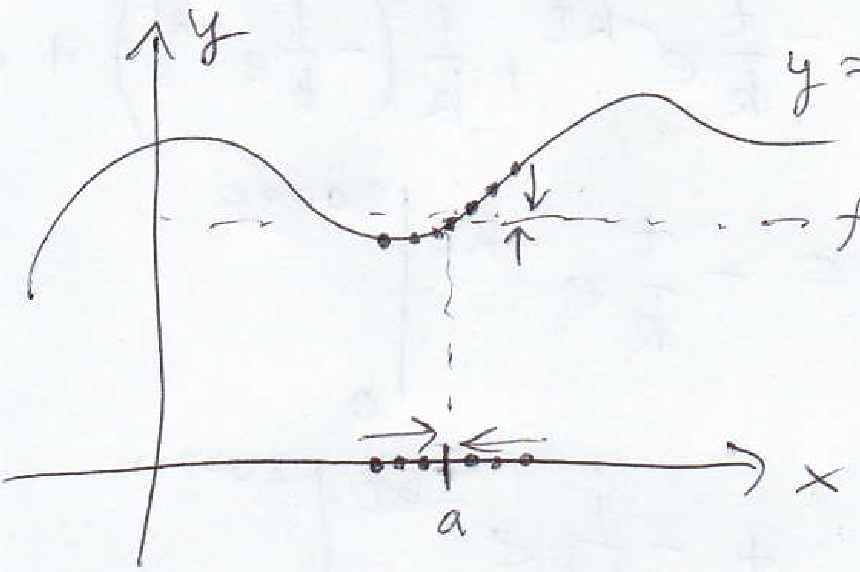
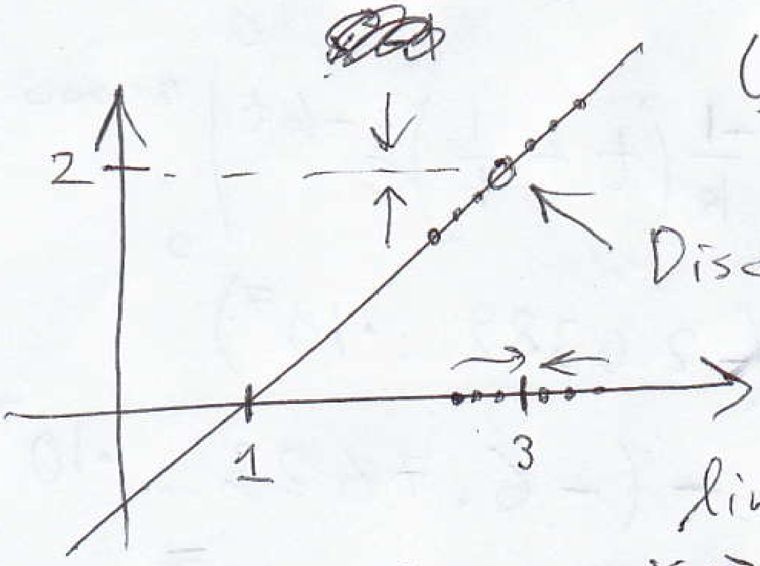
Continuity (10-3)

$f(x)$ is continuous at $x=a$

if $\lim_{x \to a} f(x) = f(a)$



$$\frac{(x-3)(x-1)}{x-3}$$



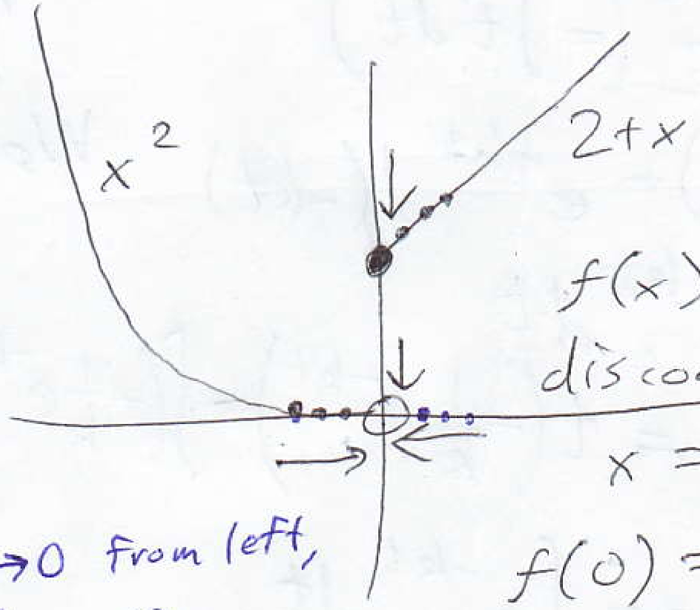
Discontinuous at $x=3$

$$\lim_{x \to 3} \frac{(x-3)(x-1)}{x-3} = 2$$

but $\frac{(3-3)(3-1)}{3-3}$ is undefined.

