing waves @ $\frac{2L}{n} = \lambda$ when both ends are fixed.