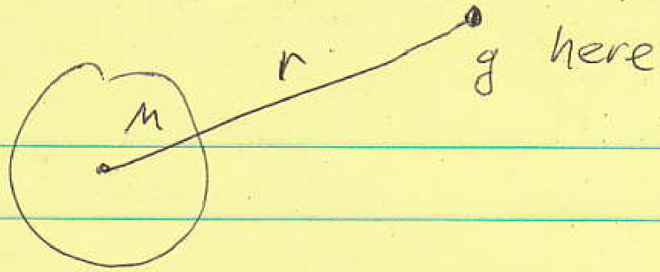
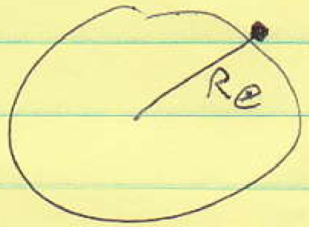
$$g = \frac{GM}{r^2}$$

↑
gravitational Field strength



Near earth's surface:



$g = 9.8\, m/s^2$

$R_e = \cancel{1.49 \times 10^{8}\, km}$ $6.378 \times 10^6\, m$

$\cancel{g = GM_e/R_e^3}$

$$g = \frac{GM_e}{R_e^2}$$

↑
Newton

$G = ?$ $M_e = ?$

Cavendish found G.

$$G = 6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$$

$$M_e = \frac{g R_e^2}{G}$$

$$g = \frac{G M_e}{R_e^2} \rightarrow m/s^2 = \frac{(?)\, kg}{m^2}$$

J

$$\frac{m \cdot kg\, m^2/s^2}{kg^2} = \frac{kg}{kg} \cdot \frac{m^3}{kg \cdot s^2} = \frac{m^3}{kg \cdot s^2} = (?)$$

$$= \frac{J \cdot m}{kg^2}$$

$$G = 6.67 \times 10^{-11} \frac{J \cdot m}{kg^2}$$

Energy is in J. G is in $\frac{J \cdot m}{kg^2}$

$$G \propto \frac{U \cdot r}{m_1 m_2}$$

energy from gravity

distance

masses

$$\Downarrow$$

$$U \propto \frac{G m_1 m_2}{r}$$

$\propto$ means "is proportional to"

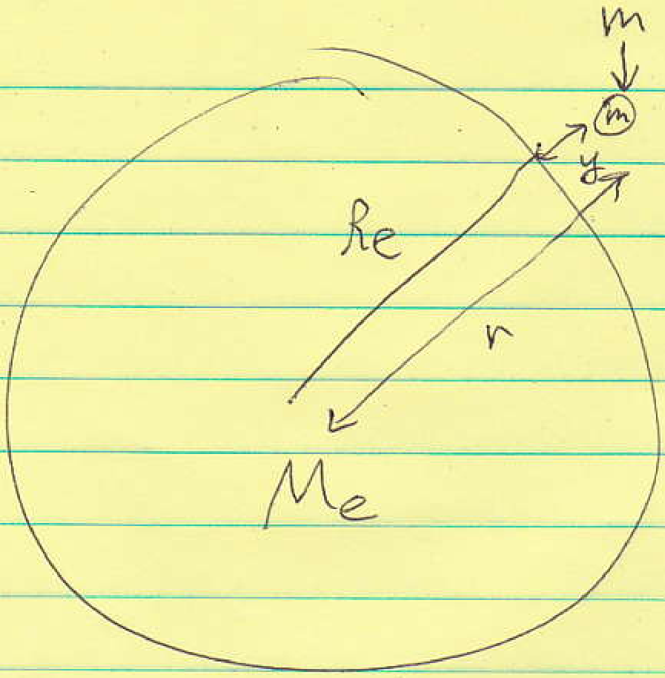
$$U = -\frac{G m_1 m_2}{r}$$

gravitational potential energy caused by masses m_1 & m_2 interacting gravitationally.

Near earth surface:

$U = mgy$ (looks very different)

$$g = \frac{G M_e}{R_e^2} \qquad U = \frac{G M_e m y}{R_e^2}$$



$$U = -\frac{G M_e m}{r}$$

$$r = R_e + y \approx R_e$$

Compare to:

$$U = m g y = \frac{G M_e m y}{R_e^2}$$

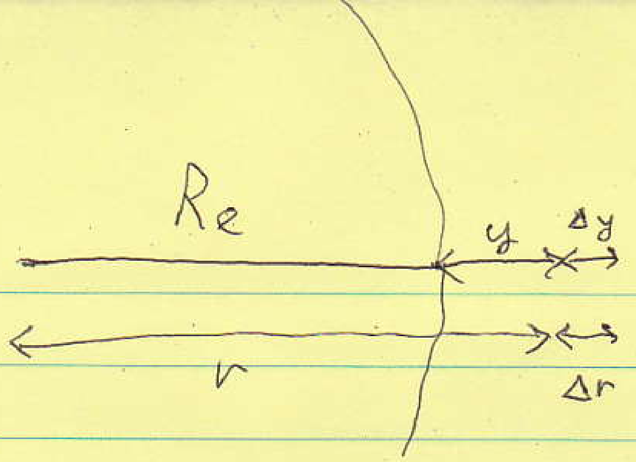
U depends on a coordinate system
Only changes in U are real.

