1 joule = energy needed to
accelerate 2 kg from
rest to 1 m/s
$(KE = \frac{1}{2} m v^2)$

1 joule = $kg \cdot m / s^2$

1 erg = $g \cdot cm / s^2$

= energy needed to
accelerate 2 g from
rest to 1 cm/s

1 calorie = energy needed to
heat 1 g of H_2O (liquid)
by 1°C.

1 watt = 1 joule per second

Heat is energy, and anything we can convert to heat or from heat is also energy.

"Heat" = "Thermal energy"

and

"Heat" = "change in thermal energy"

$\Delta \mathcal{H}$ = change in $\mathcal{H}$

$d\mathcal{H}$ = small change in $\mathcal{H}$

$$\frac{dK}{dv} = \left(\frac{1}{2}mv^2\right)' = \frac{1}{2}m(2v) = mv$$

$$dK = mv\,dv$$

energy conserved (in a closed system)

$$0 = \Delta E = mc\Delta T + \Delta K + \Delta U_g + \cdots$$

$mc\Delta T$: heat ← random atomic motion

ΔK: kinetic energy ← motion

ΔU_g: gravitational energy ← relationship between objects

$$0 = dE = mc\,dT + dK + dU_g + \cdots$$

$$0 = dE = mc\,dT + mv\,dv + mg\,dy$$

$U_g = \underline{mgy}$ near earth's surface

← y is height

$y=0$ is arbitrarily chosen

$$\Delta U_g = mg\Delta y$$

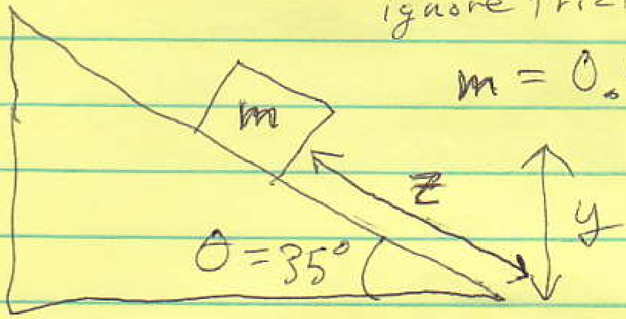
↑ not arbitrary

$$dU_g = mg\,dy$$

For simplicity, suppose $dT = 0$

↑ ignore friction

below:



$m = 0.5\,kg$

small change in energy of block

↓

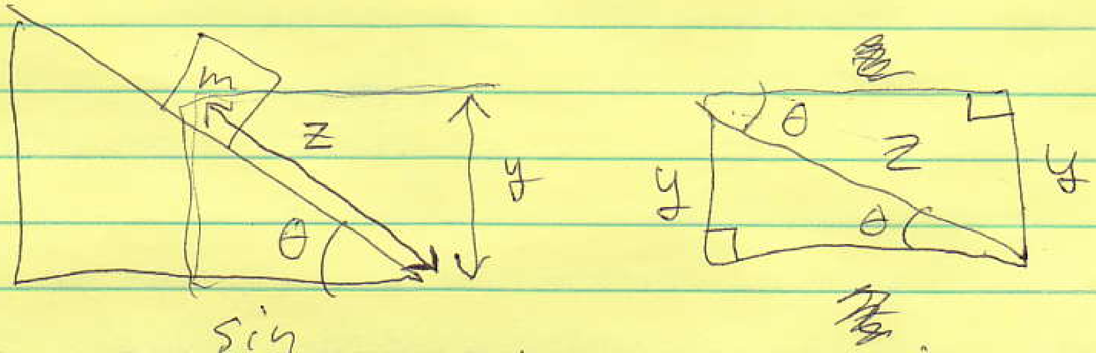
$$0 = dE = mc\underbrace{dT}_{0} + \underbrace{mv\,dv}_{dK} + \underbrace{mg\,dy}_{dU_g}$$

Find acceleration of block, in the z direction

$$v = \frac{dz}{dt} \qquad a = \frac{dv}{dt} = ?$$

$$\frac{0}{dt} = \frac{\overbrace{mv\,dv}^{dK} + \overbrace{mg\,dy}^{dU_g}}{dt}$$

$$0 = mv\frac{dv}{dt} + mg\frac{dy}{dt}$$



$$\sin\theta = \frac{y}{z} \Rightarrow z\sin\theta = y$$

$$dz\sin\theta = dy$$

$$0 = mv\frac{dv}{dt} + mg\frac{dz}{dt}\sin\theta$$

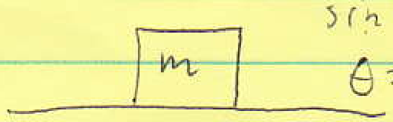
$$\frac{0}{mv} = \frac{mva + mgv\sin\theta}{mv}$$

