Today: more polar coordinates (10.3)

Tomorrow: $\begin{cases} 11.1 \\ \text{HW Due} \end{cases}$

Graph $r = \cos\theta$

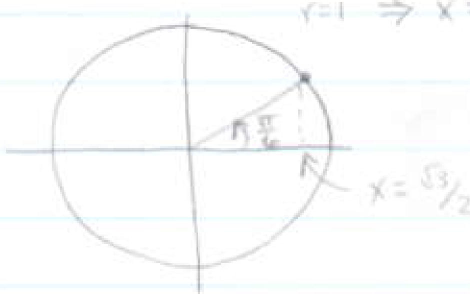
Radians are better

$$\frac{d}{dx}\sin x = \cos x$$

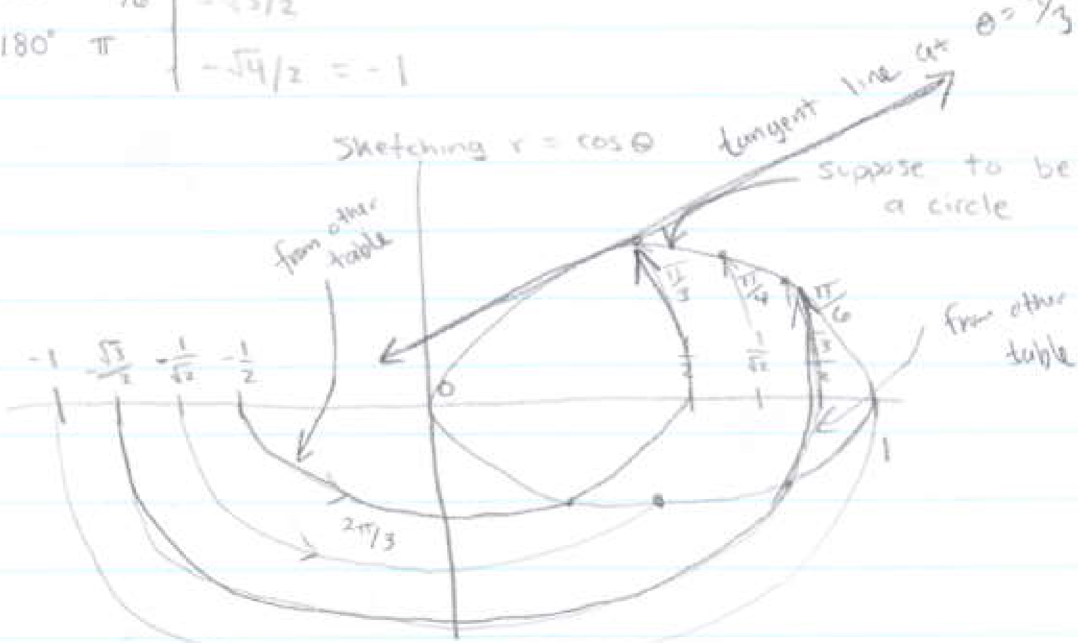
$$\frac{d}{dx}\sin x^\circ = \frac{d}{dx}\sin\frac{180x}{\pi} = \frac{180}{\pi}\cos\frac{180x}{\pi} = \frac{180}{\pi}\cos x^\circ$$

	θ	$\cos\theta$
0°	0	$\sqrt{4}/2 = 1$
30°	$\pi/6$	$\sqrt{3}/2$
45°	$\pi/4$	$\sqrt{2}/2 = 1/\sqrt{2}$
60°	$\pi/3$	$\sqrt{1}/2 = \frac{1}{2}$
90°	$\pi/2$	0
120°	$2\pi/3$	$-\sqrt{1}/2 = -\frac{1}{2}$
135°	$3\pi/4$	$-\sqrt{2}/2 = -\frac{1}{\sqrt{2}}$
150°	$5\pi/6$	$-\sqrt{3}/2$
180°	π	$-\sqrt{4}/2 = -1$

$r = 1 \Rightarrow x = r\cos\theta = \cos\theta$



Sketching $r = \cos\theta$



	θ	$\cos\theta$
	π	-1
$210^\circ =$	$7\pi/6$	$-\sqrt{3}/2$
$225^\circ =$	$5\pi/4$	$-1/\sqrt{2}$
240°	$4\pi/3$	$-1/2$
$270^\circ =$	$3\pi/2$	0
	$5\pi/3$	$1/2$
$315^\circ =$	$7\pi/4$	$1/\sqrt{2}$
$330^\circ =$	$11\pi/6$	$\sqrt{3}/2$
$360^\circ =$	2π	1

