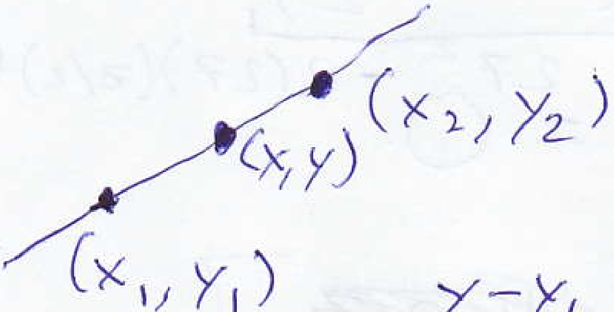
Today: Volume & double integrals

(15-6, 15-7)

Tuesday: Q&A / review

Thurs. 12/8: 11AM - 2PM final.

↳ (Bring calculator & 2 sheets of notes)



2 points determine a line

$$\frac{y-y_1}{x-x_1} = m = \underbrace{\frac{\Delta y}{\Delta x}}_{\Delta y = m\Delta x} = \frac{y_2-y_1}{x_2-x_1}$$

$y-y_1 = m(x-x_1)$

(Like $dy = f'(x)dx$)

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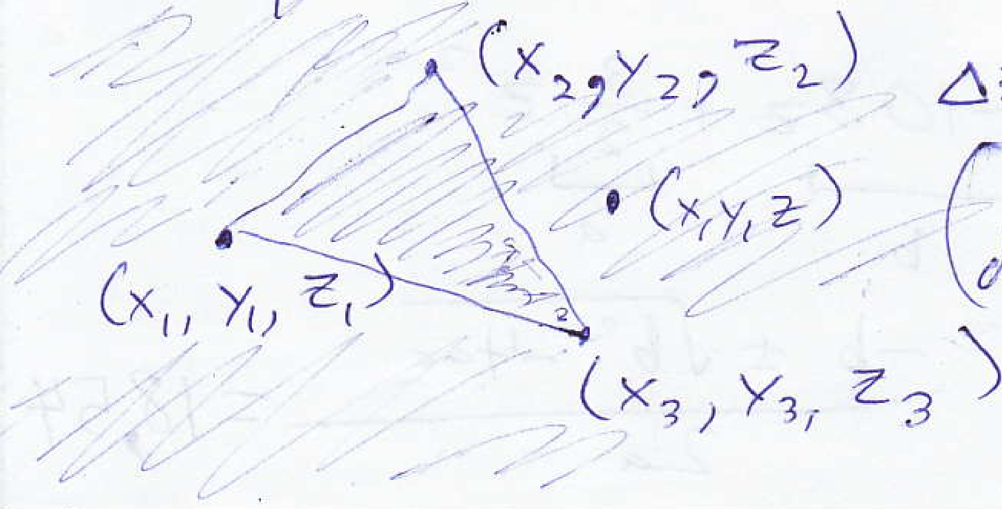
3 points determine a plane

