Derivatives as slopes (10-4)

Δ: "Deltas" = changes

d: Differentials = approximations of these changes

$$\Delta(x^2) \approx \underbrace{2x\,dx}_{d(x^2)} \quad \text{with } \underbrace{\Delta x = dx}_{\substack{\Delta x \approx dx \\ \text{would be} \\ \text{good enough}}}$$

Derivatives:

$$(x^2)' = \frac{d(x^2)}{dx} = 2x$$

The technical definition of a differential is that as Δx gets close to 0, the error of the approximation (e.g. $d(x^2) - \Delta(x^2)$) from using the differential gets close to 0, even if you divide the error by Δx.

E.g. $\lim_{\Delta x \to 0} \frac{\Delta(x^2) - d(x^2)}{\Delta x} = 0$

The error $\Delta(x^2) - d(x^2)$ is small, even compared to small Δx.

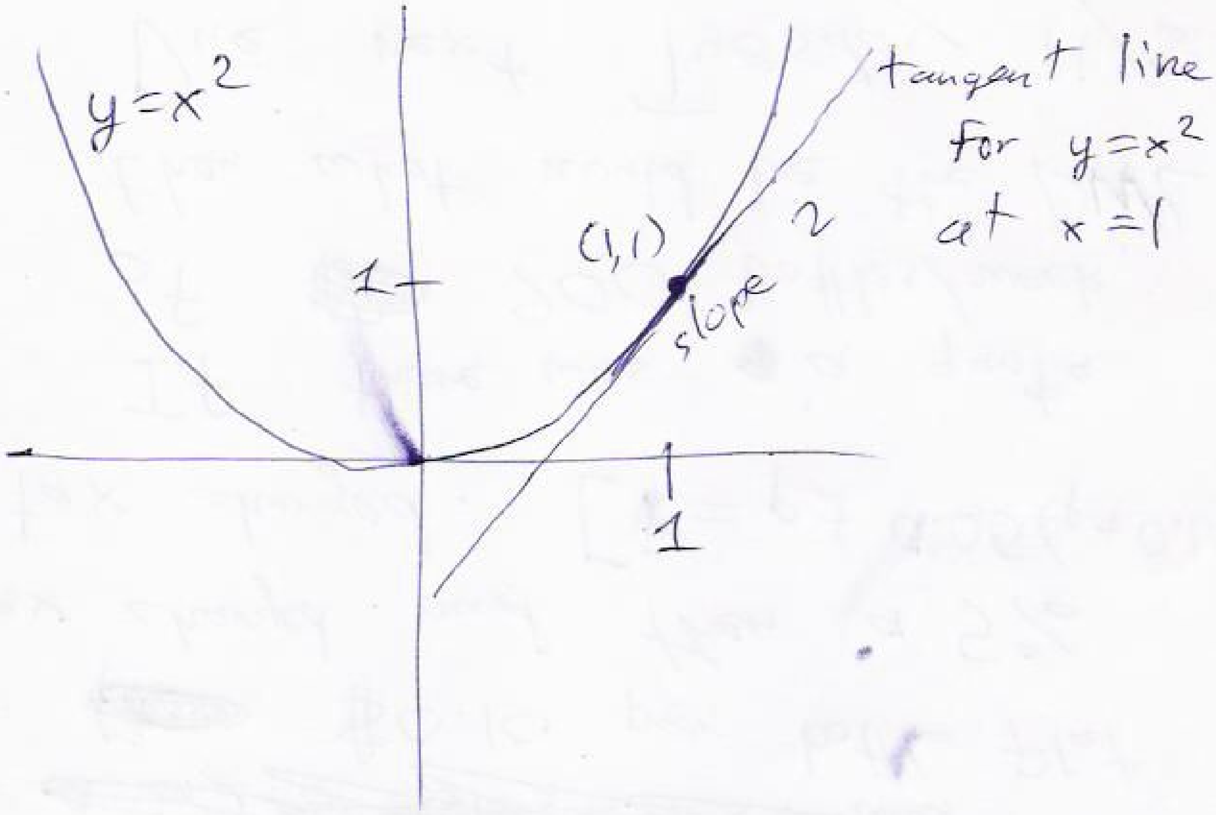
$$\lim_{\Delta x \to 0} \frac{\Delta(x^2)}{\Delta(x)} = \lim_{\Delta x \to 0} \frac{d(x^2)}{dx} = 2x$$

with $dx = \Delta x$

As the change in x gets close to 0, the rate of change of x^2 with respect to x $(\Delta(x^2)/\Delta x)$ gets close to $2x$.

x	$y=x^2$	$x+\Delta x$	Δx	$y+\Delta y$ $(x+\Delta x)^2$	Δy	$\Delta y/\Delta x$ slope
1	1	.9	−.1	.81	−.19	1.9
1	1	.99	−.01	.9801	−.0199	1.99
1	1	.999	−.001	.998001	−.001999	1.999
1	1	1.001	.001	1.002001	.002001	2.001
1	1	1.01	.01	1.0201	.0201	2.01
1	1	1.1	.1	1.21	.21	2.1

$2 = 2(1) = 2x$

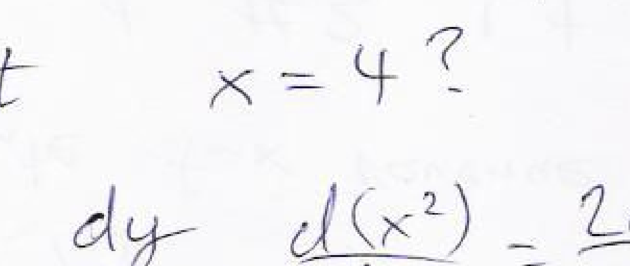


What is the slope of $y=x^2$ at $x=4$?

