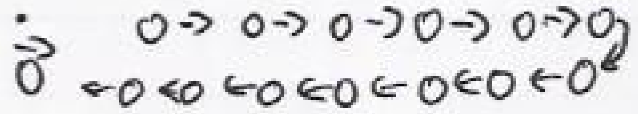
Chapter 14

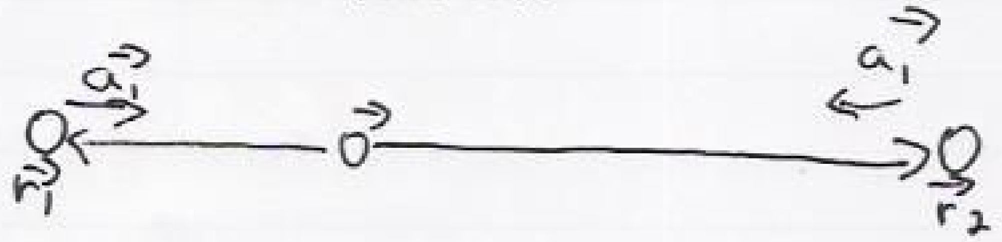
- Simple harmonic motion:

[acceleration is proportional and opposite to position] equivalent: force is proportional and opposite to position

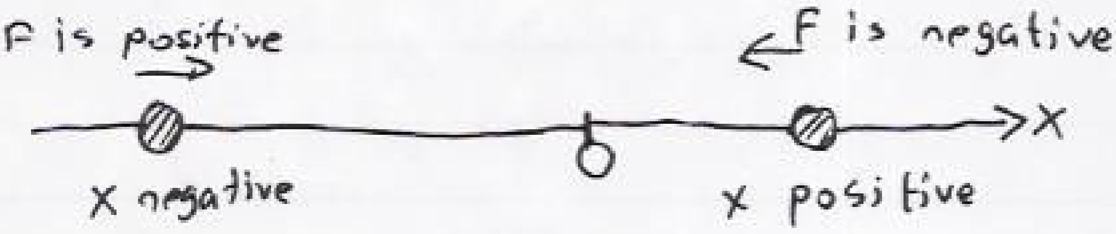


One dimensional oscillation:

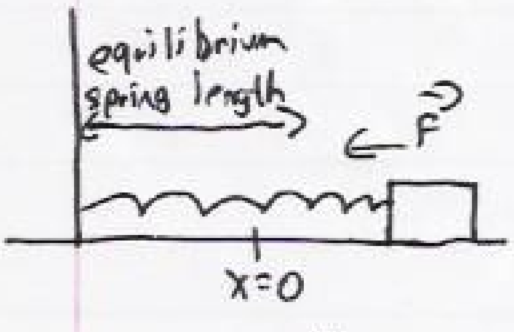
$\vec{a} = -\text{constant} \cdot \vec{r}$ $\qquad \vec{F} = m\vec{a} = \underbrace{\text{constant}}_{K} \cdot m \cdot \vec{r}$



Since this is one-dimensional motion, just use x instead of $\vec{r}$.



$F = -Kx$



$F = -Kx$

$ma = F = -kx$

$m \frac{dv}{dt}$

$m \frac{d^2x}{dt^2} = -kx$

$mx''(t) = -k\,x(t)$

Solutions to this differential equation:

$x''(t) = -\frac{k}{m}(t) \Longrightarrow x''(t) + \underbrace{\frac{k}{m}}_{w^2 = \frac{k}{m}} x(t) = 0$

