Ch. 10 & 11 intro. to Calculus.

$\left.\begin{matrix}(10-1)\\(10-2)\end{matrix}\right\}$ Limits

$$APY = \left(1+\frac{r}{m}\right)^m - 1$$

compounded m times per year

$$APY = e^r - 1 \quad \text{continuously compounded}$$

$$e^r = \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^m.$$

As m gets large (and positive),

$\left(1+\frac{r}{m}\right)^m$ gets close to e^r,

in fact, as close as you want.

Example:
$r = .12 = 12\%$

m	$\left(1+\frac{.12}{m}\right)^m$
1	1.12
10	1.1266917779 ...
100	1.12741573961 ...
1000	1.12748873428 ...
10000	1.12749603979 ...
100000	1.1274967704 ...
1000000	1.12749684346 ...
∞	$e^{.12} = 1.12749685158...$
1,000,000,000	1.12749685157 ...
10,000,000,000	1.12749685158 ...

HW #1 Estimate $\lim\limits_{x\to\infty} \frac{3x+1}{\sqrt{2x^2+5}}$

and $\lim\limits_{x\to-\infty} \frac{3x+1}{\sqrt{2x^2+5}}$

$$\lim_{x \to 0} \left(\frac{2e^x - 2 - 2x - x^2}{x^3} \right)$$

What happens to $f(x) = \dfrac{2e^x - 2 - 2x - x^2}{x^3}$

as x gets close to 0?

$$f(0) = \frac{2\overbrace{e^0}^{1} - 2 - 2(0) - 0^2}{0^3} = \frac{0}{0} = ?$$

$f(x)$ is undefined at exactly $x = 0$.

$\lim_{x \to 0} f(x)$ is between $.333$ & $.334$

(It's actually $\frac{1}{3}$

because $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{2 \cdot 3} + \frac{x^4}{2 \cdot 3 \cdot 4} + \cdots$)

$$\lim_{x \to 0} \frac{\ln(x+1)}{x} = 1$$

because $\log_{10} x = \dfrac{\ln x}{\ln 10}$

$$\lim_{x \to 0} \frac{\log_{10}(x+1)}{x} = \frac{1}{\ln(10)}$$

At exactly $x=0$,

$$\frac{\ln(0+1)}{0} = \frac{\ln 1}{0} = \frac{0}{0}$$ ← undefined

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HW #2 Estimate $\lim\limits_{x \to 0} \frac{\ln(x^2+1)}{x}$

$$\lim_{x \to 0^+} \frac{\sqrt{x^2}}{x} = 1$$ (from the right)

$$\lim_{x \to 0^-} \frac{\sqrt{x^2}}{x} = -1$$ (from the left)

$\lim\limits_{x \to 0} \frac{\sqrt{x^2}}{x}$ does not exist (from both sides)

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$

x	$1/x$	$1/x^2$
.1	10	100
-.1	-10	100
.01	100	10000
-.01	-100	10000
.001	1000	1000000
-.001	-1000	1000000

HW #3 Estimate

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1}$$

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x + 1}$$

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1}$$

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x + 1}$$

$$\lim_{x \to 1^+} \frac{x^2 - 1}{(x - 1)^2}$$

$$\lim_{x \to 1^-} \frac{x^2 - 1}{(x - 1)^2}$$