Simplex method (part III)

HW due Monday:

(6-2) #1, 2, 3, 4 ← (these are quick)

#15, 43, 51

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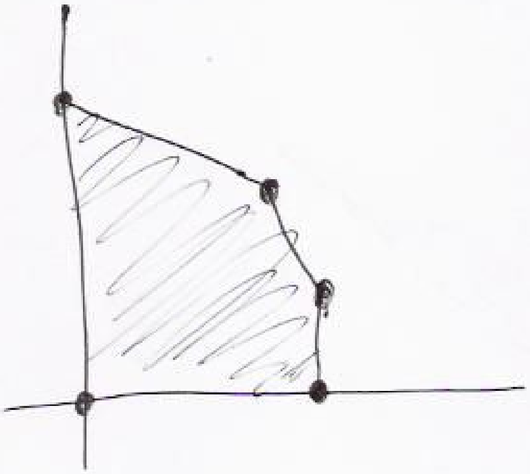
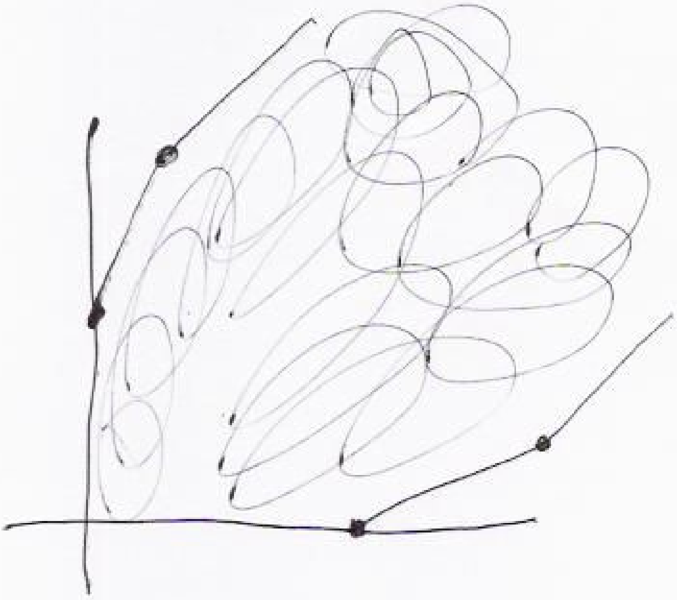


In a standard maximization ~~optimization~~ problem with <u>bounded</u> feasible set, there is a maximum at a corner.



If the feasible set is unbounded, there might not be a maximum.

Formulas for constraints:

$$2a + 3b + 2c \leq 1000$$
$$a + b + 2c \leq 800$$
$$a, b, c \geq 0$$

Introduce slack variables:
Also: rearrange objective equation

$$\begin{aligned} 2a + 3b + 2c + s_1 \quad\quad\quad &= 1000 \\ a + b + 2c \quad\quad + s_2 \quad &= 800 \\ -7a - 8b - 10c \quad\quad\quad + P &= 0 \end{aligned}$$

$$a, b, c, s_1, s_2 \geq 0$$

$$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & s_1 & s_2 & P & = \end{array} \\ \left[\begin{array}{cccccc|c} 2 & 3 & 2 & 1 & 0 & 0 & 1000 \\ 1 & 1 & 2 & 0 & 1 & 0 & 800 \\ \hline -7 & -8 & \boxed{-10} & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ 1000/2 = 500 \\ 800/2 = 400 \\ \text{smallest} \end{array}$$

−10 most negative from bottom row except last

$R_2/2 \to R_2$:

$$\begin{array}{c} \\ s_1 \\ \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & s_1 & s_2 & P & = \end{array} \\ \left[\begin{array}{cccccc|c} 2 & 3 & 2 & 1 & 0 & 0 & 1000 \\ 1/2 & 1/2 & 1 & 0 & 1/2 & 0 & 400 \\ \hline -7 & -8 & -10 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$R_1 - 2R_2 \to R_1;\ R_3 + 10R_2 \to R_3$

$$
\begin{array}{c|c}
 & \begin{array}{cccccc|c} a & b & c & s_1 & s_2 & P & = \end{array} \\
\begin{array}{c} s_1 \\ c \\ P \end{array} &
\left[\begin{array}{cccccc|c}
1 & 2 & 0 & 1 & -1 & 0 & 200 \\
1/2 & 1/2 & 1 & 0 & 1/2 & 0 & 400 \\
\hline
-2 & -3 & 0 & 0 & 5 & 1 & 4000
\end{array}\right]
\end{array}
\quad
\begin{array}{l}
\frac{200}{2} = 100 \\
\frac{400}{1/2} = 800
\end{array}
$$

$-3$ most negative

$R_1/2 \to R_1$

$$
\begin{array}{c|c}
 & \begin{array}{cccccc|c} a & b & c & s_1 & s_2 & P & = \end{array} \\
\begin{array}{c} b \\ c \\ P \end{array} &
\left[\begin{array}{cccccc|c}
1/2 & 1 & 0 & 1/2 & -1/2 & 0 & 100 \\
1/2 & 1/2 & 1 & 0 & 1/2 & 0 & 400 \\
\hline
-2 & -3 & 0 & 0 & 5 & 1 & 4000
\end{array}\right]
\end{array}
$$

$R_2 - \frac{1}{2}R_1 \to R_2;\ R_3 + 3R_1 \to R_3$

smallest

$$
\begin{array}{c|c}
\begin{array}{c} b \\ c \\ P \end{array} &
\left[\begin{array}{cccccc|c}
1/2 & 1 & 0 & 1/2 & -1/2 & 0 & 100 \\
1/4 & 0 & 1 & -1/4 & 3/4 & 0 & 350 \\
\hline
-1/2 & 0 & 0 & 3/2 & 7/2 & 1 & 4300
\end{array}\right]
\end{array}
\quad
\begin{array}{l}
\frac{100}{1/2} = 200 \\
\frac{350}{1/4} = 1400
\end{array}
$$

$-1/2$ biggest negative

$2R_1 \to R_1$

$$
\begin{array}{c|c}
 & \begin{array}{cccccccc} a & b & c & s_1 & s_2 & P & = & \end{array} \\
\begin{array}{c} \\ c \\ P \end{array} &
\left[\begin{array}{cccccc|c}
1 & 2 & 0 & 1 & -1 & 0 & 200 \\
1/4 & 0 & 1 & -1/4 & 3/4 & 0 & 350 \\
\hline
-1/2 & 0 & 0 & 3/2 & 7/2 & 1 & 4300
\end{array}\right]
\end{array}
$$

$R_2 - \frac{1}{4}R_1 \to R_2$; $R_3 + \frac{1}{2}R_1 \to R_3$

$$
\begin{array}{c|c}
 & \begin{array}{ccccccc} a & b & c & s_1 & s_2 & P & \end{array} \\
\begin{array}{c} a \\ c \\ P \end{array} &
\left[\begin{array}{cccccc|c}
1 & 2 & 0 & 1 & -1 & 0 & 200 \\
0 & -1/2 & 1 & -1/2 & 1 & 0 & 300 \\
\hline
0 & 1 & 0 & 2 & 3 & 1 & 4400
\end{array}\right]
\end{array}
$$

↑ no negatives.

Making 200 of component A & 300 of component C maximizes profit at \$4400.