Big M method (6-4) HW #18, 19

Reminder: Test Friday Nov. 4

Set-up:

If it's a minimization problem that you can't use dual method for (because of negative coefficients in the objective), then instead of minimizing C, maximize $P = -C$.

For constraints like

$$3x_1 + 4x_2 \le 5,$$

add slack: $3x_1 + 4x_2 + s_1 = 5$.

For constraints like

$$6x_1 + 7x_2 \ge 8,$$

add surplus & artificial variables:

$$6x_1 + 7x_2 - s_2 + a_2 = 8$$

For constraints like

$$9x_1 + 10x_2 = 11,$$

add an artificial variable:

$$9x_1 + 10x_2 + a_3 = 11$$

Add penalties to the objective function:

Example if trying to maximize $7x_1 - x_2$, then change the objective to maximizing

$$P = 7x_1 - x_2 - Ma_2 - Ma_3$$

$$x_1, x_2, s_1, s_2, a_2, a_3 \geq 0$$

$$3x_1 + 4x_2 + s_1 = 5$$

$$6x_1 + 7x_2 - s_2 + a_2 = 8$$

$$9x_1 + 10x_2 + a_3 = 11$$

$$-7x_1 + x_2 + Ma_2 + Ma_3 + P = 0$$

never negative

$$\begin{array}{ccccccccc} x_1 & x_2 & s_1 & s_2 & a_2 & a_3 & P & = & \\ \end{array}$$

$$\left[\begin{array}{ccccccc|c} 3 & 4 & 1 & 0 & 0 & 0 & 0 & 5 \\ 6 & 7 & 0 & -1 & \boxed{1} & 0 & 0 & 8 \\ 9 & 10 & 0 & 0 & 0 & \boxed{1} & 0 & 11 \\ -7 & 1 & 0 & 0 & M & M & 1 & 0 \end{array}\right]$$

Turn into 0s to make artificial variable basic

$R_4 - MR_2 \rightarrow R_4$; then $R_4 - MR_3 \rightarrow R_4$

Equivalently: $R_4 - MR_2 - MR_3 \rightarrow R_4$

New R_4: $[-7-15M, 1-17M, 0, M, 0, 0, 1, -19M]$

$$
\begin{array}{c|c}
 & \begin{matrix} x_1 & x_2 & s_1 & s_2 & a_2 & a_3 & P & = \end{matrix} \\
\begin{matrix} s_1 \\ a_2 \\ a_3 \\ P \end{matrix} &
\left[\begin{matrix} 3 & 4 & 1 & 0 & 0 & 0 & 0 & 5 \\ 6 & 7 & 0 & -1 & 1 & 0 & 0 & 8 \\ 9 & \boxed{10} & 0 & 0 & 0 & 1 & 0 & 11 \\ -7-15M & \boxed{1-17M} & 0 & M & 0 & 0 & 1 & -19M \end{matrix}\right]
\end{array}
\begin{matrix} 5/4 \\ 8/7 \\ 11/10 \\ \end{matrix}
$$

Now we can use the simplex method as usual.

M big $\Rightarrow 1-17M < -7-15M$

In general, if $p<q$ and M big compared to r, t, then

$r + pM < t + qM$.

$1 + (-17)M < 7 + (-15)M$

$R_3/10 \to R_3$

$$\left[\begin{array}{ccccccc|c} 3 & 4 & 1 & 0 & 0 & 0 & 0 & 5 \\ 6 & 7 & 0 & -1 & 1 & 0 & 0 & 8 \\ 9/10 & \boxed{1} & 0 & 0 & 0 & 1/10 & 0 & 11/10 \\ \hline -7-15M & 1-17M & 0 & M & 0 & 0 & 1 & -19M \end{array}\right]$$

$R_1 - 4R_3 \to R_1$; $R_2 - 7R_3 \to R_2$

$R_4 - (1-17M)R_3 \to R_4$

$$\begin{array}{cccccccc} x_1 & x_2 & s_1 & s_2 & a_2 & a_3 & P = & \\ \end{array}$$

$$\left[\begin{array}{ccccccc|c} -3/5 & 0 & 1 & 0 & 0 & -2/5 & 0 & 3/5 \\ -3/10 & 0 & 0 & -1 & 1 & -7/10 & 0 & 3/10 \\ 9/10 & 1 & 0 & 0 & 0 & 1/10 & 0 & 11/10 \\ \underbrace{-79/10 + (3/10)M}_{\text{positive}} & 0 & 0 & M & 0 & \underbrace{-\frac{1}{10} + \frac{17}{10}M}_{\text{positive}} & 1 & -11/10 - \frac{3}{10}M \end{array}\right]$$

$\boxed{-\frac{11}{10} - \frac{3}{10}M}$

(You can plug in, say, $M = 1000$ to cut down on the algebra.)

IF the simplex finishes but one or more artificial variables is basic, then there is no solution.
(The feasible set is empty in this case.)