

# Syllabus

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|-------------|---------------------------------------|
| Title       | Real Analysis I                       |
| Number      | MATH 5305-261                         |
| Time        | MW 5:30–6:45                          |
| Place       | BH 224                                |
| Instructor  | David Milovich                        |
| Email       | david.milovich@tamiu.edu              |
| Phone       | (956) 326-2570                        |
| Office      | BVC 321                               |
| Hours       | MTWR 11:00–11:45, 1:45–2:30           |
| Department  | Engineering, Mathematics, and Physics |
| College     | Arts and Sciences                     |
| Institution | Texas A&M International University    |
| Term        | Spring 2013                           |

**Course description.** This is a course on Lebesgue measure and integration. The classical  $L^p$  spaces will be defined and basic results established, such as the Holder and Minkowski inequalities and completeness of the spaces. Prerequisites: Graduate standing and permission of instructor.

**Student learning outcomes.** Upon successful completion of this course, the student will be able to:

- explain the concepts of measurable sets, measurable function and measure;
- define the concept of Lebesgue Measure and Lebesgue Integral;
- explain the difference between the Lebesgue Integral and the Riemann Integral;
- distinguish between uniform convergence and pointwise convergence, and apply the Lebesgue dominated convergence theorem to prove convergence of integrals and the continuity of basic integral transforms, such as the Fourier Transform;
- explain the definition of  $L^p$  spaces, the Riesz representation theorem, and the completeness of the  $L^p$  spaces;
- explain the definition of the derivative of a measurable function; and
- explain how to construct measure theory in spaces other than the real line.

**Textbook.** Required: *Real & Complex Analysis* (1987). 3rd edition. Walter Rudin. McGraw-Hill. ISBN: 0-07-054234-1.

**Homework.** Homework is worth 40% of your grade and will be roughly weekly. This grade is based on completeness, not correctness, though I will comment where I see problems.

**Tests.** The two take-home tests (see the schedule below) are each worth 20% of your grade; the take-home final exam is also worth 20% of your grade. The final will be comprehensive, though emphasizing topics covered after the second test. Tests are open-book and open-note, but must be your own work. That is, you may consult written, printed, and electronic resources, but you may not ask another person, either directly or indirectly (such as through an internet forum) questions for your test.

**Grading.**

As this is a graduate course, I will give A's for very good work, B's for satisfactory work, and C's or worse for unsatisfactory work.

### Approximate Schedule of Topics

| Date   | Chapter | Book section   | Notes            |
|--------|---------|--|------------------|
| 23-Jan |         | Introduction   |                  |
| 28-Jan | 1       | Set-theoretic notation and terminology                             |                  |
| 30-Jan | 1       | The concept of measurability                                       |                  |
| 4-Feb  | 1       | Simple functions; Arithmetic in $[0, \infty]$                      |                  |
| 6-Feb  | 1       | Elementary properties of measures                                  |                  |
| 11-Feb | 1       | Integration of positive functions                                  |                  |
| 13-Feb | 1       | Integration of complex functions                                   |                  |
| 18-Feb | 1       | The role played by sets of measure zero                            |                  |
| 20-Feb | 2       | Vector spaces  |                  |
| 25-Feb | 2       | Topological preliminaries  |                  |
| 27-Feb | 2       | The Riesz representation theorem                                   | Test I assigned  |
| 4-Mar  | 2       | Regularity properties of Borel measures                            | Test I due       |
| 6-Mar  | 2       | Lebesgue measure   |                  |
| 18-Mar | 2       | Continuity properties of measurable functions                      |                  |
| 20-Mar | 3       | Convex functions and inequalities; the $L^p$ -spaces               |                  |
| 25-Mar | 3       | Approximation by continuous functions                              |                  |
| 27-Mar | 6       | Total variation  |                  |
| 1-Apr  | 6       | Absolute continuity; consequences of Radon-Nikodym Theorem         |                  |
| 3-Apr  | 6       | Bounded linear functionals on $L^p$ ; Riesz representation theorem |                  |
| 8-Apr  | 7       | Derivatives of measures  |                  |
| 10-Apr | 7       | The fundamental theorem of Calculus                                |                  |
| 15-Apr | 8       | Measurability on cartesian products                                |                  |
| 17-Apr | 8       | Product measures   | Test II assigned |
| 22-Apr | 8       | The Fubini theorem   | Test II due      |
| 24-Apr | 8,9     | Convolutions; formal properties of Fourier transform               |                  |
| 29-Apr | 9       | The inversion theorem  |                  |
| 1-May  | 9       | The Plancherel theorem   |                  |
| 6-May  |         | Review   |                  |