

$$\begin{bmatrix} 1 & 0 & 3 & 32 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x    y    z =

reduced  
form

Translate to the solution set:

$$\left. \begin{array}{l} 1x + 0y + 3z = 32 \\ 0x + 1y - 1z = 7 \\ 0x + 0y + 0z = 0 \end{array} \right\} \Rightarrow \begin{array}{l} \textcircled{x} + 3z = 32 \\ \textcircled{y} - z = 7 \\ 0 = 0 \end{array}$$

leading  
variables

z free

$$\begin{aligned} x &= 32 - 3z \\ y &= 7 + z \end{aligned}$$

$$\begin{cases} x = 32 - 3t \\ y = 7 + t \\ z = t \text{ (arbitrary)} \end{cases}$$

If we require  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ ,  
then this places restrictions on  $t$ :

$$0 \leq 32 - 3t \Rightarrow 3t \leq 32 \Rightarrow t \leq \frac{32}{3}$$

$$0 \leq 7 + t \Rightarrow -7 \leq t$$

$$0 \leq t$$

Restricted solution set:

$$\begin{cases} x = 32 - 3t \\ y = 7 + t \\ z = t \text{ where } 0 \leq t \leq \frac{32}{3} \end{cases}$$

If  $x, y, z$  have to be whole numbers,  
then  $t$  has to be a whole number:

$$\begin{cases} x = 32 - 3t \\ y = 7 + t \\ z = t \text{ where } t = 0, 1, 2, \dots, 10 \end{cases}$$

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~~#67~~ #67 (4-3)

see next page

$x = \#$  tank cars leased w/ 8K gal. capacity  
 $y = \#$  tank cars leased w/ 16K gal. capacity  
 $z = \#$  tank cars leased w/ 24K gal. capacity

$$x + y + z = 24 \quad (\text{total \# cars})$$

$$8000x + 16000y + 24000z = 520,000 \quad (\text{total \# gal})$$

$$\left[ \begin{array}{cccc|c} 1 & \checkmark & 1 & 1 & 24 \\ 8000 & 16000 & 24000 & & 520,000 \end{array} \right] R_2 - 8000R_1$$

$$\left[ \begin{array}{cccc|c} 1 & \checkmark & 1 & 1 & 24 \\ 0 & \checkmark & 8000 & 16000 & 328,000 \end{array} \right] \begin{array}{l} R_2 / 8000 \\ \downarrow \\ R_2 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & \checkmark & 1 & 1 & 24 \\ 0 & \checkmark & 1 & 2 & 41 \end{array} \right] R_1 - R_2 \rightarrow R_1$$

$$\left[ \begin{array}{cccc|c} 1 & \checkmark & 0 & -1 & -17 \\ 0 & \checkmark & 1 & 2 & 41 \end{array} \right] \text{reduced form}$$

$$\begin{array}{l} \textcircled{x} \\ \textcircled{y} \end{array} + \begin{array}{l} -z \\ +2z \end{array} = \begin{array}{l} -17 \\ 41 \end{array} \Rightarrow \begin{cases} x = z - 17 \\ y = 41 - 2z \end{cases}$$

leading                  free

$$\begin{cases} x = t - 17 \\ y = 41 - 2t \\ z = t \text{ (arbitrary)} \end{cases}$$

You can't lease fractional or negative #s of tanks cars.

$x, y, z \geq 0$  &  $t$  is whole #:

$$0 \leq t - 17 \Rightarrow 17 \leq t$$

$$0 \leq 41 - 2t \Rightarrow \cancel{41} 2t \leq 41 \Rightarrow t \leq 20.5$$

$$0 \leq t$$

$$\begin{cases} x = t - 17 \\ y = 41 - 2t \\ z = t \text{ where } t = 17, 18, 19, 20 \end{cases} \quad \checkmark$$

$$(x, y, z) = (0, 7, 17), (1, 5, 18), (2, 3, 19), (3, 1, 20) \quad \checkmark$$

$$\text{Cost} = 450x + 650y + 1150z \quad (\$/\text{month})$$

(#69, 4-3)

$(x, y, z)$	Cost
$(0, 7, 17)$	24100 ← cheapest
$(1, 5, 18)$	24400
$(2, 3, 19)$	24700
$(3, 1, 20)$	25000

$$\begin{bmatrix} 1 & -5 & 0 & -8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x \quad y \quad z =$$

$$0 = 1$$

No solution

$$\begin{bmatrix} \textcircled{1} & -5 & 0 & 0 & -14 \\ 0 & 0 & \textcircled{1} & 0 & 8 \\ 0 & 0 & 0 & \textcircled{1} & 3 \end{bmatrix} \begin{array}{l} x_1 - 5x_2 = -14 \\ x_3 = 8 \\ x_4 = 3 \end{array}$$

$$x_1 \quad x_2 \quad x_3 \quad x_4 =$$

$$\begin{array}{l} \textcircled{x_1} - 5x_2 = -14 \Rightarrow x_1 = 5x_2 - 14 \\ \underbrace{\phantom{x_1 - 5x_2}}_{x_2 \text{ free}} \quad \textcircled{x_3} = 8 \\ \textcircled{x_4} = 3 \end{array}$$

$$x_1 = 5t - 14$$

$$x_2 = t \text{ (arbitrary)}$$

$$x_3 = 8$$

$$x_4 = 3$$

If  $x_1$  must be  $\geq 0$ , then

$$0 \leq 5t - 14 \Rightarrow 14 \leq 5t \Rightarrow \frac{14}{5} \leq t.$$

If  $x_2$  must be a whole #, then

$$3 \leq t.$$

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No HW due<sub>m</sub> on Monday or Wednesday

Test on Monday. (Bring calculator & 1 sheet of notes.)

~~Review~~ Practice with exercises from 4-3

and similar exercises at the end of chapter 4. Study the

lecture notes & HW solutions too.