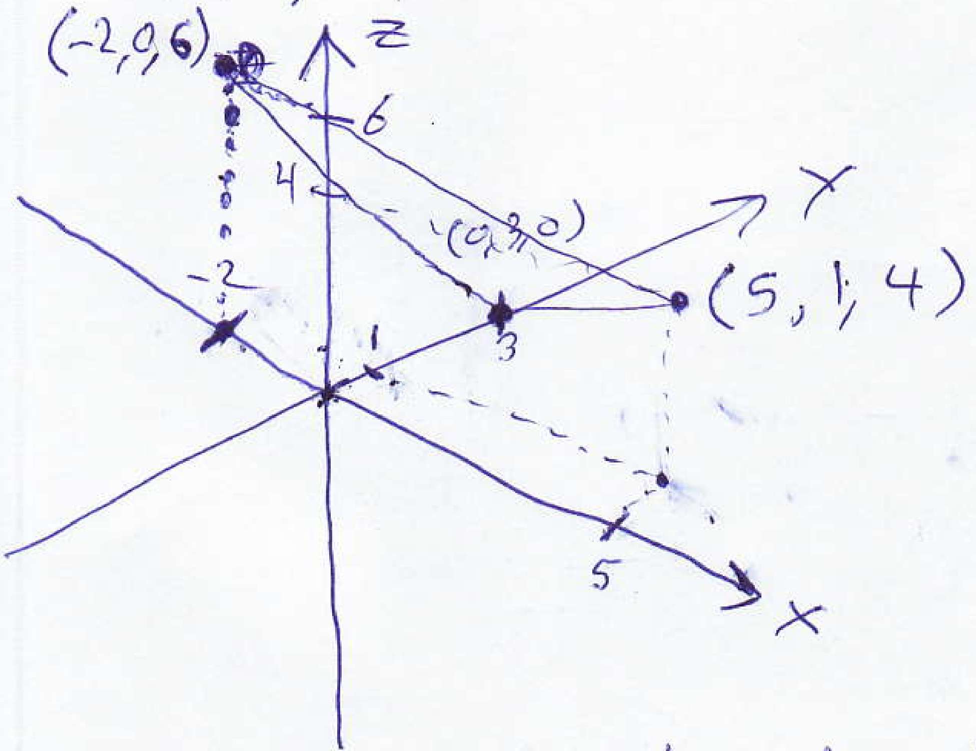


$\Delta z = a\Delta x + b\Delta y$

(Like $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$)

Given a, b, $\boxed{z - z_1 = a(x - x_1) + b(y - x_1)}$

Find an equation for the plane through the points $(0,3,0)$, $(5,1,4)$ and $(-2,0,6)$.



$\Delta z = a\Delta x + b\Delta y$

start	end	Δx	Δy	Δz	equation
$(0,3,0)$	$(5,1,4)$	5	-2	4	$4 = 5a - 2b$
$(0,3,0)$	$(-2,0,6)$	-2	-3	6	$6 = -2a - 3b$

$\frac{4}{5} + \frac{2}{5}b = a \Leftarrow 4 + 2b = 5a \Leftarrow$

$6 = -2\left(\frac{4}{5} + \frac{2}{5}b\right) - 3b = -\frac{8}{5} - \frac{19}{5}b$