As x gets close to 3, y gets close to 2

$$f(x) = \begin{cases} 2+x & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$$



As $x \to 0$ From right, $y = 2+x \to 2$.

$f(x)$ is discontinuous at $x = 0$:

$f(0) = 2$ but $\lim_{x \to 0} f(x)$ does not exist

As $x \to 0$ from left, $y = x^2 \to 0$.

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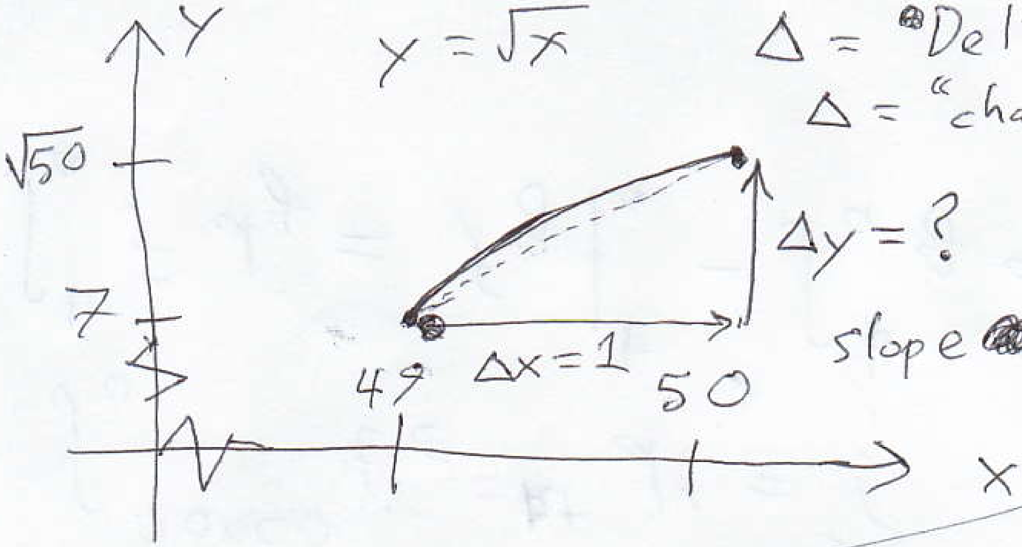
(10-4) Derivatives

(10-6) Differentials

Approximating curves by lines



Δ = Delta

Δ = "change in"

slope $= \frac{\Delta y}{\Delta x} \approx \frac{1}{2\sqrt{49}}$

$$\Delta y \approx \frac{\Delta x}{2\sqrt{49}} = \frac{1}{2 \cdot 7} = \frac{1}{14}$$

$$7.07106\ldots = \sqrt{50} \approx 7 + \frac{1}{14} = 7.07142\ldots$$

For small changes $\underset{(1)}{\Delta x}$ from $\underset{(49)}{x}$ to $\underset{(50)}{x + \Delta x}$

$$\Delta(\sqrt{x}) \approx \frac{\Delta x}{2\sqrt{x}}$$

$$\sqrt{50} - \sqrt{49} \approx \frac{1}{2\sqrt{49}}$$

Differentials: $\underbrace{d\sqrt{x}}_{\text{differential of } \sqrt{x}} = \frac{dx}{2\sqrt{x}}$ dx is the differential of x

Derivatives: $\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$ is the derivative of $\sqrt{x}$

another notation: $(\sqrt{x})'$

Estimate $\sqrt{96}$

$x = 100 \qquad y = \sqrt{100} = 10$

$\Delta x = -4 \qquad \Delta y = \sqrt{96} - \sqrt{100} = ?$

$x + \Delta x = 96 \qquad y + \Delta y = \sqrt{96} = ?$

$dx = \Delta x \qquad \Delta y \approx dy = \dfrac{dx}{2\sqrt{x}} = \dfrac{-4}{2\sqrt{100}}$

$\Delta y \approx \dfrac{-4}{2 \cdot 10} = \dfrac{-4}{20} = -\dfrac{1}{5}$

$9.7979589\ldots = \sqrt{96} = y + \Delta y \approx 10 - \dfrac{1}{5} = 9.8$

Error $dy - \Delta y \approx \dfrac{(\Delta x)^2}{8x\sqrt{x}} = \dfrac{2}{1000}$

Error close to $\dfrac{2}{1000}$

HW: Estimate $\sqrt{66}$ and $\sqrt{60}$ using differentials and compare to calculator estimates.

Formally, $d\sqrt{x} = \dfrac{dx}{2\sqrt{x}}$ means that the error $\varepsilon = \Delta(\sqrt{x}) - d\sqrt{x}$ is small enough that $\lim_{\Delta x \to 0} \dfrac{\varepsilon}{\Delta x} = 0$

For small Δx,

$\Delta\sqrt{x} - d\sqrt{x}$

is small compared to Δx.