We only need formulas for ΔU
that agree.

$$\Delta U = \boxed{-G M_e m \, \Delta\left(\frac{1}{r}\right)} \quad r = R_e + y$$

compare to:

$$\Delta U = m g \Delta y = \boxed{\frac{G M_e m}{R_e^2} \Delta y}$$



$$\Delta y = \Delta r$$

$$\Delta U = -G M_e m \, \Delta\left(\frac{1}{r}\right) \approx G M_e m \left(\frac{\Delta r}{r^2}\right)$$

compare to:

$$\Delta U = mg\,\Delta y = \frac{G M_e m}{R_e^2}\,\Delta r$$

$$g = \frac{G M_e}{R_e^2}$$

$$\Delta\left(\frac{1}{r}\right) \approx \frac{-\Delta r}{r^2}$$

For Δr small

Near earth's surface: $r \approx R_e$

$$G M_e m \left(\frac{\Delta r}{R_e^2}\right) \approx$$

Newton was right; $\Delta U = mg\,\Delta y$ is just an approximation

6, 12, 17, "42" HW from 2/17

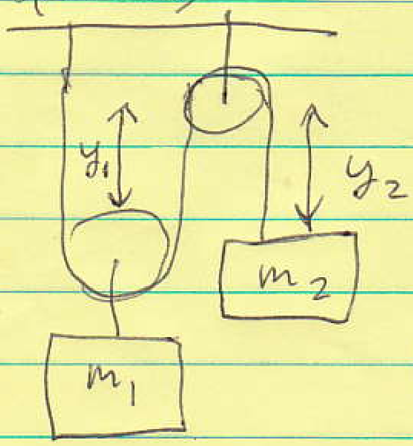
#6 The ball has maximum kinetic energy at the bottom; at the top, it has converted all its kinetic energy to ~~reach its~~ potential energy, which is highest at the top.

(Anya's to Ivan's penny)

#12 ~~At the top~~ The ratio of initial energies is 2 because the initial energy E of each is mgy, and the heights' ratio is 2. By conservation of energy, the ratio of final energies is also 2. The final (all kinetic) energy E of each is $\frac{1}{2}mv^2$, so $v = \sqrt{2E/m}$, so $v \propto \sqrt{E}$, so the ratio of final speeds is $\sqrt{2}$.

(If you took the ratio of Ivan's to Anya's penny, then you'd get a ratio of $1/2$ for final (kinetic) energy & $1/\sqrt{2}$ for final speeds.)

