Today: Review
Tuesday: MT2 - bring calculator & 1 sheet of notes

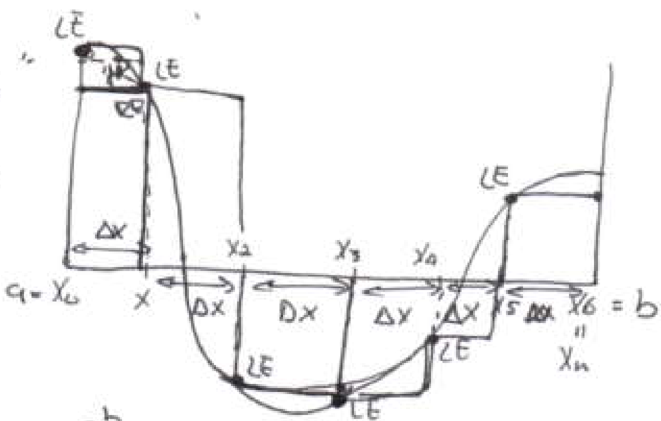
(Monday: 11.3)

MT2: Topics

7.7 approximate integration
7.8 improper integrals
8.1 arc length
8.2 surface area
9.3 separable } differential
9.5 linear } equations
10.1 parametric }
10.2 parametric } curves
10.3 polar }

7.7 → "rules"

TD



$n = 6 = \#$ subintervals

$\Delta x = \dfrac{b-a}{n}$

$x_0 = a$

$x_1 = a + \Delta x$

$x_2 = a + 2\Delta x$

$x_i = a + i\Delta x$

Leftendpoints $\int_a^b f(x)\,dx \approx \Delta x\big(f(x_0) + f(x_1) + \dots f(x_{n-1})\big)$

Right endpoints $\int_a^b f(x)\,dx \approx \Delta x\big(f(x_1) + \dots + f(x_n)\big)$

Midpoint: $\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \dots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]\Delta x$

Midpoint MP $\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x$

Trapezoid $\Delta x\left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + f(x_3) \dots + f(x_{n-1}) + \frac{f(x_n)}{2}\right)$

Simpson's rule $\frac{\Delta x}{3}\big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) \dots 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\big)$

n even only

Simpsons Pattern:

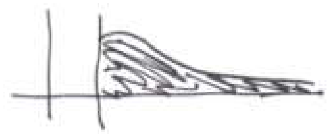
14242 .... 24 24241

More on Simpson's Pattern (added after class)

141
14241
1424241
142424241
14242424241

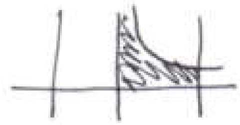
7.8 Improper Integrals

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx \longrightarrow$$



or $f(x)$ blows up at a:

$$\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx \longrightarrow$$

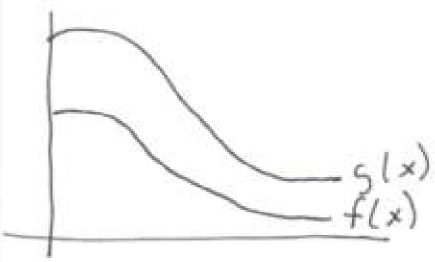


If $0 \le f(x) \le g(x)$ on $[a, \infty)$, then if $\int_a^\infty f(x)dx$ diverges

then $\int_a^b g(x)$ diverges

if $\int_a^\infty g(x)\,dx$ converges

then $\int_a^\infty f(x)dx$ converges



8.1 arc length

$$\int_a^b ds = \text{arc length}$$

$$\frac{dy}{dx} = f'(x)$$

$$y = f(x) \Rightarrow ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dx}{dy} = g'(x)$$

$$x = g(y) \Rightarrow ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$\left.\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}\right\} \Rightarrow ds = \sqrt{\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dx}\right)^2}\,dt$$

$$\int_a^b ds = \text{arc length}$$

Parametric

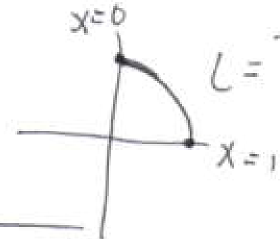
left x-value

$$y = f(x) \quad '' \quad \int_{x_A}^{x_B}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

right x-value

$$\int_{\text{lesser } t\text{-value of endpoints}}^{\text{greater } t\text{-value of endpoints}} \sqrt{\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dx}\right)^2}\,dt$$

$$L=\int_{x=0}^{x=1}\sqrt{1+\left(\frac{dy}{dy}\right)^2}\,dx$$



$L = \pi/2$

$y=\sqrt{1+x^2}$, $0\le x\le 1$

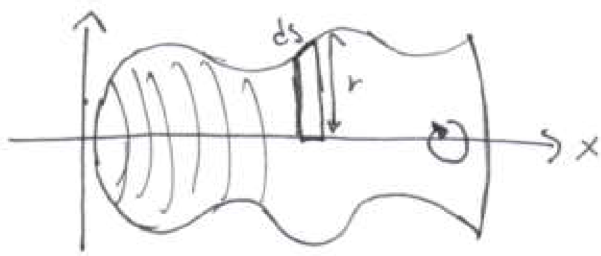
$$\left.\begin{aligned}x&=\cos t\\ y&=\sin t\\ 0&\le t\le \pi/2\end{aligned}\right\} L=\int_{t=0}^{t=\pi/2}\sqrt{\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dx}\right)^2}\,dt$$

