

General goal of G-J elimination: (4-3)

A matrix in reduced form:

$$\begin{bmatrix} \boxed{1} & - & - & 0 & - & 0 & - & - & - \\ 0 & 0 & 0 & \boxed{1} & - & 0 & - & - & - \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} & - & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The first nonzero entry of a row is called a leading entry.

A matrix is in reduced form if:

- All leading entries are 1s.
- The leading entries go the right as you go down the rows.
- The rows without leading entries are at the bottom.
- The remainder of a column with a leading entry is all 0s.

Next test:

Monday

Oct 3

$$\begin{cases} 2x_1 + 3x_2 + x_3 + 9x_4 + x_5 + 15x_6 = 13 \\ 4x_1 + 9x_2 + 3x_3 + 27x_4 + x_5 + 41x_6 = 23 \\ 3x_1 + 6x_2 + x_3 + 14x_4 + \cancel{19x_5} + 19x_6 = 8 \\ 4x_1 + 9x_2 + 2x_3 + 23x_4 + x_5 + 34x_6 = 21 \end{cases}$$

$$\begin{bmatrix} 2 & 3 & 1 & 9 & 1 & 15 & 13 \\ 4 & 9 & 3 & 27 & 1 & 41 & 23 \\ 3 & 6 & 1 & 14 & 0 & 19 & 8 \\ 4 & 9 & 2 & 23 & 1 & 34 & 21 \end{bmatrix}$$

reduced form:

$$\begin{bmatrix} \boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 5/3 & 0 & 2 & 1 \\ 0 & 0 & \boxed{1} & 4 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & \boxed{1} & 2 & 8 \end{bmatrix}$$

x_1 x_2 x_3 x_4 x_5 x_6

← free
↑

$$x_1 = 0$$

$$x_2 + \frac{5}{3}x_4 + 2x_6 = 1$$

$$x_3 + 4x_4 + 7x_6 = 2$$

$$x_5 + 2x_6 = 8$$

Solution set:

$$\left\{ \begin{array}{l} x_1 = 0 \\ x_2 = 1 - \frac{5}{3}s - 2t \\ x_3 = 2 - 4s - 7t \\ x_4 = s \text{ arbitrary} \\ x_5 = 8 - 2t \\ x_6 = t \text{ arbitrary} \end{array} \right.$$

An example where you flip rows:

$$\begin{array}{rcl} & y + z & = 3 \\ x & + z & = 4 \\ x + y & & = 5 \end{array} \quad \begin{bmatrix} 0 & 1 & 1 & 3 \\ 1 & 0 & 1 & 4 \\ 1 & 1 & 0 & 5 \end{bmatrix}$$

Goal: $\begin{bmatrix} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \end{bmatrix}$

reduced form: $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

~~x = 3~~ $x = 3$

~~y = 2~~ $y = 2$

~~z = 1~~ $z = 1$

Applications

HW #65, 68, 77 (4-3)

- Define what your variable means.
- Write down your system of equations.
- Convert that system to an augmented matrix.
- Use a calculator/computer to get the reduced form matrix.
- Find the solution set for your variables.
- Answer the question. (#65, 68, 77).