#17



$$dU = -m_1 g\,dy_1 - m_2 g\,dy_2$$

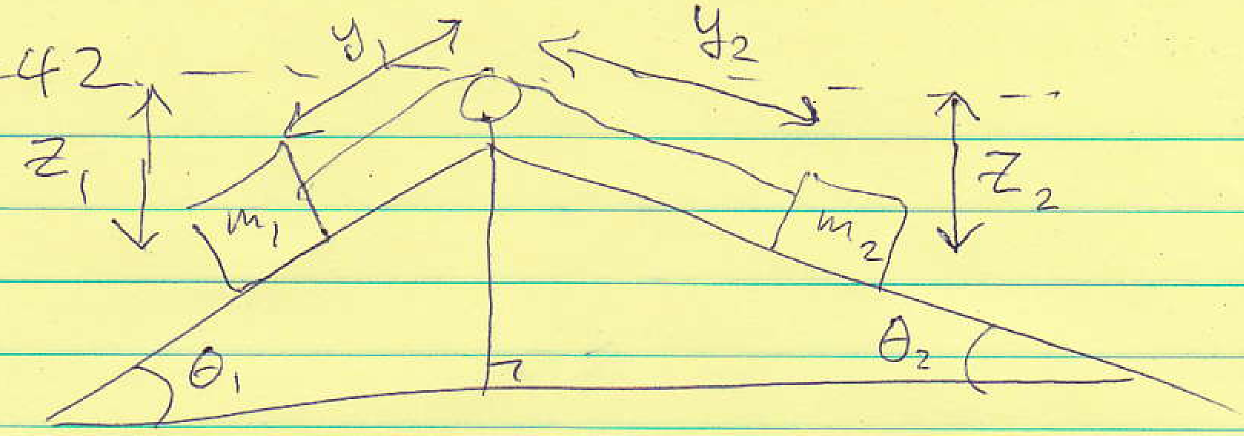
$$dy_2 = -2\,dy_1$$

$$dU = (-m_1 + 2m_2)\,g\,dy_1$$

For balance, we need $dU = 0$, so we need $-m_1 + 2m_2 = 0$, so

$$m_1/m_2 = 2$$

#42



$$a_2 = \frac{dv_2}{dt} \qquad v_2 = \frac{dy_2}{dt}$$

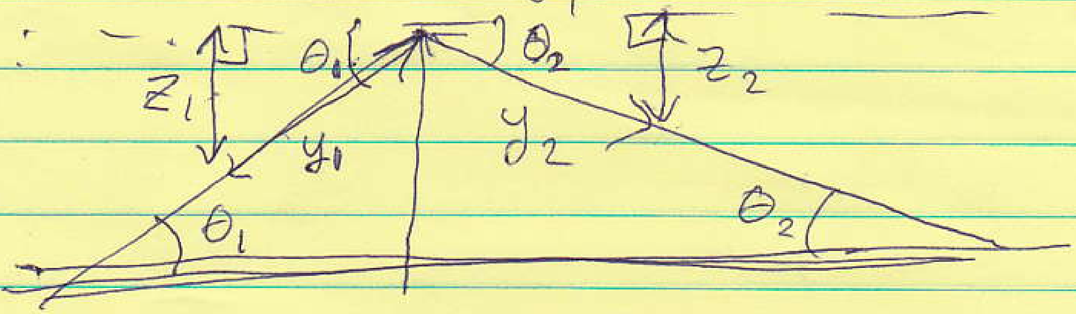
$$a_2 = ? \qquad \Delta y_1 = -\Delta y_2$$

$$v_1 = \frac{dy_1}{dt} = -\frac{dy_2}{dt} = -v_2$$

$$K = \frac{1}{2} m_1 \underbrace{v_1^2}_{(-v_2)^2 = v_2^2} + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} v_2^2 (m_1 + m_2)$$

$$\Delta U = -m_1 g \Delta z_1 - m_2 g \Delta z_2$$

$$\sin\theta_1 = z_1 / y_1 \qquad \sin\theta_2 = z_2 / y_2$$



$$z_1 = y_1 \sin\theta_1 \qquad \Delta z_1 = \Delta y_1 \sin\theta_1$$

$$\Delta U = -m_1 g \Delta y_1 \sin\theta_1 - m_2 g \Delta y_2 \sin\theta_2$$

$$\Delta y_1 = -\Delta y_2$$

$$\Delta U = -m_1 g (-\Delta y_2) \sin\theta_1 - m_2 g \Delta y_2 \sin\theta_2$$

$$\Delta U = g \Delta y_2 (m_1 \sin\theta_1 - m_2 \sin\theta_2)$$

$$\frac{dU}{dt} = g \frac{dy_2}{dt} (m_1 \sin\theta_1 - m_2 \sin\theta_2)$$

$$dU/dt = g v_2 (m_1 \sin\theta_1 - m_2 \sin\theta_2)$$

$$dK + dU = 0 \qquad \frac{dK}{dt} + \frac{dU}{dt} = 0$$

?

Conservation of energy

$$K = \frac{1}{2} v_2^2 (m_1 + m_2) \qquad dK = ?$$

$$dK = \frac{1}{2}(m_1 + m_2) \underbrace{d(v_2^2)}_{2 v_2 \, dv_2} = (m_1 + m_2) v_2 \, dv_2$$

$$\frac{dK}{dt} = (m_1 + m_2) v_2 \frac{dv_2}{dt} = (m_1 + m_2) v_2 a_2$$

$$0 = \underbrace{\frac{dK}{dt} + \frac{dU}{dt}}_{E\,=\,0} = (m_1 + m_2) v_2 a_2 + g v_2 (m_1 \sin\theta_1 - m_2 \sin\theta_2)$$

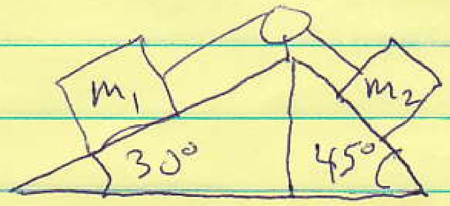
Solve for a_2

Divide both sides by $(m_1 + m_2) v_2$

$$0 = a_2 + \frac{g \cancel{v_2} (m_1 \sin\theta_1 - m_2 \sin\theta_2)}{(m_1 + m_2)\cancel{v_2}}$$

$$\frac{g(m_2 \sin\theta_2 - m_1 \sin\theta_1)}{m_1 + m_2} = a_2$$

And $a_1 = -a_2$.