$$\frac{dy}{dx}=\frac{d(x^2)}{dx}=\frac{2x\,dx}{dx}=2x$$

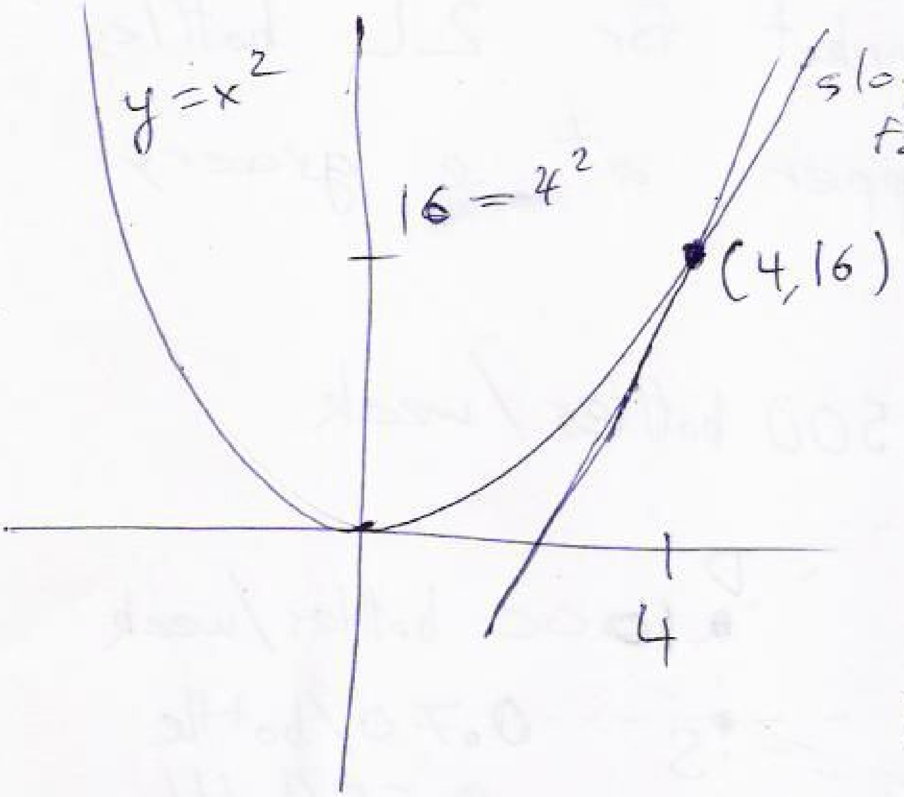
$2(4)=8$. Slope is 8.

Plotting lines given slope & 1 point:



$$m=\frac{\Delta y}{\Delta x}=\frac{y-y_0}{x-x_0}$$

$$y=y_0+m(x-x_0)$$



Equation for tangent line:

$$y=16+8(x-4)$$

$$y=y_0+m(x-x_0)$$

$$y=x^3-5x^2+x+1$$

What is the equation for the line tangent to this curve at $x=3$?

$$dy=d(x^3)-5d(x^2)+dx+d1$$
$$dy=3x^2dx-5(2x\,dx)+dx+0$$
$$d(x^n)=nx^{n-1}dx \quad [(x^n)'=nx^{n-1}]$$
$$y'=dy/dx=3x^2-10x+1$$

Now plug in $x=3$ to get slope.
(DO NOT plug in $x=3$ until after you compute dy/dx.)

$$y' = dy/dx = \text{slope} = 3(3)^2 - 10(3) + 1$$
$$= -2 \quad \text{at} \quad x = 3.$$

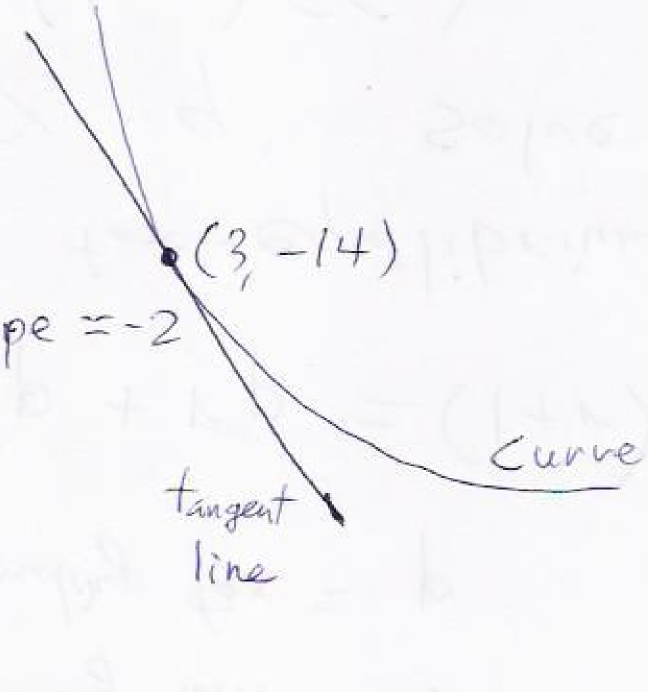
$$y = (3)^3 - 5(3)^2 + 3 + 1 \quad \text{at } x = 3$$
$$y = -14$$

Equation for tangent line is:

$$y = -14 - 2(x-3)$$

$$y = y_0 + m(x - x_0)$$

~~Actually picture more like:~~



HW #1: Find an equation for the line tangent to $y = 3\sqrt{x} - 1 + \frac{1}{x}$

at $x = 1$.

Sketch the curve & the tangent line.

#2: Repeat #1 with

$$y = \frac{x^3 + 1 - 2x}{\sqrt[3]{x}} \quad \text{at } x = 1$$