

More Gauss-Jordan elimination: (4-3)

- Empty solution sets
- ~~Empty solution sets~~ Revising the goal.

$$\begin{cases} x + y + z = 3 \\ y + z + w = 4 \\ 3x - 2z = 1 \\ -2x + 2y + 4z + w = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 & 4 \\ 3 & 0 & -2 & 0 & 1 \\ -2 & 2 & 4 & 1 & 0 \end{bmatrix} R_3 - 3R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & -3 & -5 & 0 & -8 \\ -2 & 2 & 4 & 1 & 0 \end{bmatrix}$$

Goal:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & - \\ 0 & 1 & 0 & 0 & - \\ 0 & 0 & 1 & 0 & - \\ 0 & 0 & 0 & 1 & - \end{bmatrix}$$

$$R_4 + 2R_1 \rightarrow R_4$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & -3 & -5 & 0 & -8 \\ 0 & 4 & 6 & 1 & 6 \end{bmatrix} R_1 - R_2 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & -3 & -5 & 0 & -8 \\ 0 & 4 & 6 & 1 & 6 \end{bmatrix} R_3 + 3R_2 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & -2 & 3 & 4 \\ 0 & 4 & 6 & 1 & 6 \end{bmatrix} R_4 - 4R_2 \rightarrow R_4$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & -2 & 3 & 4 \\ 0 & 0 & 2 & -3 & -10 \end{bmatrix} -R_3/2 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & 1 & -3/2 & -2 \\ 0 & 0 & 2 & -3 & -10 \end{bmatrix} R_2 - R_3 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 5/2 & 6 \\ 0 & 0 & 1 & -3/2 & -2 \\ 0 & 0 & 2 & -3 & -10 \end{bmatrix} R_4 - 2R_3 \rightarrow R_4$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 5/2 & 6 \\ 0 & 0 & 1 & -3/2 & -2 \\ 0 & 0 & 0 & 0 & -6 \end{bmatrix}$$

$$0x + 0y + 0z + 0w = -6$$

$$0 = -6$$

No solutions.

$$\begin{bmatrix} 3x_1 + x_2 - 2x_3 + x_4 - 5x_5 = 1 \\ -3x_1 - x_2 + x_4 = 0 \\ 4x_3 - x_4 + x_5 = 3 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{3} & 1 & -2 & 1 & -5 & 1 \\ -3 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & -1 & 1 & 3 \end{bmatrix}$$

$R_1/3 \rightarrow R_1$

Goal: $\begin{bmatrix} 1 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & - & - & - \\ 0 & 0 & 1 & - & - & - \end{bmatrix}$

$$\begin{bmatrix} 1 \checkmark & 1/3 & -2/3 & 1/3 & -5/3 & 1/3 \\ \textcircled{-3} & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & -1 & 1 & 3 \end{bmatrix}$$

$R_2 + 3R_1 \rightarrow R_2$

$$\begin{bmatrix} 1 \checkmark & 1/3 & -2/3 & 1/3 & -5/3 & 1/3 \\ 0 \checkmark & \textcircled{0} & \textcircled{-2} & 2 & -5 & 1 \\ 0 \checkmark & 0 & 4 & -1 & 1 & 3 \end{bmatrix}$$

Can't get 1 without damaging 1st column

Goal: $\begin{bmatrix} 1 & 0 & 0 & - & - \\ 0 & 0 & 1 & 0 & - \\ 0 & 0 & 0 & 1 & - \end{bmatrix}$

$-R_2/2 \rightarrow R_2$

$$\begin{bmatrix} 1 & \sqrt{3} & -2/3 & \sqrt{3} & -5/3 & \sqrt{3} \\ 0 & 0 & 1 & -1 & +5/2 & -1/2 \\ 0 & 0 & 4 & -1 & 1 & 3 \end{bmatrix}$$

$$R_1 + \frac{2}{3}R_2 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 1/3 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & -1 & 5/2 & -1/2 \\ 0 & 0 & 4 & -1 & 1 & 3 \end{bmatrix}$$

$$R_3 - 4R_2 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 1/3 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & -1 & 5/2 & -1/2 \\ 0 & 0 & 0 & 3 & -9 & 5 \end{bmatrix}$$

$$R_3 / 3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 1/3 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & -1 & 5/2 & -1/2 \\ 0 & 0 & 0 & 1 & -3 & 5/3 \end{bmatrix}$$

~~$$R_1 + \frac{1}{3}R_3 \rightarrow R_1$$~~

$$\begin{bmatrix} 1 & 1/3 & 0 & 0 & -1 & 5/9 \\ 0 & 0 & 1 & -1 & 5/2 & -1/2 \\ 0 & 0 & 0 & 1 & -3 & 5/3 \end{bmatrix}$$

$$R_2 + R_3 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 1/3 & 0 & 0 & -1 & 5/9 \\ 0 & 0 & 1 & 0 & -1/2 & 7/6 \\ 0 & 0 & 0 & 1 & -3 & 5/3 \end{bmatrix}$$

$$\left\{ \begin{array}{l} \textcircled{x_1} + \frac{1}{3}x_2 \\ \textcircled{x_3} \\ \textcircled{x_4} - 3x_5 = 5/3 \end{array} \right. \quad \begin{array}{l} -x_5 = 5/9 \\ -\frac{1}{2}x_5 = 7/6 \end{array}$$

$$\left\{ \begin{array}{l} x_1 = -\frac{1}{3}x_2 + x_5 + 5/9 \\ x_3 = (\frac{1}{2})x_5 + 7/6 \\ x_4 = 3x_5 + 5/3 \end{array} \right.$$

x_2 & x_5 are free:

$$\left\{ \begin{array}{l} x_1 = -\frac{1}{3}s + t + 5/9 \\ x_2 = s \text{ arbitrary} \\ x_3 = (\frac{1}{2})t + 7/6 \\ x_4 = 3t + 5/3 \\ x_5 = t \text{ arbitrary} \end{array} \right\} \begin{array}{l} \text{Solution} \\ \text{set} \end{array}$$

Example: $s = 1$, $t = -1$:

$$(x_1, x_2, x_3, x_4, x_5) = \left(-\frac{1}{3}, 1, \frac{2}{3}, -4/3, -1\right)$$

HW

Solve these systems:

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 + 3x_2 + 4x_3 = 0 \\ 3x_1 + 4x_2 + 5x_3 = 2 \end{cases}$$

$$\begin{cases} x_1 - x_2 + x_3 - x_4 + x_5 = 6 \\ x_1 - x_2 + x_3 - 2x_4 = 2 \\ x_4 + 3x_5 = -1 \end{cases}$$