Marginal cost, revenue, profit, etc...

(10-7)

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Reminder: $d(f \pm g) = df \pm dg$

k constant $\Rightarrow d(kf) = k\,df$

$d(x^k) = kx^{k-1}dx$

$dk = 0$

$d(e^x) = e^x dx$

$d(\ln(x)) = \frac{dx}{x}$

$d(fg) = df \cdot g + f \cdot dg$

$d\left(\frac{f}{g}\right) = \frac{df \cdot g - f \cdot dg}{g^2}$

x = quantity　R = revenue

p = price　C = cost

P = proft

$MR = \frac{dR}{dx}$　$MC = \frac{dC}{dx}$

$MP = \frac{dP}{dx}$　M for "marginal"

10-7 #38 Example (not HW):

$x = 9{,}000 - 30p$　$C = 150{,}000 + 30x$

$R = px = (9000 - 30p)p = 9000p - 30p^2$

$\frac{dR}{dx} = MR = \frac{9000\,dp - 30(2p\,dp)}{0 - 30dp} = \frac{9000 - 60p}{-30}$

$$\frac{dR}{dx} = -300 + 2p = MR$$

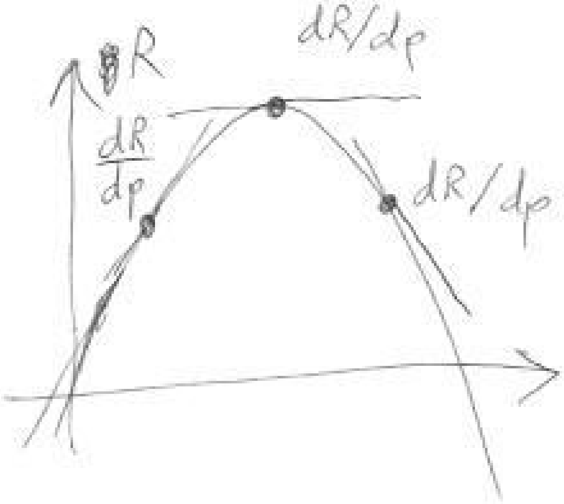
Alternative: $\boxed{x = 9000 - 30p} \Rightarrow x - 9000 = -30p$

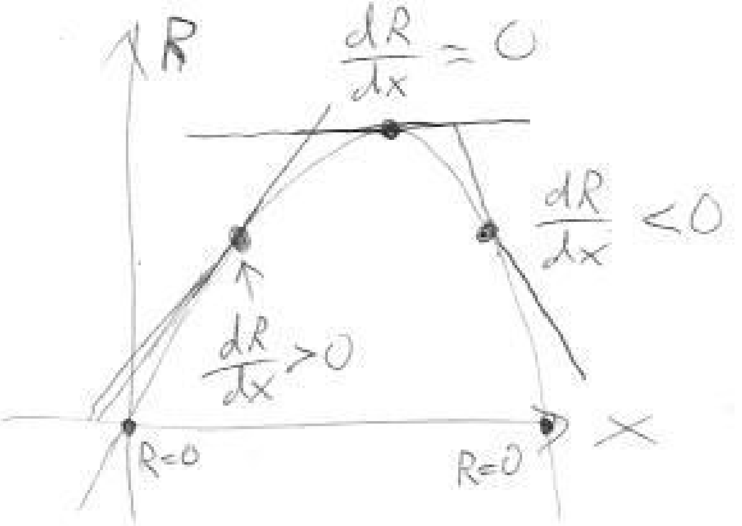
$$\Rightarrow \frac{x - 9000}{-30} = p \Rightarrow R = px = x\left(\frac{9000 - x}{30}\right)$$

$$R = 300x - \frac{x^2}{30} \Rightarrow \frac{dR}{dx} = \frac{300\,dx - 2x\,dx/30}{dx}$$

$$= \underbrace{300 - \frac{2x}{30}}_{= MR} = 300 - \frac{2(9000 - 30p)}{30}$$

$$= 300 - 600 + 2p = -300 + 2p$$




$$\frac{dR}{dp} = \frac{9000\,dp - 60p\,dp}{dp}$$

$$\frac{dR}{dx} = 300 - \frac{2x}{30}$$

$$\frac{dR}{dp} = 9000 - 60p$$

E.g. At $x=10$, $dR/dx = 300 - 20/30$,

so selling one more raises revenue by

$\approx 300 - 2/3$.