$x(t) = A \sin(wt - \phi)$

A, ϕ arbitrary constants

Angular frequency $= w = \sqrt{\frac{k}{m}}$

$x''(t) + w^2 x(t) = 0$

has solutions of form

$A \sin(wt - \phi)$

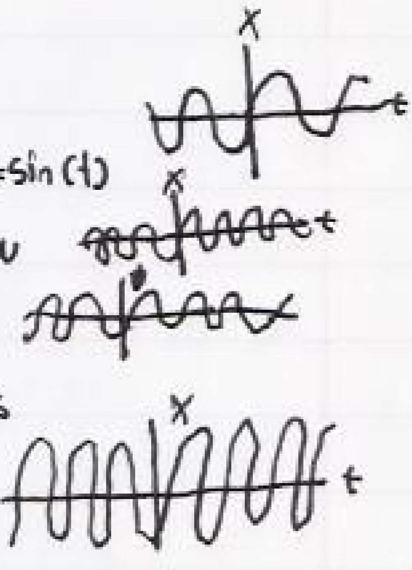
Graph: ① Start with $y = \sin(t)$

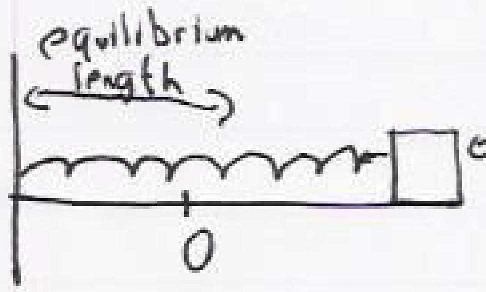
②. Compress by factor of w

③. Shift right by ϕ

in radians

④. Stretch by A





Ignoring friction, the block undergoes cyclic motion. How long is one cycle?

$$T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

$\boxed{t=0}$

Initially, the block, with mass 55 grams, is at $X = 3.0\,cm$ with $v = 0.0\,cm/s$
If $K = \frac{64\,N}{m}$, then find A, w, ϕ where $x(t) = A\sin(wt - \phi)$. <Ignoring friction>

$$w^2 = \frac{K}{m} = \frac{64\,N/m}{.055\,Kg} = \frac{64\,(Kg \cdot m/s^2)/m}{.055\,Kg} = 1163.63/s^2$$

$$w = \sqrt{1163.63/s^2} = 34.1121/s = 34.1121\,rad/s$$

$$T = \frac{2\pi}{w} = .18\,s$$

$$v(t) = x'(t) = A(\cos(wt - \phi))\,(wt - \phi)' = Aw\cos(wt - \phi)$$

Traditionally you pick $A > 0$

$$t=0 \begin{cases} 3.0\,cm = x = A\sin(w \cdot 0 - \phi) = A\sin(-\phi) = -A\sin\phi \\ 0.0\,cm/s = Aw\cos(w \cdot 0 - \phi) = Aw\cos(-\phi) = Aw\cos\phi \Rightarrow \cos\phi = 0 \Rightarrow \phi \text{ has many solutions} \end{cases}$$

$A = -3.0\,(cm/s) \Leftarrow 3.0\,(cm/s) = -A\sin\frac{\pi}{2} = -A \longleftarrow$ Pick $\phi = 90° = \frac{\pi}{2}$

$$x(t) = (-3.0\,(cm/s))\sin((34/s)t - \tfrac{\pi}{2})$$

$A = 3.0\,(cm/s) \Leftarrow -A\sin -\frac{\pi}{2} = A \longleftarrow$ Pick $\phi = -90° = -\frac{\pi}{2}$

$$x(t) = (3.0\,(cm/s))\sin((34/s)t + \tfrac{\pi}{2})$$

$\Rightarrow$ Both equal $(3.0\,(cm/s))\cos((34/s)t)$.

Example: At $t = 0.035\,s$

$$X = 3.0\,(cm/s)\cos(\overbrace{34.1121 \cdot 0.035}^{radians})$$

$= $ [redacted] $1.1\,cm$

At $t = 0.0355$: $A = 3.0\,cm$ $\phi = -90°$

$v = A\omega\cos(\omega t - \phi) = 3.0\,cm \cdot 34.1121/s \cdot \cos((34.1121/s)\cdot(0.0355) + \frac{\pi}{2})$

$= -95\ cm/s$

$X(t) = A\sin(\omega t + \phi)$
$v(t) = A\omega\cos(\omega t + \phi)$

Book uses "+"
It doesn't matter much

How much energy does the block have?

$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$ $\omega^2 = \frac{K}{m}$

$E = \frac{1}{2}m\underbrace{A^2\omega^2}_{\frac{K}{m}}\cos^2(\omega t+\phi) + \frac{1}{2}KA^2(\omega t+\phi) = \frac{1}{2}KA^2(\cos^2(\omega t+\phi) + \sin^2(\omega t+\phi))$

$32\frac{N}{m}\cdot(.030m)^2 = \frac{1}{2}\cdot 64\frac{N}{m}\cdot(3.0cm)^2 = \underbrace{\frac{1}{2}KA^2}_{constant}$

$32\frac{N}{m}\cdot .00090\,m^2 = 2.9\times10^{-2}\ N\cdot m = \boxed{2.9\times10^{-2}\ J}$