Today: $\begin{cases} \text{Alternating series (continued)} \\ \text{§ 11.5} \end{cases}$

Monday 4/19: HW 11 due: 11.4 # 12, 16, 18, 20, 24
11.5 # 6, 10, 14, 28, 30

:|

Suppose that $b_0 \geq b_1 \geq b_2 \geq b_3 \geq \dots \geq 0$.

If $\lim_{n\to\infty} b_n = 0$, then $\sum_{n=0}^{\infty} (-1)^n b_n = b_0 - b_1 + b_2 - b_3 + \dots$ converges

Alternating Series test

Divergence Test Applied to alternating series

If $\lim_{n\to\infty} b_n \neq 0$, then $\lim_{n\to\infty} (-1)^n b_n \neq 0$ too, so $\sum_{n=0}^{\infty} (-1)^n b_n$ diverges.

or...

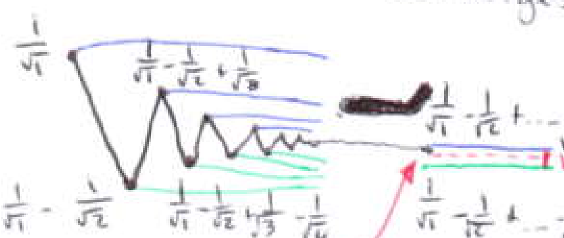
Today: estimating alternating series...

$\frac{1}{\sqrt{1}} \geq \frac{1}{\sqrt{2}} \geq \frac{1}{\sqrt{3}} \geq \dots \geq 0$ and $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, so $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = \frac{(-1)^{1+1}}{\sqrt{1}} + \frac{(-1)^{2+1}}{\sqrt{2}} + \frac{(-1)^{3+1}}{\sqrt{3}} \dots$

$= \frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots$

converges

Let's estimate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$.



$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \dots - \frac{1}{\sqrt{998}} + \frac{1}{\sqrt{999}} \approx .620714$

h

$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \dots - \frac{1}{\sqrt{998}} + \frac{1}{\sqrt{999}} - \frac{1}{\sqrt{1000}} \approx .589091$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \approx 0.604903$

$|\text{error}| = 0.015811$

$h = \text{height} = \frac{1}{\sqrt{1000}} \approx 0.031623$

Best guess $\approx \frac{0.620714 + 0.589091}{2} \approx 0.604903$

error is at worst $\pm \frac{h}{2} \approx 0.015811$ $= \pm \frac{1}{2\sqrt{1000}}$

$S = \sum_{n=3}^{\infty} \frac{(-1)^n}{n^3} = \frac{(-1)^3}{3^3} + \frac{(-1)^4}{4^3} + \frac{(-1)^5}{5^3} + \dots$

$= \frac{-1}{27} + \frac{1}{64} - \frac{1}{125} + \dots$

Estimate S with error at worst ± 0.0001

We pick a # K and compute

$A = \sum_{n=3}^{K-1} \frac{(-1)^n}{n^3}$ and $B = \sum_{n=3}^{K} \frac{(-1)^n}{n^3}$

Best guess is $\frac{A+B}{2}$ and worst possible error $\pm \frac{1}{2} \cdot \frac{1}{K^3}$

We need $\frac{1}{2K^3} \leq 0.0001$

$2K^3 \geq 10^4$ $\quad K^3 = \frac{10^4}{2}$ $\quad K \geq \sqrt[3]{5000} \approx 17.0998\dots$

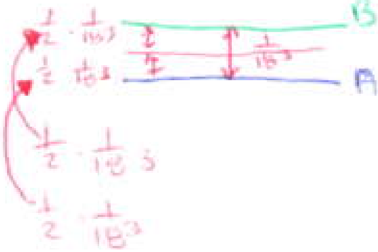
whole number $\quad$ Pick $K = 18$

$$A = \sum_{n=3}^{17} \frac{(-1)^n}{n^3} = \frac{-1}{3^3} + \frac{1}{4^3} - \frac{1}{5^3} + \frac{1}{6^3} - \dots - \frac{1}{17^3} \approx -0.026636$$

$$B = \sum_{n=3}^{18} \frac{(-1)^n}{n^3} = A + \frac{1}{18^3} \approx -0.026464$$

$$\frac{A+B}{2} \approx -0.026550$$

$s = -0.025550$



error at worst $\pm\, 0.0001$

Actually, error at worst

$$\frac{\pm 1}{2 \cdot 18^3} \approx \pm\, 0.000086$$

Estimate $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^6+1}$

with error at worst $\pm\, 0.0001$.

Pick whole # K

$$A = \sum_{n=0}^{K-1} \frac{(-1)^n}{n^6+1}$$

$$B = \sum_{n=0}^{K} \frac{(-1)^n}{n^6+1} = A + \frac{(-1)^K}{K^6+1}$$

Estimate $\frac{A+B}{2}$ error at worst $\frac{1}{2(K^6+1)}$

We need

$$\frac{1}{2(K^6+1)} \le 0.0001 = 10^{-4}$$

$$2(K^6+1) \ge 10^4 = 1000$$

$$K^6+1 \ge 5000$$

$$K^6 \ge 4999$$

$$K \ge \sqrt[6]{4999} \approx 4.13505$$

$$A = \frac{(-1)^0}{0^6+1} + \frac{(-1)^1}{1^6+1} + \frac{(-1)^2}{2^6+1} + \frac{(-1)^3}{3^6+1} + \frac{(-1)^4}{4^6+1} \approx 0.514259$$

$$B = A + \frac{(-1)^5}{5^6+1} = A - \frac{1}{5^6+1} \approx 0.514195$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^6+1} \approx s = \approx \frac{A+B}{2} \approx 0.514227$$

$|\text{error}| \le \pm\, 0.0001.$