* sorry about the messy handwriting. if you have any questions ask me in class. *

Today: 11.6 Absolute and conditional convergence

Monday HW due @ 5

Recall p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is $\boxed{\begin{cases} \text{convergent}: p>1 \\ \text{divergent}: p \le 1 \end{cases}}$

Alternating p-series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p} = \frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} \dots$

is $\boxed{\begin{cases} \text{convergent}: p>0 \\ \text{divergent}: p \le 0 \end{cases}}$

Alt. series test: if $b_1 \ge b_2 \ge b_3 \dots \ge 0$ and $\lim_{n\to\infty} b_n = 0$ then $b_1 - b_2 + b_3 - b_4 \dots$ $\boxed{\text{converges}}$

Divergence test: if $\lim_{n\to\infty} a_n \ne 0$, then $a_1 + a_2 + a_3 \dots$ $\boxed{\text{diverges}}$

ex. $\boxed{p = \frac{1}{2}}$ $\quad \frac{1}{\sqrt{1}} \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{3}}$

alt. series test: $\frac{1}{1^{1/2}} \ge \frac{1}{2^{1/2}} \ge \frac{1}{3^{1/2}}$

$\lim_{n\to\infty} \frac{1}{n^{1/2}} = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = \frac{1}{+\text{big}} \to 0$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/2}}$ $\boxed{\text{converges}}$

This same argument works for any $p>0$

ex. $\boxed{p = -3}$

$\frac{1}{1^{-3}} - \frac{1}{2^{-3}} + \frac{1}{3^{-3}} \dots$

$1^3 - 2^3 + 3^3 - 4^3 + 5^3 \dots \lim_{n\to\infty} (-1)^{n+1} n^3$ DNE

cont. →

Divergence test: $\frac{1}{1^{-3}} - \frac{1}{2^{-3}} + \dots$ $\boxed{\text{diverges}}$

ex $\boxed{p=0}$ $\frac{1}{1^0} - \frac{1}{2^0} + \frac{1}{3^0} - \frac{1}{4^0} + \dots = 1 - 1 + 1 - 1 + 1 -$

$\boxed{\text{diverges}}$

$$\sum_{n=1}^{\infty} a_n \text{ is } \begin{cases} \text{absolutely convergent if } \sum_{n=0}^{\infty} |a_n| \text{ converges} \\ \text{conditional convergent if } \sum_{n=0}^{\infty} |a_n| \text{ diverges} \\ \quad \text{but } \sum_{n=0}^{\infty} a_n \text{ converges} \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p} = \begin{cases} \text{absolutely convergent: } p > 1 \\ \text{conditional convergent: } 0 < p \le 1 \\ \text{divergent } p \le 0 \end{cases}$$

Note: if $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges too

if $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} |a_n|$ diverges too

* If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then reordering the terms (like $a_3 + a_2 + a_1 + a_6 + a_5 + a_4$) doesn't change the sum.

E.g. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$

$\frac{1}{1^2} + \frac{1}{3^2} - \frac{1}{2^2} + \frac{1}{5^2} + \frac{1}{7^2} - \frac{1}{4^2} + \dots$

* if $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, then you can make it convergent to any real number you want by reordering the terms.

ex. $\ln 2 = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \cdots$

$\frac{3}{2} \ln 2 = \frac{1}{1} + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} \cdots$