balanced if:

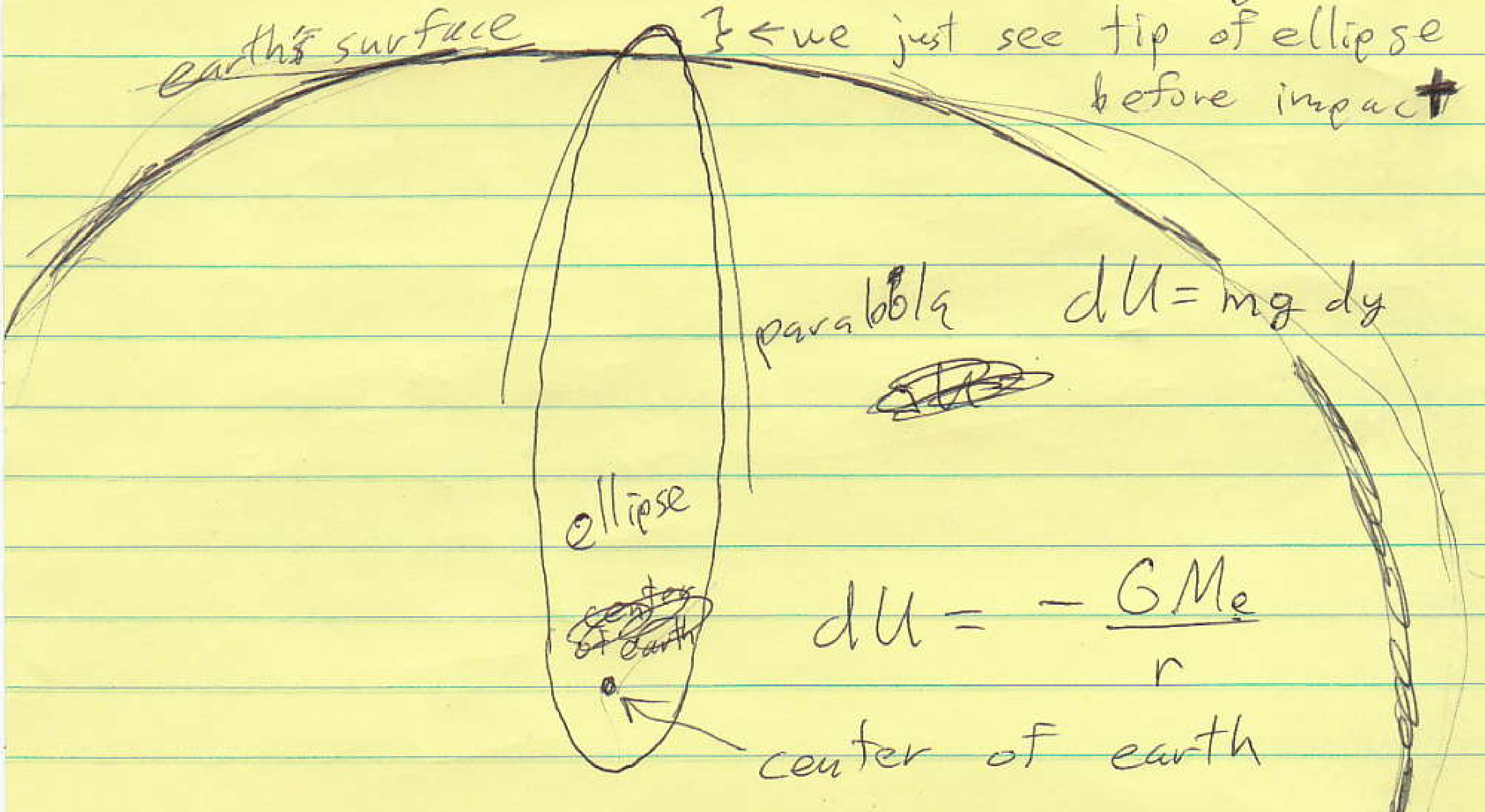
$m_1 = \sqrt{2}\, m_2$

because

$\sin 30° = \frac{1}{\sqrt{2}} \sin 45°$

Back to universal gravitation:

Parabolic trajectories from gravity are just approximations of actual elliptical trajectories.



$dU = mg\,dy$

$dU = -\frac{GM_e}{r}$

HW due 2/24: 21, 22, 24, 25, 27

Reading due 2/24: 2.4, 2.5