Derivatives of e^x & $\ln x$ (11-2)

$$d(e^x) = e^x\,dx \qquad d(\ln x) = \frac{dx}{x}$$

$$(e^x)' = \frac{d(e^x)}{dx} = e^x \qquad (\ln x)' = \frac{d(\ln x)}{dx} = \frac{1}{x}$$

E.g. $e^{-0.1}$ can be approximated with differentials:

$x = 0$ $\qquad e^x = e^0 = 1$

$x + \Delta x = -0.1 \qquad e^x + \Delta(e^x) = e^{-0.1}$

$dx = \Delta x = -0.1 \qquad \Delta(e^x) = e^{-0.1} - e^0 = e^{-0.1} - 1$

$$\Delta(e^x) \approx d(e^x) = \boxed{e^x\,dx} = e^0(-.1)$$

$$\Delta(e^x) \approx (1)(-.1) = -0.1$$

$$e^{-0.1} \approx e^x + \Delta(e^x) = 1 - .1 = .90$$

$$e^{-0.1} \approx .9048374180\,36$$

(calculator)

$x = 1$ $\qquad \ln x = \ln 1 = 0$

$x + \Delta x = 1.3$ $\qquad \ln x + \Delta(\ln x) = \ln(1.3)$

$dx = \Delta x = .3$ $\qquad \Delta(\ln x) = \ln(1.3) - \ln(1)$

$$\Delta(\ln x) \approx d(\ln x) = \boxed{\frac{dx}{x}} = \frac{.3}{1} = .3$$

$$\ln 1.3 \approx \underbrace{\ln x}_{0} + \underbrace{d(\ln x)}_{.3} = .3$$

$$\ln 1.3 \approx .262364264467$$

Estimate gets better with smaller Δx:

$$\overset{.0300}{}$$

$$\ln(1.03) \approx .0295588022415 \approx .03$$

$$\ln(1.003) \approx .0029955089798 \approx .003$$

$$.003000$$

HW #1 Estimate $\ln(e + .04)$ using differentials.

Continuously compounded interest: $\qquad r$ = interest rate

$$\boxed{A = Pe^{rt}}$$

A = amount in account at time t (in year)

P = amount at time 0

What is the rate of change of A with respect to t when $t=2$? Let $P = \$10{,}000$ and $r = 2.1\% = .021$.

$A = Pe^{rt}$ $\qquad \frac{dA}{dt} = ?$ when $t=2$

$dA = d(Pe^{rt}) = P\,d(e^{rt}) = Pe^{rt}\,d(rt)$

P constant multiple

r constant multiple

$d(e^{x}) = e^{x}dx$

$\underbrace{dA}_{\text{Differential}} = Pe^{rt}r\,dt$ $\qquad \underbrace{\frac{dA}{dt}}_{\text{Derivative}} = Pe^{rt}r$

Plug in $t=2$ <u>after</u> compute dA.

At $t=2$: $dA/dt = (10{,}000)e^{(.021)2}(.021)$

$\frac{dA}{dt} \approx \underbrace{219.01 \text{ dollars/year}}$

≈ 0.60 dollars/day

$$d(e^{5x}) = e^{5x}\,d(5x) = 5e^{5x}\,dx$$

$$d\left(e^{(x^2)}\right) = e^{(x^2)}\,d(x^2) = e^{(x^2)}(2x\,dx)$$

$$d\left((e^x+1)^2\right) \underset{\uparrow}{=} 2(e^x+1)\underbrace{d(e^x+1)} = 2(e^x+1)e^x dx$$

$d(x^2) = 2x\,dx$

$$\boxed{d\left(e^{f(x)}\right) = e^{f(x)}\,d(f(x))} \quad \underbrace{d(e^x)}_{e^x dx} + \underbrace{d1}_{0}$$

Note that $(e^x)' = e^x$ but $(e^{5x})' \neq e^{5x}$

$(e^{5x})' = 5e^{5x}$

$$d(\ln(x^2+3)) \underset{\uparrow}{=} \frac{d(x^2+3)}{x^2+3} = \frac{2x\,dx + 0}{x^2+3}$$

$d(\ln x) = \frac{dx}{x}$

$$(\ln(x^2+3))' = \frac{d(\ln(x^2+3))}{dx} = \frac{2x}{x^2+3} \neq \frac{1}{x^2+3}$$

$$(\ln x)' = \frac{1}{x}$$

$$d((\ln x)^3) \underset{\uparrow}{=} 3(\ln x)^2 \underbrace{d(\ln x)} = 3(\ln x)^2 \frac{dx}{x}$$

$d(x^n) = nx^{n-1}dx$ $\quad$ dx/x

$d(kf) = k\,df \qquad d(f \pm g) = df \pm dg$

$\uparrow$ k constant $\qquad dk = 0$

$d(x^n) = nx^{n-1}dx$; $\boxed{d(e^x) = e^x dx}$; $\boxed{d(\ln x) = \frac{dx}{x}}$

$$\left.\begin{aligned} d(x^2) &= 2x\,dx \\ d(x^3) &= 3x^2 dx \\ d(\sqrt{x}) &= \frac{dx}{2\sqrt{x}} \end{aligned}\right] \text{Examples of power rule}$$

$$f'(x) = \frac{df}{dx}$$

HW #2 ~~Compute the~~

Find an equation for the line tangent to $y = (e^{3x} + \ln(x+2))^2$ at $x = 0$.