4-15-10

Today the ratio TEST (11.6)

$$\sum_{n=0}^{\infty} r^n = \begin{cases} \frac{1}{1-r} : \boxed{-1 < r < 1} & \text{SAME AS } |r| < 1 \\ \text{(Divergent)} : \boxed{\text{otherwise}} & \text{SAME AS } |r| \geq 1 \end{cases}$$

GEOMETRIC SERIES

Because $\lim_{n\to\infty} r^n \neq 0$ in this case

The ratio takes $\sum_{n=0}^{\infty} a_n$ and tries to find a_n $r \geq 0$ such that $\sum_{n=0}^{\infty} |a_n|$ is more like $\sum_{n=0}^{\infty} r^n$ than $\sum_{n=0}^{\infty}$ for any other $s \geq 0$

$$\sum_{k=0}^{\infty} r^k = r^0 + r^1 + r^2 + r^3 + \cdots + r^n + r^{n+1} + \cdots$$

$$r = \frac{r^1}{r^0} = \frac{r^2}{r^1} = \frac{r^3}{r^2} = \cdots = \frac{r^{n+1}}{r^n} = \cdots$$

For the ratio test, find $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ (if it exists)

IF $L > 1$, then, like $\sum_{n=0}^{\infty} L^n$, $\sum_{n=0}^{\infty} a_n$ Diverges

IF $0 \leq L < 1$, then, like $\sum_{n=0}^{\infty} L^n$, $\sum_{n=0}^{\infty} a_n$ converges, (and $\underbrace{\sum_{n=0}^{\infty} |a_n| \text{ converges too}}_{\text{"Absolute convergence"}}$)

($L < 0$ never happens)

$L=1 \Rightarrow$ ratio test inconclusive

EXAmple : $\sum_{n=0}^{\infty} \frac{2^n}{n!} \Rightarrow \sum_{n=0}^{\infty} 2^n$

$a_n = \frac{2^n}{n!}$ $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}}\right| = \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}}$

$= \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \frac{2(n!)}{(n+1)!} = \frac{2 \cdot \cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5} \cdot \cancel{6} \cdots (n-1) \cdot n}{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5} \cdot \cancel{6} \cdots (n-1) \cdot n \cdot (n+1)} = \frac{2}{n+1}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{2}{n+1} = \frac{2}{+\text{big}+1} = \frac{2}{\text{big}} = +s$

$\sum_{n=5}^{\infty} \frac{4^n + 3^{n+2}}{\sqrt{10^n+1}}$ conveges or diverges ?

$a_n = \frac{4^n + 3^{n+2}}{\sqrt{10^n+1}} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{\frac{4^{n+1}+3^{n+3}}{\sqrt{10^{n+1}+1}}}{\frac{4^n+3^{n+2}}{\sqrt{10^n+1}}} = \frac{4^{n+1}+3^{n+3}}{\sqrt{10^{n+1}+1}} \cdot \frac{\sqrt{10^n+1}}{4^n+3^{n+2}}$

$= \frac{(4^{n+1}+3^{n+3})\sqrt{10^n+1}}{(4^n+3^{n+2})\sqrt{10^{n+1}+1}} = \frac{(4^{n+1}+3^{n+3})\sqrt{10^n+1} \,/\, 4^n}{(4^n+3^{n+2})\sqrt{10^{n+1}+1} \,/\, 4^n}$

$= \frac{(4 + 3^{n+3}/4^n)\sqrt{10^n+1}}{\left(1+\frac{3^{n+2}}{4^n}\right)\sqrt{10^{n+1}+1}} = \frac{4 + \frac{3}{4}^n \cdot 27 \sqrt{10^n+1}}{(1+(3/4)^n \cdot 9)\sqrt{10^{n+1}+1}}$

$$= \frac{\left(4+\left(\frac{3}{4}\right)^n \cdot 27\right)\sqrt{10^n+1} \Big/ \sqrt{10^n}}{\left(1+\left(\frac{3}{4}\right)^n \cdot 9\right)\sqrt{10^{n+1}+1} \Big/ \sqrt{10^n}}$$

$$= \frac{\left(4+\left(\frac{3}{4}\right)^n \cdot 27\right)\sqrt{1+\frac{1}{\sqrt{10^n}}}}{\left(1+\left(3/4\right)^n \cdot 9\right)\sqrt{10+1/10^n}} = \frac{(4+0)\sqrt{1+0}}{(1+0)\sqrt{10+0}} \text{ as } n \Rightarrow \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges } a_n = \frac{1}{n} \Rightarrow \left|\frac{a_{n+1}}{a_n}\right| = \frac{1/(n+1)}{1/n} = \frac{n}{n+1} =$$

$$= \frac{n/n}{(n+1)/n} = \frac{1}{1+\frac{1}{n}} \Rightarrow \frac{1}{1+0} = 1$$

$$\frac{1}{n} \to \frac{1}{\infty} = 0 \qquad \text{as } n \Rightarrow \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ converges } b_n = \frac{1}{n^2} \Rightarrow \left|\frac{b_{n+1}}{b_n}\right| = \frac{\frac{1}{(n+1)^2}}{1/n^2} =$$

$$= \frac{n^2}{(n+1)^2} = \frac{n^2/n^2}{(n+1)^2/n^2} = \frac{1}{(1+1/n)^2} \to \frac{1}{(1+0)^2} = 1$$

so it can Diverge or Converge

if the limit of ratios is 1.