$\Rightarrow 30 = -8 - 19b \Rightarrow 38 = -19b \Rightarrow b = -2$

$$a = \frac{4}{5} + \frac{2}{5}(-2) = \frac{4}{5} - \frac{4}{5} = 0$$

$$\Delta z = 0 \,\Delta x + (-2)\,\Delta y$$

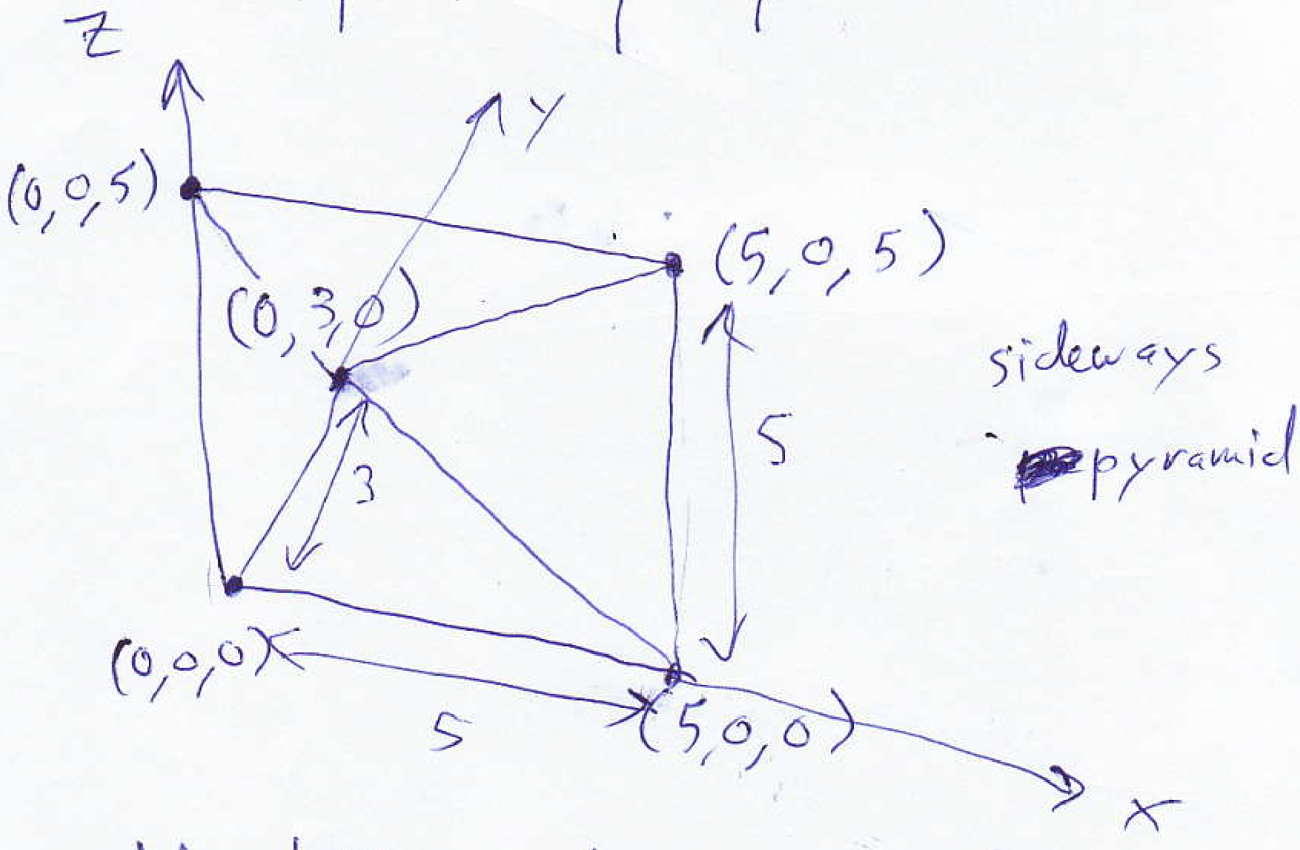
$$z - 0 = 0(x-0) + (-2)(y-3)$$

start: $(0,3,0)$ end: (x,y,z)

$$z = -2(y-3) = -2y + 6 = 6 - 2y$$

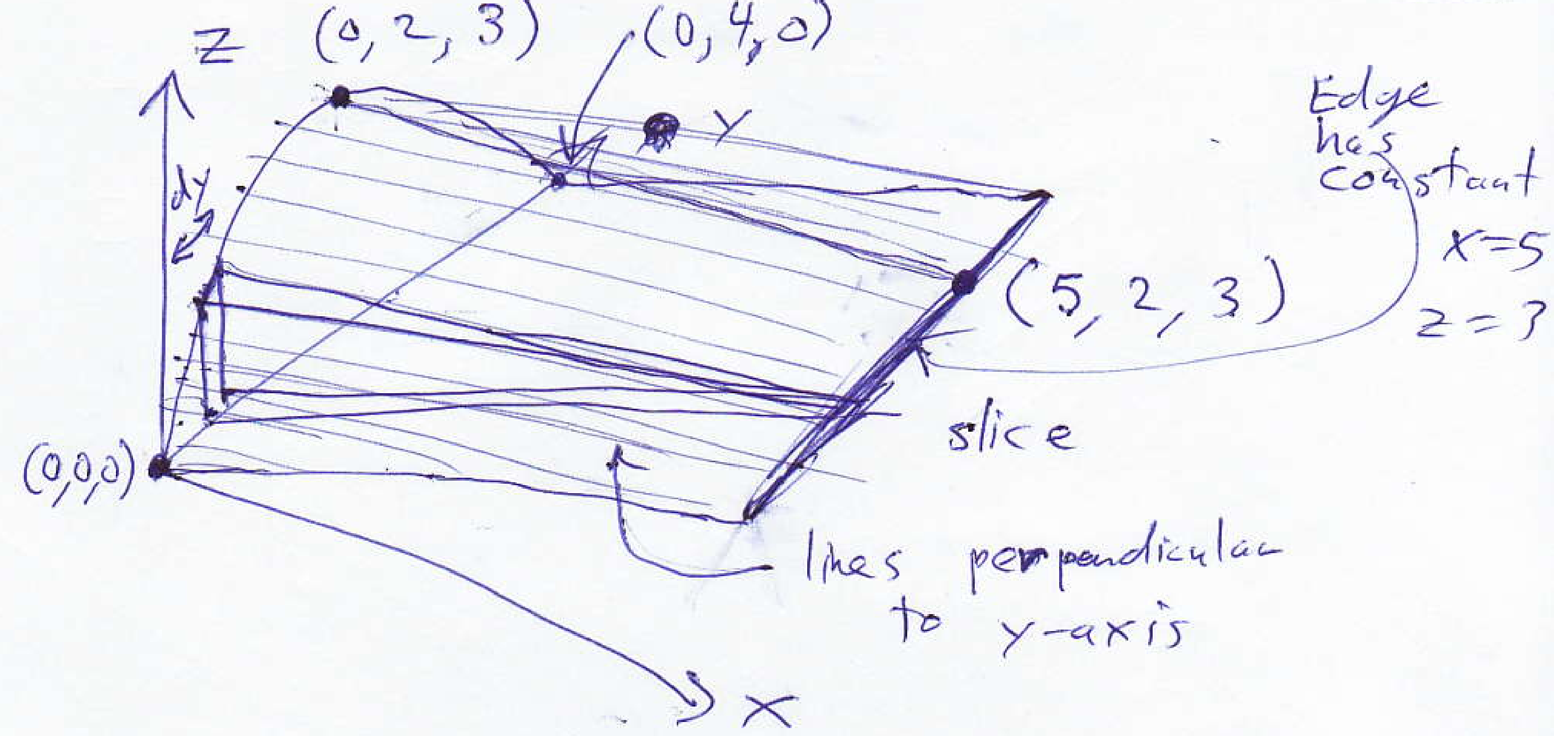
Check:

point	z	$6-2y$
$(0,3,0)$	0	0 ✓
$(5,1,4)$	4	4 ✓
$(-2,0,6)$	6	6 ✓



$$V = \frac{1}{3} B h \quad \text{(Appendix C)}$$

$V = \frac{1}{3}(5^2)3 = 25$



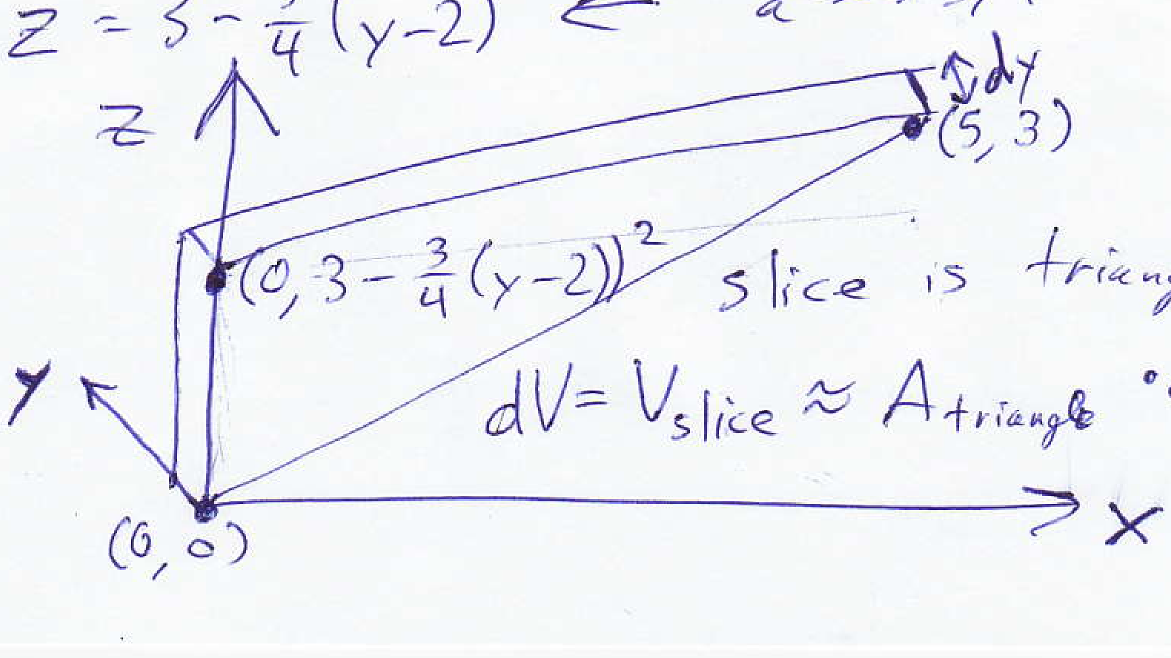
parabola: $x=0$ & $z = 3 - a(y-2)^2$

$0 = 3 - a(0-2)^2$

$0 = 3 - 4a$

$4a = 3$

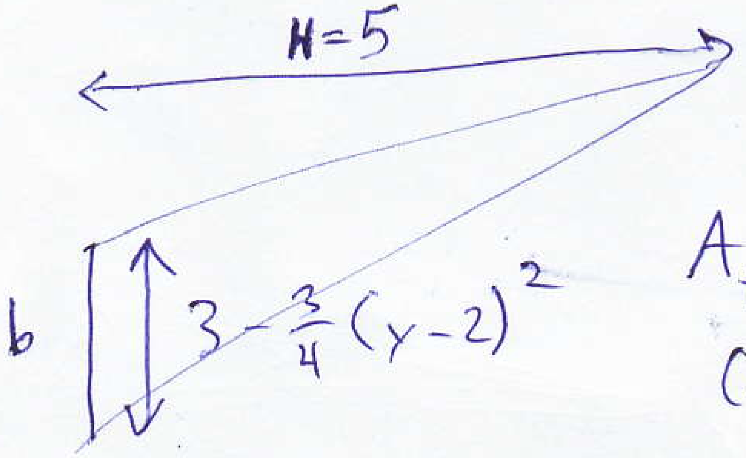
$z = 3 - \frac{3}{4}(y-2)^2 \leftarrow \quad a = 3/4$



slice is triangle

$dV = V_{slice} \approx A_{triangle} \cdot dy$

$$V_{total} = \int_{y=0}^{y=4} dV = \int_{y=0}^{y=4} A_{triangle}\, dy$$



$$A_{tri} = \frac{1}{2} b H$$

(Appendix C)

$$V = \int_0^4 \frac{1}{2}\left(3 - \frac{3}{4}(y-2)^2\right) 5\, dy$$

$$V = \frac{5}{2}\int_{y=0}^{x=4} \left(3 - \frac{3}{4}(y-2)^2\right) d(y-2)$$

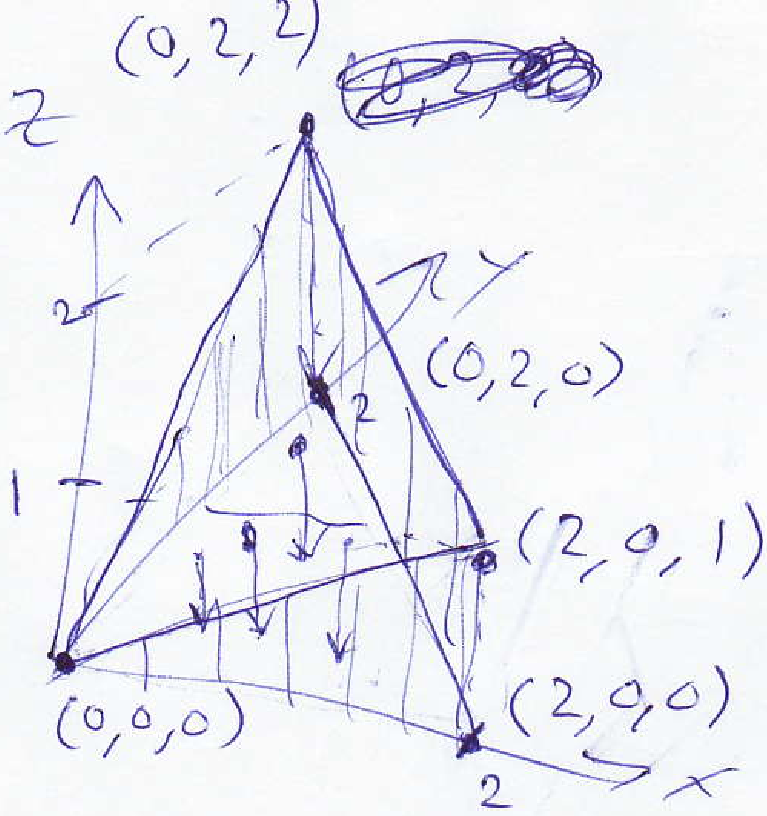
$$dy = dy - 0 = d(y-2)$$

$$V = \frac{5}{2}\left[3\int_{y=0}^{y=4} d(y-2) - \frac{3}{4}\int_{y=0}^{y=4} (y-2)^2 d(y-2)\right]$$