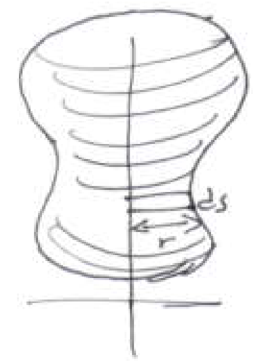
$x=\sqrt{1-y^2}$; $0\le y\le 1$

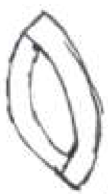
$$L=\int_{y=0}^{y=1}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

8.2 Surface area of solids of revolution

$$\int 2\pi r\,ds$$



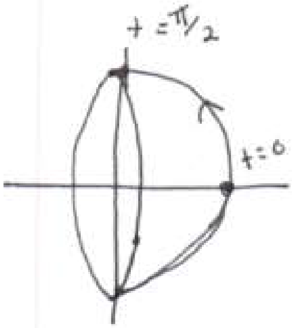




area of slice $= 2\pi r\,ds$

Rotating about x-axis : $r=y$

" " y-axis $r=x$



$$2\pi = A = \int_{t=0}^{t=\pi/2} y = \sqrt{\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dx}\right)^2}\,dt$$

$\boxed{2*Pi*y*Sqrt(...)dt}$

9.3 Separable differential equations
can be put in form

$$f(x)\,dx + g(y)\,dy = 0$$

$$\frac{dy}{dx}=x\cos y \qquad \int\frac{dy}{\cos y}-\int x\,dx = C$$

$$\frac{dy}{\cos y}=x\,dx$$

$$\frac{dy}{\cos y} - x\,dx = 0$$

$\ln|\sec y + \tan y| - \frac{1}{2}x^2 = C$

If our solution passes through $(4,0)$, then

$\ln|\sec 0 + \tan 0| - \frac{1}{2}(4)^2 = C$

$\Downarrow$

$C = -8 \Rightarrow \ln|\sec y + \tan y| - \frac{1}{2}x^2 = -8$

9.5 linear differential equations
can be put in form

$y' + p'y = q \quad \big\} \quad y'(x) + p'(x)y(x) = q(x)$

$u = e^p y = e^{p(x)} y(x) \Rightarrow u' = e^p q$

① Choose p, given p'

E.g., $p = \ln|x|$ if $p' = \frac{1}{x}$

② Find general $u = \int e^p q\,dx \;\underline{+\,C}$
important

E.g., $p = \ln|x|$ & $q = x$

③ $y = e^{-p}u = e^{-p}\int e^p q\,dx + Ce^{-p}$

10.1 & 10.3 Parametric & Polar curves

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ or $\frac{dy/d\theta}{dx/d\theta}$

for slopes

parametric
$x = f(t)$
$y = f(t)$

polar
$r = f(\theta)$
$\Downarrow$
$x = f(\theta)\cos\theta$
$y = f(\theta)\sin\theta$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \quad \text{or} \quad \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{dx/d\theta}$$

for ~~slopes~~ concavity

9.5

$$y' + \frac{y}{x} = 3 \quad \& \quad x > 0$$

$$y' + p'y = q$$

$$p' = \frac{1}{x} = p = \int \frac{1}{x}\,dx = \ln|x| + C$$

(choose $p = \ln|x|$)

$$x > 0 \Rightarrow p = \ln x$$

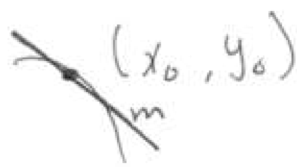
$$u = e^{p} y \Rightarrow u' = e^{p} q$$

$$u = \int e^{p} q\,dx = \int e^{\ln x}\, 3\,dx$$

$$= \int 3x\,dx = \frac{3}{2}x^2 + C \Rightarrow y = e^{-p}\,u = \frac{u}{e^{p}} = \frac{u}{x} = \frac{\frac{3}{2}x^2 + C}{x}$$

$$\boxed{\frac{3}{2}x + \frac{C}{x}}$$

Tangent lines



slope m, point (x_0, y_0)

$$y - y_0 = m(x - x_0)$$ line's equations