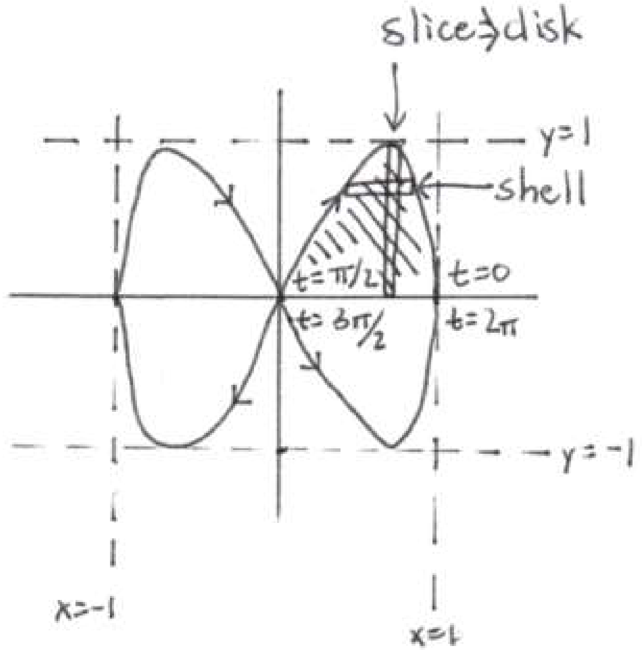
Today: more from 10.2

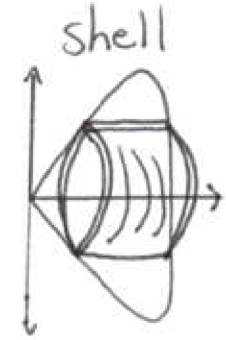
Tomorrow: / Wednesday: { 10.3

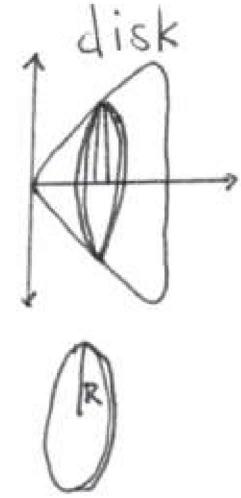
Thursday: { 11.1, HW8 due

Last time:
- Area under parametric curves,
- Length of parametric curves.

This time:
- Volumes of solids of revolution defined by parametric curves
- Areas of surfaces of revolution







Volume=?

Surface Area=?

$V_{disk} = \pi R^2 \Delta x$

$R = y = \sin 2t$

$$\text{Volume} = \int_{x=0}^{x=1} \pi R^2 dx = \int_{x=0}^{x=1} \pi y^2 dx = \int_{t=\pi/2}^{t=0} \sin^2 2t(-\sin t\, dt) = \int_0^{\pi/2} \pi(\sin^2 2t)(\sin t)dt$$

$$dx \frac{dt}{dt} = \frac{dx}{dt} dt = -\sin t\, dt$$

$\sin 2t = 2\cos t \sin t$

$$V = \int_0^{\pi/2} \pi(2\cos t \sin t)^2 \sin t\, dt = \pi \int_0^{\pi/2} 4\cos^2 t \sin^2 t \sin t\, dt$$

$$= 4\pi \int_0^{\pi/2} \cos^2 t \sin^3 t\, dt \quad (7.2)$$

$u = \sin t \Rightarrow du = \cos t\, dt \Rightarrow V = 4\pi \int^{t=\pi/2} \cos t \sin^3 t\, du$ ($\sin^3 t = u^3$, $\cos t = \sqrt{1-u^2}$) ☹

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★ $u = \cos t \Rightarrow du = -\sin t\,dt \Rightarrow V = 4\pi \int_{t=0}^{t=\pi/2} \underbrace{\cos^2 t}_{u^2} \underbrace{(-\sin^2 t)}_{u^2-1} du$

⇓

$V = 4\pi \int_1^0 u^2(u^2-1)\,du = 4\pi \int_0^1 u^2(1-u^2)\,du$

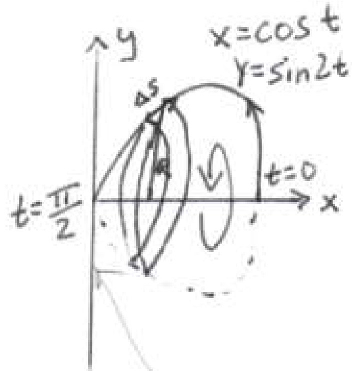
$\cos^2 + \sin^2 = 1$

$\cos^2 = 1 - \sin^2$

$\cos^2 - 1 = \sin^2$

$= 4\pi \int_0^1 (u^2 - u^4)\,du = 4\pi \left(\frac{u^3}{3} - \frac{u^5}{5}\right)\Big|_0^1$

$V = 4\pi \left[\left(\underbrace{\frac{1^3}{3}}_{5/15} - \underbrace{\frac{1^5}{5}}_{3/15}\right) - \left(\frac{0^3}{3} - \frac{0^5}{5}\right)\right] = 4\pi \cdot \frac{2}{15} = \frac{8\pi}{15}$



Surface Area = ? $A_{slice} = 2\pi R \Delta s$ $R = y = \sin 2t$

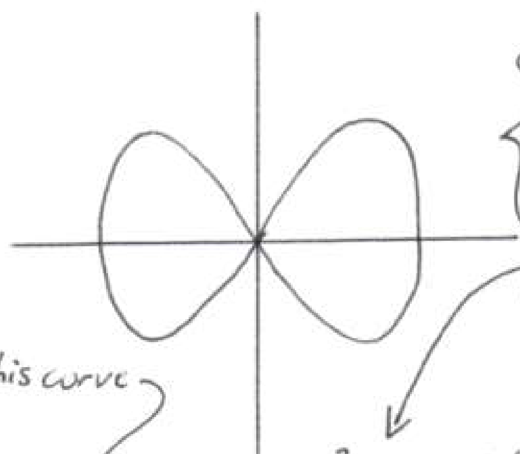
Go from left ($t=\frac{\pi}{2}$) to right ($t=0$)

$A = \int_{t=0}^{t=\pi/2} 2\pi R\,ds$ flip (Milovich's mistake)

$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Last topic for today:

"elimination of parameter"



$\begin{cases} x = \cos t \\ y = \sin 2t \\ \quad = 2\sin t \cos t \end{cases}$

use $\cos^2 t + \sin^2 t = 1$

Get a "cartesian" equation for this curve

↑

x & y only; no t

$y^2 = 4\underbrace{\sin^2 t}_{\underbrace{1-\cos^2 t}_{1-x^2}} \underbrace{\cos^2 t}_{x^2} = 4(1-x^2)x^2$

$y^2 = \boxed{4x^2 - 4x^4}$

$$\begin{cases} x = 5 + \ln t \Rightarrow t = e^{x-5} \Rightarrow y = (e^{x-5})^2 + 5 \\ y = t^2 + 5 \\ t > 0 \end{cases} \Rightarrow t = \sqrt{y-5} \Rightarrow x = 5 + \ln\sqrt{y-5}$$

Two solutions (both fine).

Eliminate the parameter.

$$y = (e^{x-5})^2 + 5$$

$$y - 5 = (e^{x-5})^2$$

$$\sqrt{y-5} = e^{x-5}$$

$$\ln\sqrt{y-5} = x - 5$$

$$5 + \ln\sqrt{y-5} = x$$