$$V = \frac{5}{2}\left[3(y-2)\Big|_{y=0}^{y=4} - \frac{3}{4}\left((y-2)^3/3\right)\Big|_0^4\right]$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

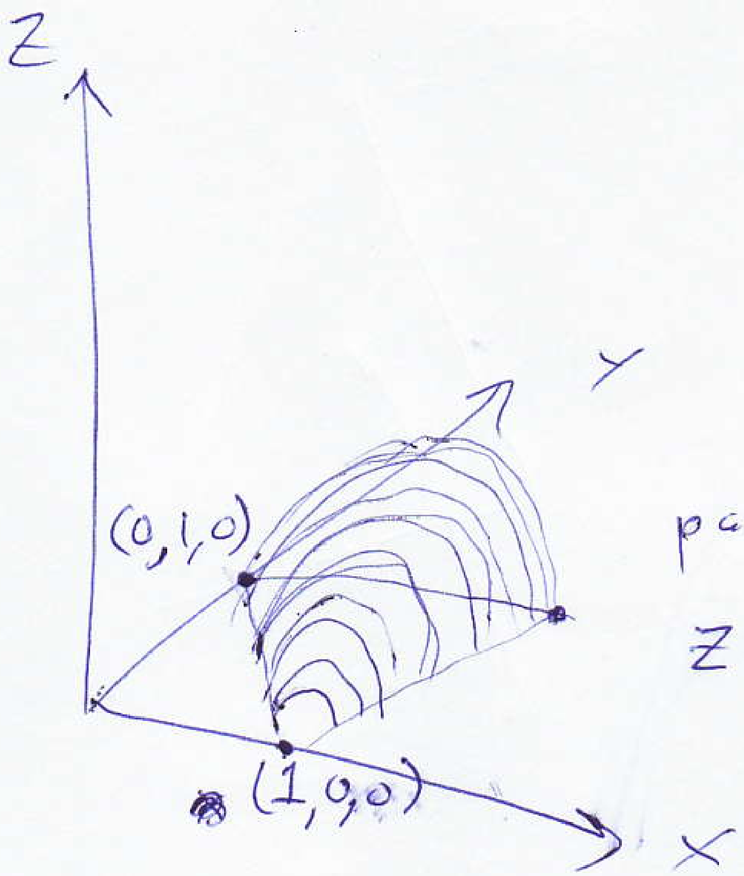
$$V = \frac{5}{2}\left[\underbrace{3(2-(-2))}_{12} - \underbrace{\frac{3}{4}\left(\frac{8}{3} - \frac{-8}{3}\right)}_{4}\right] = 20$$



HW #1 Find an equation for the plane through those 3 points: (0,0,0), (0,2,2), (2,0,1).

#2 Find the volume of the region under that triangle and above the xy plane.

Hint: either use pyramid geometry or slice it (similar to example 7, 15-7, p. 860)

