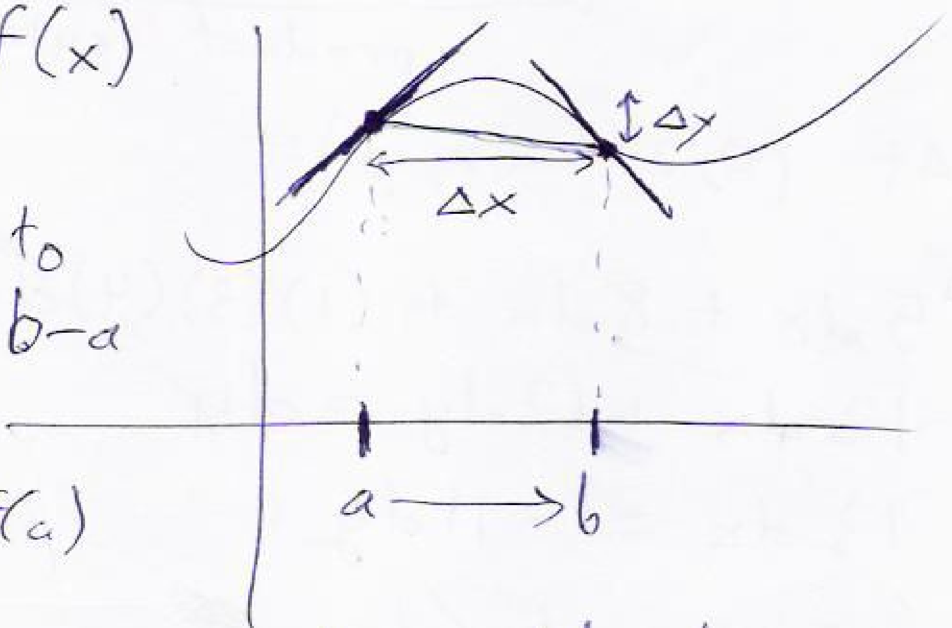
(10-4, 10-5) Derivatives as rates of change.

(Last time: derivatives as slopes.)

$y = f(x)$



From $x = a$ to $x = b$, $\Delta x = b - a$

and

$\Delta y = f(b) - f(a)$

Δ = change = new value − old value

The average rate of change of y with respect to x from $x = a$ to $x = b$ is $\Delta y / \Delta x = (f(b) - f(a))/(b - a)$

The rate of change of y with respect to x at $x = a$ is

dy/dx at $x = a$.

[Alternative notation $y' = dy/dx$]

y = height in feet

t = time in seconds

t	y
0	6
.612	0

$y(t)$ should be

$6 - 16t^2$

(from physics)

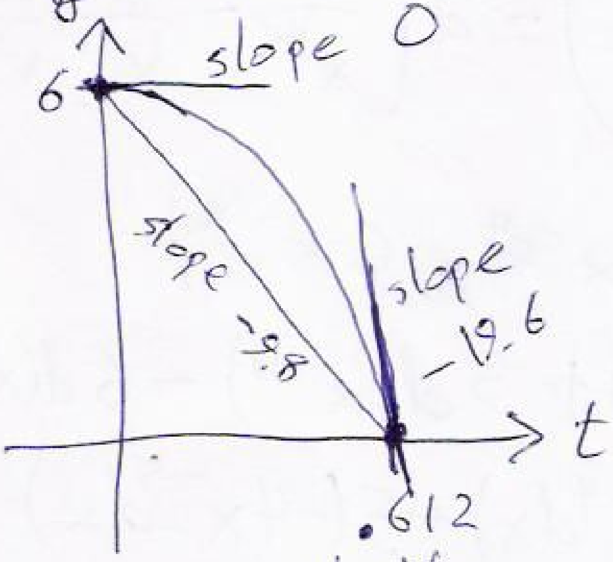
Solve $y = 0$ for t:

$0 = 6 - 16t^2$

$16t^2 = 6$

$t^2 = 3/8$

$t = \sqrt{3/8} \approx .612$



average ~~speed~~ velocity of fall = average rate of change of y with respect to t from $t = 0$ to $t = .612$ is

$\Delta y / \Delta t = (0 - 6)/(.612 - 0) = -9.80 \frac{ft}{s}$

$y = 6 - 16t^2 \Rightarrow dy = d6 - 16\, d(t^2)$

$dy = 0 - 16(2t\, dt) = -32t\, dt$

differential $y' = dy/dt = -32t$

derivative

t	y	dy/dt
0	6	0 ← initial velocity
.612	0	−19.6 ← velocity of impact

Another example: If t = time
& O = odometer reading on car
& S = speedometer reading on car,

then $S = \dfrac{dO}{dt}$

Suppose you drive 1000 miles in a month (say, 30 days), then

$\dfrac{\Delta O}{\Delta t}$ = average speed $= \dfrac{1000 \text{ miles}}{30 \cdot 24 \text{ hours}} = 1.3889$ mph

but $S = \dfrac{dO}{dt}$ was lots of different speeds at different times during the month.

HW: (10-5) # 83, 87, 89