Prove $r=\cos\theta$ is a circle using elimination of parameter

$r=\cos\theta \longrightarrow$ $x = r\cos\theta = \cos^2\theta$

$y = r\sin\theta = \cos\theta\sin\theta$

$$\begin{cases} x=\cos^2 t \\ y=\cos t\sin t \end{cases} \quad t=\theta$$

parametric curve

$x=\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$2y = 2\cos\theta\sin\theta = \sin 2\theta$

$2x-1=\cos 2\theta$

$1=\cos^2(2\theta)+\sin^2(2\theta)$

$1=(2x-1)^2+(2y)^2$

$1=(2x-1)^2+(2y)^2$

$\left(\frac{1}{2}\right)^2 = \left(\frac{2x-1}{2}\right)^2 + \left(\frac{2y}{2}\right)^2$

$\left(\frac{1}{2}\right)^2 = \left(x-\frac{1}{2}\right)^2 + y^2$

$\left(\frac{1}{2}\right)^2 = \left(x-\frac{1}{2}\right)^2 + (y-0)^2$

circle with radius $\frac{1}{2}$

+ center $(x,y)=\left(\frac{1}{2},0\right)$

Tangent lines

Find an (x, y) equation for the line tangent to $r = \cos\theta$ at $\theta = \frac{\pi}{3}$

(x_0, y_0)

$$y - \overset{\sqrt{3}/4}{y_0} = m(x - \overset{1/4}{x_0})$$

$x = r\cos\theta = \cos^2\theta$

$y = r\sin\theta = \cos\theta\sin\theta$

$\theta = \frac{\pi}{3} \Rightarrow \quad x_0 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$y_0 = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$

$m = \frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(\cos\theta\sin\theta)}{\frac{d}{d\theta}(\cos^2\theta)}$$

$$\frac{dy}{dx} = \frac{\cos'\theta\sin\theta + \cos\theta\sin'\theta \;]\text{ product rule}}{2\cos\theta\,\cos'\theta\;]\text{ chain rule}} = \frac{-\sin^2\theta + \cos^2\theta}{-2\cos\sin\theta}$$

$$\frac{dy}{dx} = \frac{\sin^2\theta - \cos^2\theta}{2\cos\theta\sin\theta} \quad \text{At } \theta = \frac{\pi}{3}$$

$$\begin{cases} \cos\theta = \frac{1}{2} \\ \sin\theta = \sqrt{3}/2 \end{cases}$$

$$M = \frac{(\sqrt{3}/2)^2 - (1/2)^2}{2(1/2)(\sqrt{3}/2)} = \frac{2/4}{\sqrt{3}/2} = \frac{1/2}{\sqrt{3}/2}$$

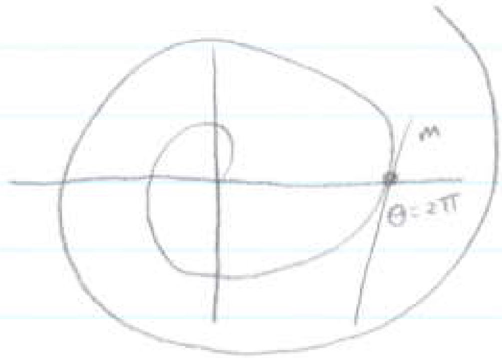
$$\boxed{M = \frac{1}{\sqrt{3}}}$$

$y - y_0 = m(x - x_0)$

tangent line

$$y - \frac{\sqrt{3}}{4} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{4}\right)$$

$r = \sqrt{\theta}$



What is $\frac{dy}{dx}$ at $\theta = 2\pi$?

$y = r\sin\theta = \sqrt{\theta}\sin\theta$

$x = r\cos\theta = \sqrt{\theta}\cos\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(\sqrt{\theta}\sin\theta\right) = \frac{1}{2\sqrt{\theta}}\sin\theta + \sqrt{\theta}\cos\theta$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(\sqrt{\theta}\cos\theta\right) = \frac{1}{2\sqrt{\theta}}\cos\theta + \sqrt{\theta}(-\sin\theta)$$

at $\theta = 2\pi$

$$\frac{1}{2\sqrt{2\pi}}\,0 + \sqrt{2\pi}\,(1)$$

$$\frac{1}{2\sqrt{2\pi}}\,1 + \sqrt{2\pi}\,(-0)$$

at $\theta = 2\pi$

$$\frac{dy}{d\theta} = \sqrt{2\pi} \qquad \frac{dx}{d\theta} = \frac{1}{2\sqrt{2\pi}} \qquad \frac{dy}{dx} = \frac{\sqrt{2\pi}}{\frac{1}{2\sqrt{2\pi}}}$$

$$\boxed{M = 4\pi}$$