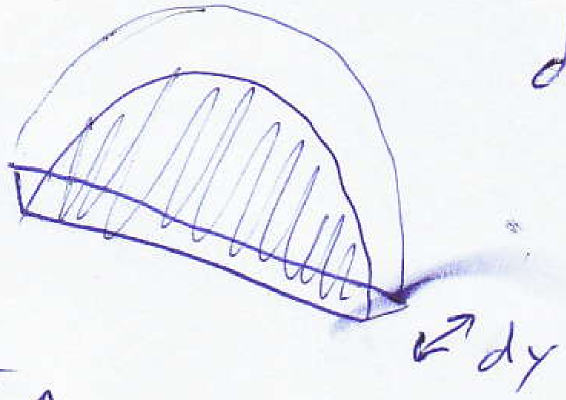


parabolas

$$z = y^2 - (x-1)^2$$

$$0 \le y \le 1$$

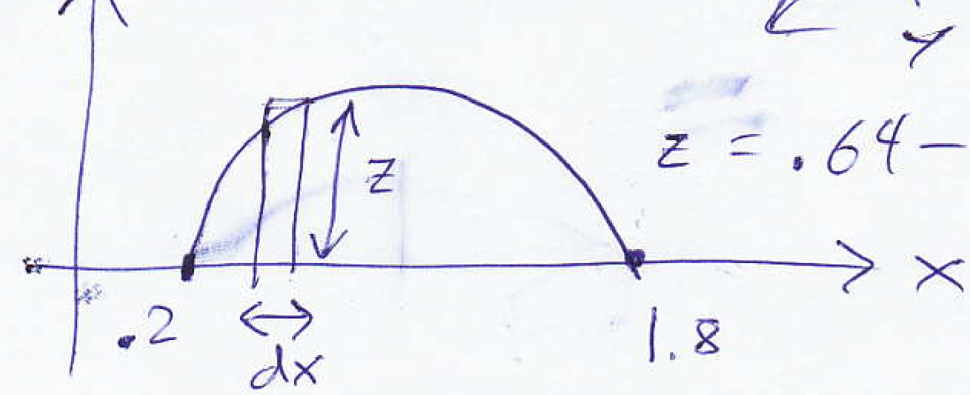
To find volume, slice:



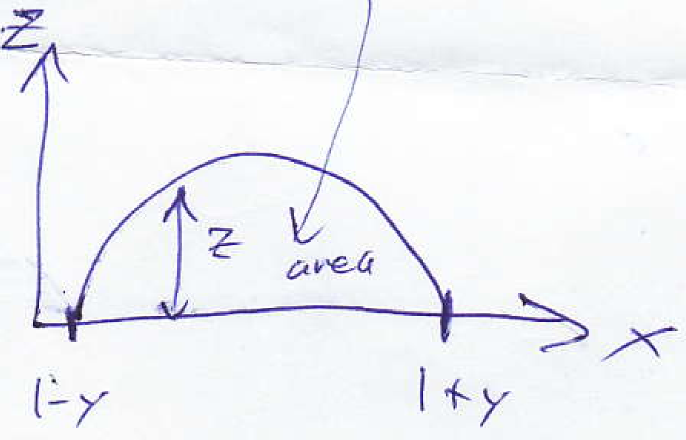
$$dV \approx \underbrace{dy}_{\text{thickness}} \cdot \underbrace{\int_{1-y}^{1+y} z \, dx}_{\text{area}}$$

$y = .8$

$$z = .64 - (x-1)^2$$



$$V = \int_{y=0}^{y=1} \left(\int_{(x=1-y)}^{(x=1+y)} z \cdot dx \right) dy \quad \text{(double integral)}$$



$$\underbrace{\int_{1-y}^{1+y} \left(y^2 - (x-1)^2 \right) dx}_{\text{In this slice, } y \text{ is constant}}$$

$$\int_{1-y}^{1+y} y^2 \, dx = x y^2 \Big|_{1-y}^{1+y}$$

$$= (1+y)y^2 - (1-y)y^2 = 2y^3$$

$$\int_{1-x}^{1+y} (x-1)^2 dx = \int_{1-x}^{1+x} (x-1)^2 d(x-1)$$

$$d(x-1) = dx - 0 = dx$$

$$\int_{1-y}^{1+y} (x-1)^2\, d(x-1) = (x-1)^3/3 \Big|_{1-y}^{1+y}$$

$$= (1+y-1)^3/3 - (1-y-1)^3/3$$

$$= (y^3 - -y^3)/3 = 2y^3/3$$

$$V = \int_{y=0}^{y=1} \frac{7}{3} y^3\, dy = \frac{7}{3} \cdot \frac{y^4}{4} \Big|_0^1$$

$$= \frac{7}{3}\left(\frac{1}{4} - \frac{0}{4}\right) = \frac{7}{12}$$

HW # 36 (15-7)