(6-3) Dual Problem HW # 10, 51, 53

Minimization: #45 (6-3)

Minimize $C =$ ~~cost~~ cost per day

$$C = 70x_1 + 75x_2 + 90x_3$$

x_1 = # hours to operate Cedarburg plant

x_2 = # hours to operate Grafton plant

x_3 = # hours to operate West Bend plant

subject to the constraints:

$$x_1, x_2, x_3 \geq 0$$

$$20x_1 + 10x_2 + 20x_3 \geq 300$$

↑ gallons reg. ice cream per day

deluxe: $10x_1 + 20x_2 + 20x_3 \geq 200$

Standard form for minimization

$$20x_1 + 10x_2 + 20x_3 \geq 300$$
$$10x_1 + 20x_2 + 20x_3 \geq 200$$
$$70x_1 + 75x_2 + 90x_3 = C$$

$$
\begin{bmatrix}
20y_1 + 10y_2 \le 70 \\
10y_1 + 20y_2 \le 75 \\
20y_1 + 20y_2 \le 90 \\
300y_1 + 200y_2 = P
\end{bmatrix}
$$

Dual Problem

maximize

$$
\begin{bmatrix}
20y_1 + 10y_2 + x_1 = 70 \\
10y_1 + 20y_2 + x_2 = 75 \\
20y_1 + 20y_2 + x_3 = 90 \\
-300y_1 - 200y_2 + P = 0
\end{bmatrix}
$$

20	10	1	0	0	0	70	70/20 = 3.5
10	20	0	1	0	0	75	75/10 = 7.5
20	20	0	0	1	0	90	90/20 = 4.5
-300	-200	0	0	0	1	0	

$R_1/20 \to R_1$

1	1/2	1/20	0	0	0	7/2
10	20	0	1	0	0	75
20	20	0	0	1	0	90
-300	-200	0	0	0	1	0

$R_2 - 10R_1 \to R_2$

$R_3 - 20R_1 \to R_3$

$R_4 + 300R_1 \to R_4$

$$\begin{bmatrix} 1 & 1/2 & 1/20 & 0 & 0 & 0 & 7/2 \\ 0 & 15 & -1/2 & 1 & 0 & 0 & 45 \\ 0 & 10 & -1 & 0 & 1 & 0 & 20 \\ 0 & -50 & 15 & 0 & 0 & 1 & 1050 \end{bmatrix}$$

$(7/2)/(1/2) = 7$

$45/15 = 3$

$20/10 = 2$

$R_3/10 \to R_3$

$$\begin{bmatrix} 1 & 1/2 & 1/20 & 0 & 0 & 0 & 7/2 \\ 0 & 15 & -1/2 & 1 & 0 & 0 & 45 \\ 0 & 1 & -1/10 & 0 & 1/10 & 0 & 2 \\ 0 & -50 & 15 & 0 & 0 & 1 & 1050 \end{bmatrix}$$

$R_1 - \frac{1}{2}R_3 \to R_1$, $R_2 - 15R_3 \to R_2$, $R_4 + 50 \cdot R_3 \to R_4$

$$\begin{bmatrix} 1 & 0 & 1/10 & 0 & -1/20 & 0 & 5/2 \\ 0 & 0 & 1 & 1 & -3/2 & 0 & 15 \\ 0 & 1 & -1/10 & 0 & 1/10 & 0 & 2 \\ 0 & 0 & 10 & 0 & 5 & 1 & 1150 \end{bmatrix}$$

no negatives

Y_1 Y_2 | x_1 x_2 x_3 | P

Cost C is minimized at \$1150/day

with $x_1 = 10$ hours/day @ Cedarburg

and $x_3 = 5$ hours/day @ West Bend