Simplex method (6-2)

① Recall the "standard form" of a maximization problem (10/12; 6-1)

Maximize $5x_1 + 4x_2 = P$

subject to: $4x_1 + x_2 \le 28$, $2x_1 + x_2 \le 16$, $x_1 + x_2 \le 13$ } Problem constraints

$x_1, x_2 \ge 0$ } nonnegative constraints

① In standard form, ✓ (Crucially, $28, 16, 13 \ge 0$.)

② Add slack variables:

$$\begin{aligned} 4x_1 + x_2 + s_1 \quad\quad\quad\quad &= 28 \\ 2x_1 + x_2 \quad\quad + s_2 \quad\quad &= 16 \\ x_1 + x_2 \quad\quad\quad\quad + s_3 &= 13 \end{aligned}$$

$$x_1, x_2, s_1, s_2, s_3 \ge 0$$

③ Rearrange $P = 5x_1 + 4x_2$:

Standard form for maximization:

Maximize $P = c_1 x_1 + c_2 x_2 + c_3 x_3 + \cdots c_n x_n$

subject to constraints of the form

$a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots + a_n x_n \le b$,

where $b \ge 0$, and subject to the "nonnegative constraints": $x_1, x_2, \ldots, x_n \ge 0$

$$\begin{aligned}
4x_1 + x_2 + s_1 \qquad\qquad\qquad &= 28\\
2x_1 + x_2 \qquad + s_2 \qquad\qquad &= 16\\
x_1 + x_2 \qquad\qquad + s_3 \qquad &= 13\\
-5x_1 - 4x_2 \qquad\qquad\qquad + P &= 0
\end{aligned}$$

$$\begin{array}{c|c}
 & \begin{array}{cccccccc} x_1 & x_2 & s_1 & s_2 & s_3 & P & & \end{array} \\
\text{basic variables}\left\{\begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array}\right. \\ P &
\left[\begin{array}{cccccc|c}
4 & 1 & 1 & 0 & 0 & 0 & 28 \\
2 & 1 & 0 & 1 & 0 & 0 & 16 \\
1 & 1 & 0 & 0 & 1 & 0 & 13 \\
\hline
-5 & -4 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}$$

"Where we are": $x_1, x_2 = 0$, $P = 0$,

$s_1 = 28$, $s_2 = 16$, $s_3 = 13$

Pick direction that increases P "fastest":

4) Pick largest negative # in the bottom row (but not in the last column).

The column of what you picked is called the <u>pivot column</u>; the corresponding variable is called the <u>entering variable</u>.

4) Increase x_1 until we hit a constraint and can go no further:
(the entering variable)

5) We divide the entries in the last column by the entries in the pivot column:

$28/4 = 7$
$16/2 = 8$
$13/1 = 13$

If an entry in the pivot column is 0 or negative, then skip that row.

The row with the smallest quotient is called the <u>pivot row</u>.

	x_1	x_2	s_1	s_2	s_3	P	
pivot row s_1	4	1	1	0	0	0	28
s_2	2	1	0	1	0	0	16
s_3	1	1	0	0	1	0	13
P	-5	-4	0	0	0	1	0

pivot column

pivot

s_1 is the <u>exiting variable</u>

We're going to "go" to the corner where $x_1 = 7$ & $s_1 = 0$

To do this on the matrix, we do what's called the pivot operation:

(6) Turn the pivot into a $\underline{1}$ by dividing the pivot row by 4.

(7) Then turn the rest of the pivot column into 0's by adding multiples of the pivot row to the other rows. (Do not swap rows.)

$R_1/4 \to R_1$

$$\begin{bmatrix} 1 & 1/4 & 1/4 & 0 & 0 & 0 & 7 \\ 2 & 1 & 0 & 1 & 0 & 0 & 16 \\ 1 & 1 & 0 & 0 & 1 & 0 & 13 \\ -5 & -4 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 - 2R_1 \to R_2$

$R_3 - R_1 \to R_3$

$R_4 + 5R_1 \to R_4$

$$
\begin{array}{cc|cccccc|c}
 & & x_1 & x_2 & s_1 & s_2 & s_3 & P & \\
\text{new basic variables} & x_1 & 1 & 1/4 & 1/4 & 0 & 0 & 0 & 7 \\
 & s_2 & 0 & 1/2 & -1/2 & 1 & 0 & 0 & 2 \\
 & s_3 & 0 & 3/4 & -1/4 & 0 & 1 & 0 & 6 \\
 & P & 0 & -11/4 & 5/4 & 0 & 0 & 1 & 35
\end{array}
$$

(7) We are at: $x_1 = 7$, $x_2 = 0$

$s_1 = 0$, $s_2 = 2$, $s_3 = 6$, $P = 35$

(8) Now repeat steps (4) & (5) & (6) & (7):
Next you would ~~[illegible]~~ have x_2 enter & s_2 exit...

$$
\begin{cases}
7/(1/4) = 28 \\
2/(1/2) = 4 \\
6/(3/4) = 8
\end{cases}
$$

(No HW ~~[illegible]~~ assigned today.)

Monday 10/17 Video lecture
Wednesday 10/19 Video lecture } HW will be assigned
Friday 10/21 Fall break
Monday 10/24 Normal class (HW due)