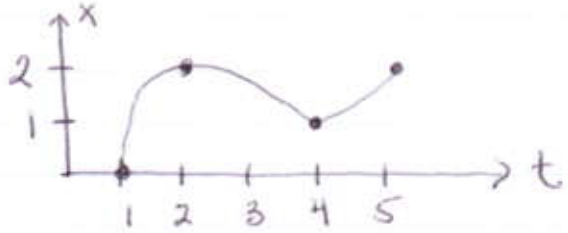
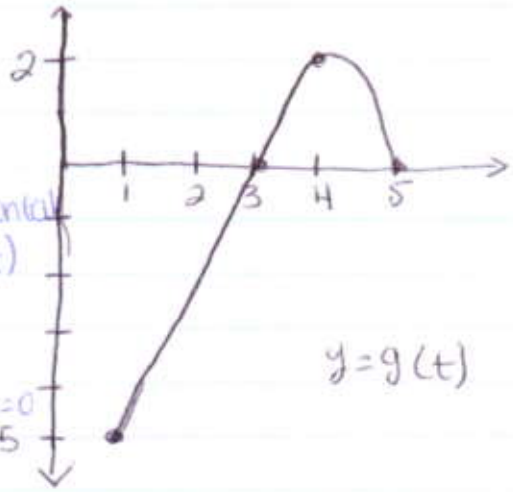
Today: more on parametric curves (10.1, 10.2?) 03/10/10



$x = f(t)$



$y = g(t)$

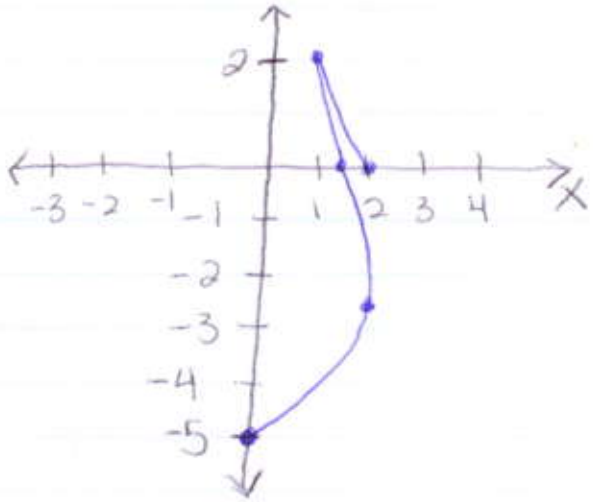
$t = 1 : f(1) = 0$

$t = 2 : f'(2) = 0$ (horizontal tangent)

$t = 3 : g(3) = 0$

$t = 4 : g'(4) = 0$

$t = 5 : g(5) = 0$



$x = f(t)$
$y = g(t)$
$1 \le t \le 5$

Curve $\begin{cases} x = f(t) \\ y = g(t) \\ 1 \le t \le 5 \end{cases}$

$y = h(x)$	$x = f(t), y = g(t)$
Important x-values:	Important t-values:
x where $h(x) = 0$ (x-intercepts)	t where: $f(t) = 0$, $g(t) = 0$, $f'(t) = 0$, or $g'(t) = 0$
x where $h'(x) = 0$ (critical points)	

$t=1: f(1)=0$

$t=2: f'(2)=0$ ← (horizontal tangent)

$t=3: g(3)=0$

$t=4: g'(4)=0$ ←

$t=5: g(5)=0$

t	x	y
1	0	-5
2	2	-2.5
3	1.3	0
4	1	2
5	2	0

t	1	2	3	4	5
x	+	+	+	+	
$\frac{dx}{dt}$	+	−	−	+	
y	−	−	+	+	
$\frac{dy}{dt}$	+	+	+	−	
(x, y)	(+, −) → IV	(+, −) → IV	(+, +) → I	(+, +) → I	
$\left(\frac{dx}{dy}, \frac{dy}{dt}\right)$	(+, +) ↴ right, up ↗	(−, +) ↴ left, up ↖	(−, +) ↴ left, up ↖	(+, −) ↴ right, down ↘	
$\frac{dy}{dx}$	+ = / +	+ = / −	+ = / −	− = / +	

