

Row reduce  $\begin{bmatrix} \textcircled{0} & 7 & 2 & 4 \\ 1 & 5 & 3 & 6 \\ 2 & 0 & 1 & -2 \\ x & y & z = \end{bmatrix}$

Something like  $\begin{bmatrix} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \end{bmatrix}$

is the goal.

3 moves you can make:

- swap rows
- multiply/divide a row by a nonzero #
- add a multiple of a row to another row  
subtract

Strategy: ~~from~~ starting from left

column, get the 1, then get the zeros

swapping rows  
and/or dividing  
by a #

~~add~~ multiples  
subtract  
of the row  
with the 1  
From the other  
rows

$$\begin{bmatrix} 1 & \checkmark & 5 & 3 & 6 \\ 0 & \checkmark & 7 & 2 & 4 \\ \textcircled{2} & 0 & 1 & -2 & \end{bmatrix} R_3 - 2R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & \checkmark & 5 & 3 & 6 \\ 0 & \checkmark & \textcircled{7} & 2 & 4 \\ 0 & \checkmark & -10 & -5 & -14 \end{bmatrix} R_2 / 7 \rightarrow R_2$$

$$\begin{bmatrix} 1 & \checkmark & \textcircled{5} & 3 & 6 \\ 0 & \checkmark & 1 & 2/7 & 4/7 \\ 0 & \checkmark & -10 & -5 & -14 \end{bmatrix} R_1 - 5R_2 \rightarrow R_1$$

$$\begin{bmatrix} 1 & \checkmark & 0 & 11/7 & 22/7 \\ 0 & \checkmark & 1 & 2/7 & 4/7 \\ 0 & \checkmark & \textcircled{-10} & -5 & -14 \end{bmatrix} R_3 + 10R_2 \rightarrow R_3$$

$$\begin{bmatrix} 1 & \checkmark & 0 & 11/7 & 22/7 \\ 0 & \checkmark & 1 & 2/7 & 4/7 \\ 0 & \checkmark & 0 & \textcircled{-15/7} & -58/7 \end{bmatrix} -\frac{7}{15}R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & \checkmark & 0 & \textcircled{11/7} & 22/7 \\ 0 & \checkmark & 1 & 2/7 & 4/7 \\ 0 & \checkmark & 0 & 1 & 58/15 \end{bmatrix} R_1 - \frac{11}{7}R_3 \rightarrow R_1$$

$$\begin{bmatrix} 1 & \checkmark & 0 & 0 & -44/15 \\ 0 & \checkmark & 1 & \textcircled{2/7} & 4/7 \\ 0 & \checkmark & 0 & 1 & 58/15 \end{bmatrix} R_2 - \frac{2}{7}R_3 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -44/15 \\ 0 & 1 & 0 & -8/15 \\ 0 & 0 & 1 & 58/15 \end{bmatrix} \begin{array}{l} x = -44/15 \approx -2.933 \\ y = -8/15 \approx 0.5333 \\ z = 58/15 \approx 3.867 \end{array}$$

x      y      z

# #43 end of Ch. 4 (p. 248)

$x = \#$  of 3000 ft<sup>3</sup> hopper cars to lease

$y = \#$  — 4500

$z = \#$  — 6000

$$x + y + z = 20 \quad (\text{total \# of cars})$$

$$3000x + 4500y + 6000z = 108000 \quad (\text{total ft}^3)$$

$$\begin{bmatrix} 1 & 1 & 1 & 20 \\ 3000 & 4500 & 6000 & 108000 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & -12 \\ 0 & 1 & 2 & 32 \end{bmatrix}$$

$x \quad y \quad z =$

$$\left. \begin{array}{l} \textcircled{x} \\ \textcircled{y} \end{array} \right\} \left. \begin{array}{l} -z = -12 \\ +2z = 32 \end{array} \right\}$$

$z$  free

augmented matrix

↓ calculator

reduced form

↓ solution set

$$\begin{cases} x = z - 12 \\ y = 32 - 2z \\ z = t \text{ (arbitrary)} \end{cases}$$

$$\begin{cases} x = t - 12 \\ y = 32 - 2t \\ z = t \text{ (arbitrary)} \end{cases}$$

$$0 \leq x \text{ whole \#}$$

$$0 \leq y \text{ whole \#}$$

$$0 \leq z \text{ whole \#}$$

$$0 \leq t - 12 \Rightarrow 12 \leq t$$

$$0 \leq 32 - 2t \Rightarrow 2t \leq 32 \Rightarrow t \leq 16$$

$$0 \leq t$$

$t$	$=$	12	13	14	15	16
$x$	$=$	0	1	2	3	4
$y$	$=$	8	6	4	2	0
$z$	$=$	12	13	14	15	16

✓

$$\begin{cases} x = t - 12 \\ y = 32 - 2t \\ z = t = 12, 13, \dots, 16 \end{cases}$$

✓

$$(x, y, z) = (0, 8, 12), (1, 6, 13), (2, 4, 14), (3, 2, 15), (4, 0, 16)$$

✓