At $x=1000$, $dR/dx = 300 - 2000/30$,

so selling one more increases revenue by

$\approx 300 - 66.666\ldots \approx 233.33$

At $x=10000$, $dR/dx = 300 - 20000/30$,

so selling one more <u>decreases</u> revenue by

$-[300 - 666.66] \approx 366.66$

Solve $\frac{dR}{dx} = 300 - \frac{2x}{30} = 0$ : $9000 = 2x$

$x = 4500$ maximizes revenue.

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What about profit?

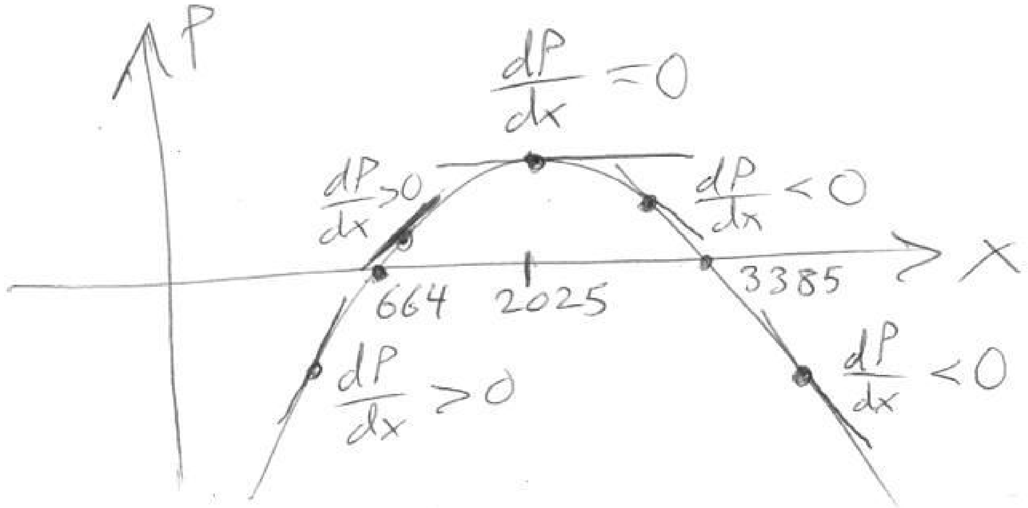
$x = 9000 - 30p \iff p = 300 - \frac{2x}{30} = 300 - \frac{x}{15}$

Cost $C = 150000 + 30x$.

Profit $P = R - C = px - C$

$P = \left(300 - \frac{x}{15}\right)x - 150000 - 30x$

$$P = 270x - \frac{x^2}{15} - 150000$$




$$\frac{dP}{dx} = \frac{270dx - 2x\,dx/15}{dx} = 270 - \frac{2x}{15}$$

Solve $\frac{dP}{dx} = 0$: $270 = \frac{2x}{15} \Rightarrow 2025 = x$

Profit maximized at $x = 2025$:

$$P = 270(2025) - \frac{(2025)^2}{15} - 150000$$
$$P = 123375$$

Where do we break even, in terms of $x$? Solve $P = 0$:

$$0 = 270x - \frac{x^2}{15} - 150000$$

$0 = ax^2 + bx + c$ $\quad a = -\frac{1}{15}$ $\quad b = 270$

$c = -150000$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 664.6, 3385.3$$

HW: Do #37 of (10-7)

and then find the price & quantity that maximize profit & the maximum profit.

Also: Do #43 of (10-7).

(You may need a calculator to estimate the answers to 43b.)