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

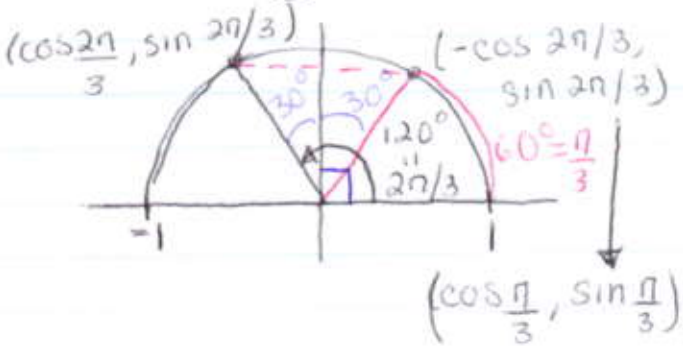
Example:

$$\begin{cases} x = \cos t \\ y = \sin 2t \\ 0 \le t < 2\pi \end{cases}$$

what is the slope $\frac{dy}{dx}$ at $t = \frac{\pi}{3}$?

$$\left.\begin{array}{l} dx/dt = -\sin t \\ dy/dt = 2\cos 2t \end{array}\right\} \Rightarrow \frac{dy}{dx} = \frac{2\cos 2t}{-\sin t}$$

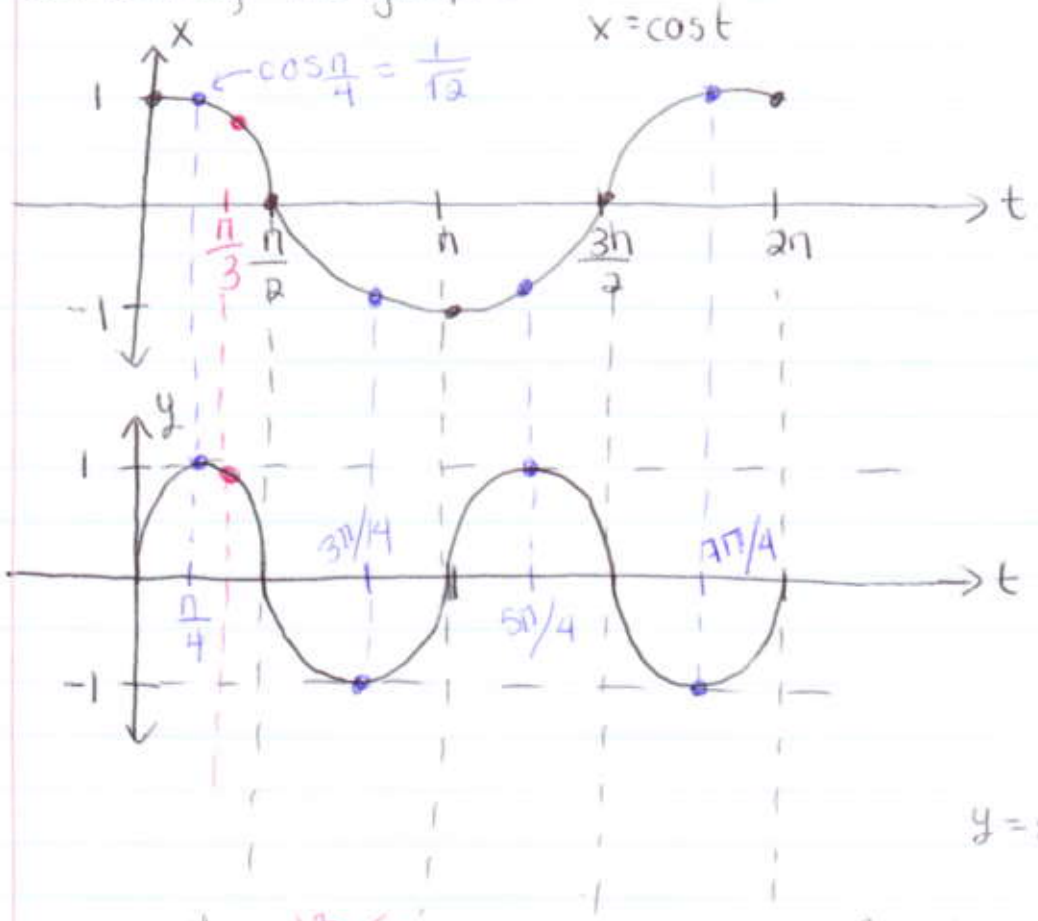
$$\left.\frac{dy}{dx}\right|_{t=\pi/3} = \frac{2\cos 2\pi/3}{-\sin \pi/3} = \frac{2(-1/2)}{-\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$ ← positive slope