$$0 = a + g\sin\theta$$

$$a = -g\sin\theta$$

$$= -9.8\,m/s^2 \cdot \sin 35^\circ$$

$a = -g \sin\theta$ makes sense



$\sin\theta = 0$

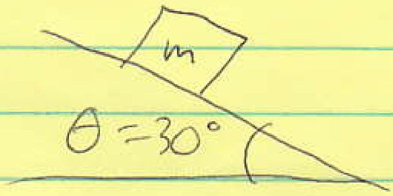
$\theta = 0$

$a = 0$

$\theta = 90^\circ$

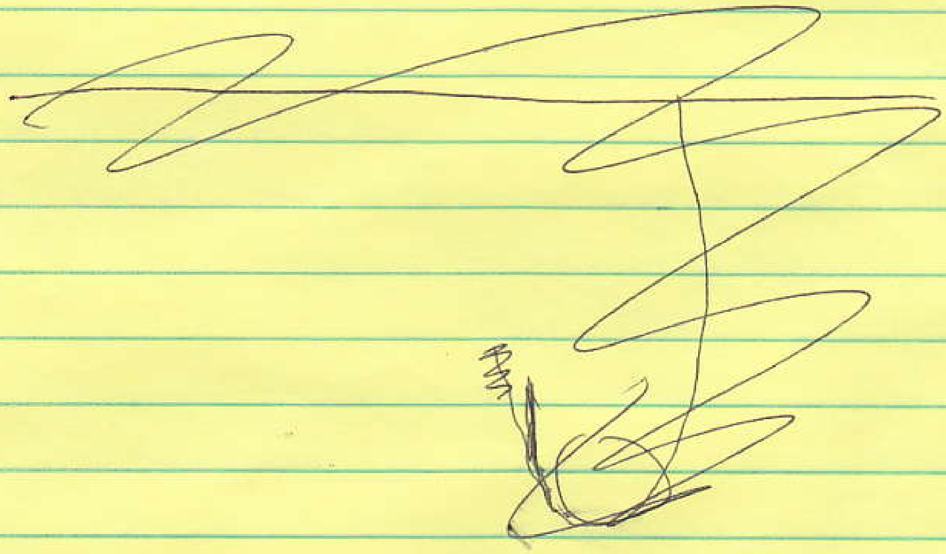
$\sin 90^\circ = 1$

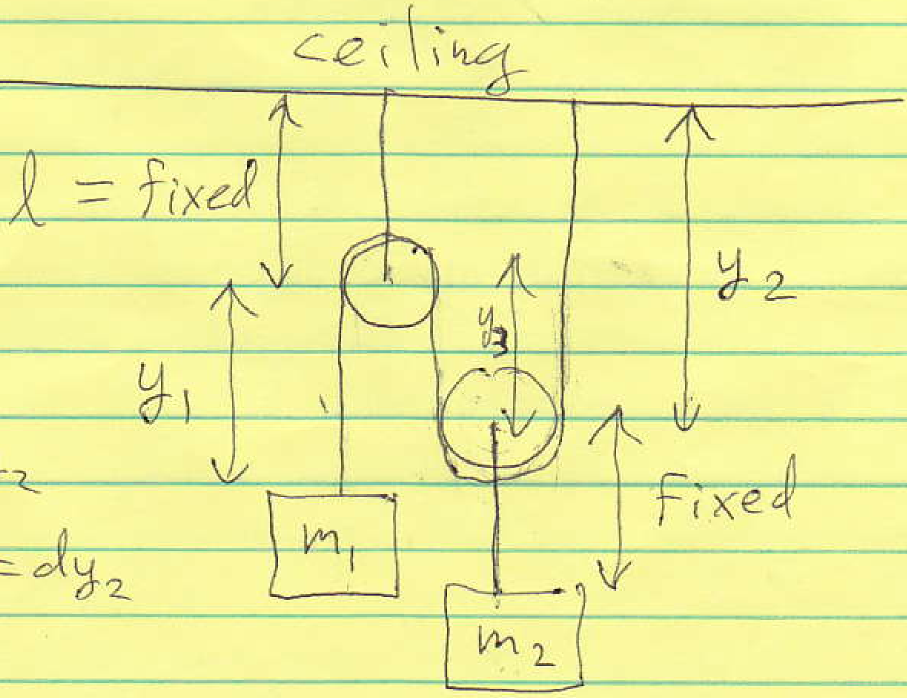
$a = -g$



$\sin\theta = \frac{1}{2}$

$a = -g/2$





$l + y_3 = y_2$

$0 + dy_3 = dy_2$

gravity: total energy of both objects

$$dU_g = -m_1 g\, dy_1 - m_2 g\, dy_2$$

$dy_3 = dy_2$

$y_1 + y_3 + y_2$ is constant

~~$dy_1 + 2dy_2$~~ $\leftarrow$ $dy_1 + dy_3 + dy_2 = 0$

$dy_1 + 2dy_2 = 0$

total string length is not changing

$dy_2 = -dy_1/2$

$$dU_g = -m_1 g\, dy_1 - m_2 g\,(-dy_1/2)$$

$$= g\, dy_1 (-m_1 + m_2/2)$$

$$0 = dK + dU_g = m_1 v_1 dv_1 + dU_g + m_2 v_2 dv_2$$

$$0 = m_1 v_1 dv_1 + m_2 v_2 dv_2 + g\, dy_1 \left(-m_1 + \frac{m_2}{2}\right)$$

$v_1 = \frac{dy_1}{dt}$ $\qquad$ $v_2 = \frac{dy_2}{dt}$

$$\frac{dy_2}{dt} = \frac{-dy_1/2}{dt} \Rightarrow v_2 = -\frac{v_1}{2}$$

$$v_2 = -\frac{v_1}{2} \Rightarrow dv_2 = -\frac{dv_1}{2}$$

$$0 = m_1 v_1 dv_1 + m_2 \left(-\frac{v_1}{2}\right)\left(-\frac{dv_1}{2}\right) + g\, dy_1 \left(-m_1 + \frac{m_2}{2}\right)$$

$$\frac{1}{v_1} \cdot \frac{0}{dt} = \left[m_1 v_1 \frac{dv_1}{dt} + \frac{1}{4} m_2 v_1 \frac{dv_1}{dt} + g \left(\frac{dy_1}{dt}\right) \left(-m_1 + \frac{m_2}{2}\right) \right] \cdot \frac{1}{v_1}$$

$\frac{dy_1}{dt} \leftarrow v_1$

$$0 = m_1 a_1 + \frac{1}{4} m_2 a_1 + g\left(-m_1 + \frac{m_2}{2}\right)$$

$$a_1 = \frac{m_1 - m_2/2}{m_1 + m_2/4}\, g$$