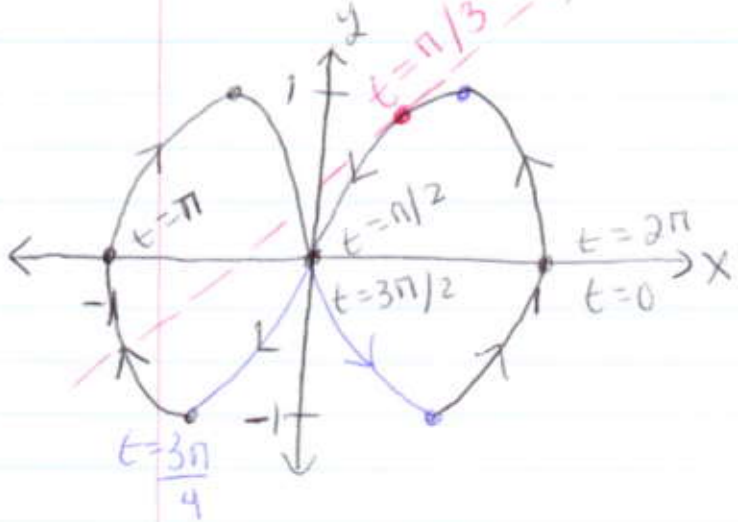
$$-\cos\frac{2\pi}{3} = \cos\frac{\pi}{3}$$

$$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

Sketching the graph



$y = \sin(2t)$

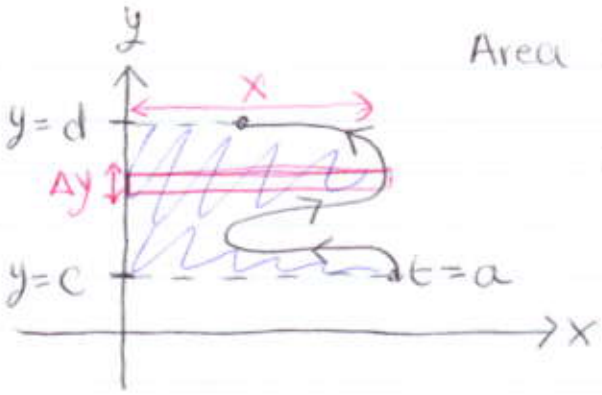


Graph is $y = \sin t$

compressed →→ ←←

by a factor of 2

Area between curves



$x = f(t)$
$y = g(t)$
$a \le t \le b$

▭ : Area

$x \Delta y$

Area = ?

$$\text{Area} \approx \sum_{n=1}^{K} x^*_n \Delta y$$

$$\text{Area} = \int_{y=c}^{y=d} x \, dy = \int_{t=a}^{t=b} f(t) dy \cdot \frac{dt}{dt}$$

$$= \int_{t=a}^{t=b} f(t) \frac{dy}{dt} dt = \boxed{\int_{t=a}^{t=b} f(t) g'(t) dt}$$