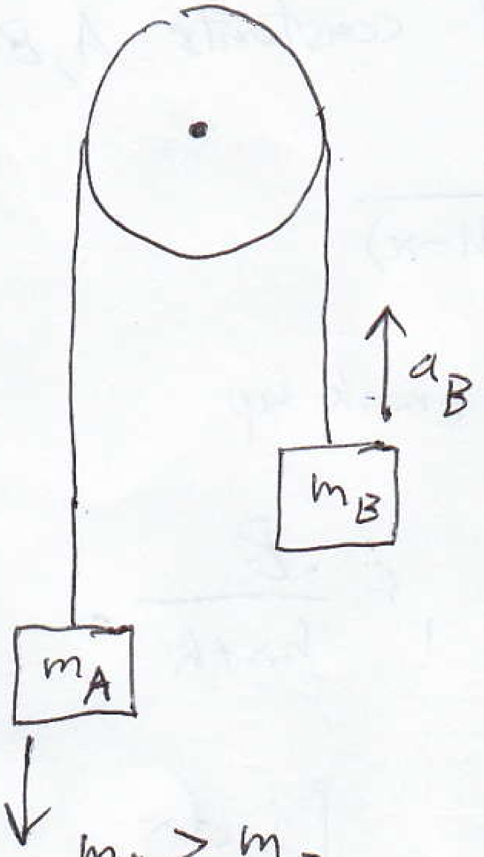


$m_A > m_B$

$$a_A = a_B$$

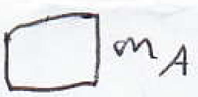
$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma$$

if m constant

$$F_A = \frac{dp_A}{dt} = m_A a_A \text{ (downward)}$$

$$F_B = \frac{dp_B}{dt} = m_B a_B \text{ (upward)}$$

Forces come in pairs. They are interactions between pairs of objects.



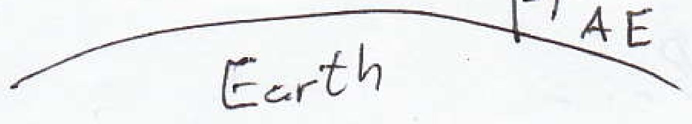
$$\downarrow F_{AE} = \frac{G m_E m_A}{r^2}$$

$$a_A = \frac{F_A}{m_A}$$

$$a_E = \frac{F_E}{m_E}$$

$\uparrow -F_{AE}$

distance between centers of E & A



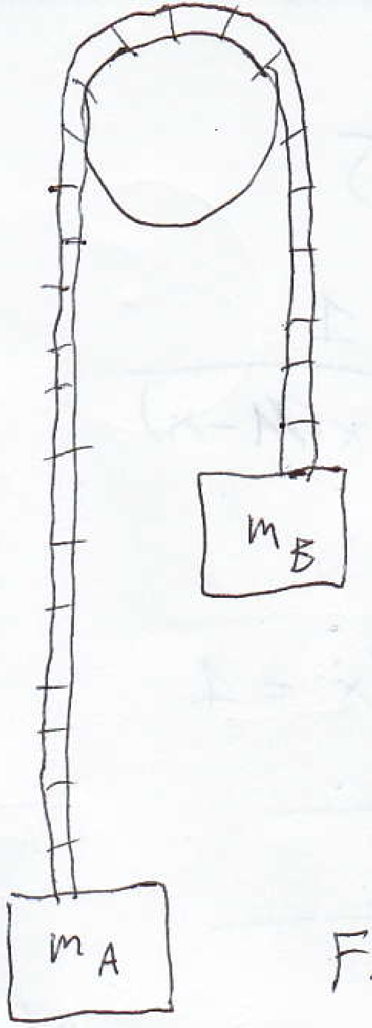
$$a_A = a_B = a$$

$$m_A a_A = F_A \text{ (downward)} = m_A g - F_T$$
$$m_B a_B = F_B \text{ (upward)} = F_T - m_B g$$

$$\begin{cases} m_A a = m_A g - F_T \\ m_B a = F_T - m_B g \end{cases}$$ Solve for a, F_T.

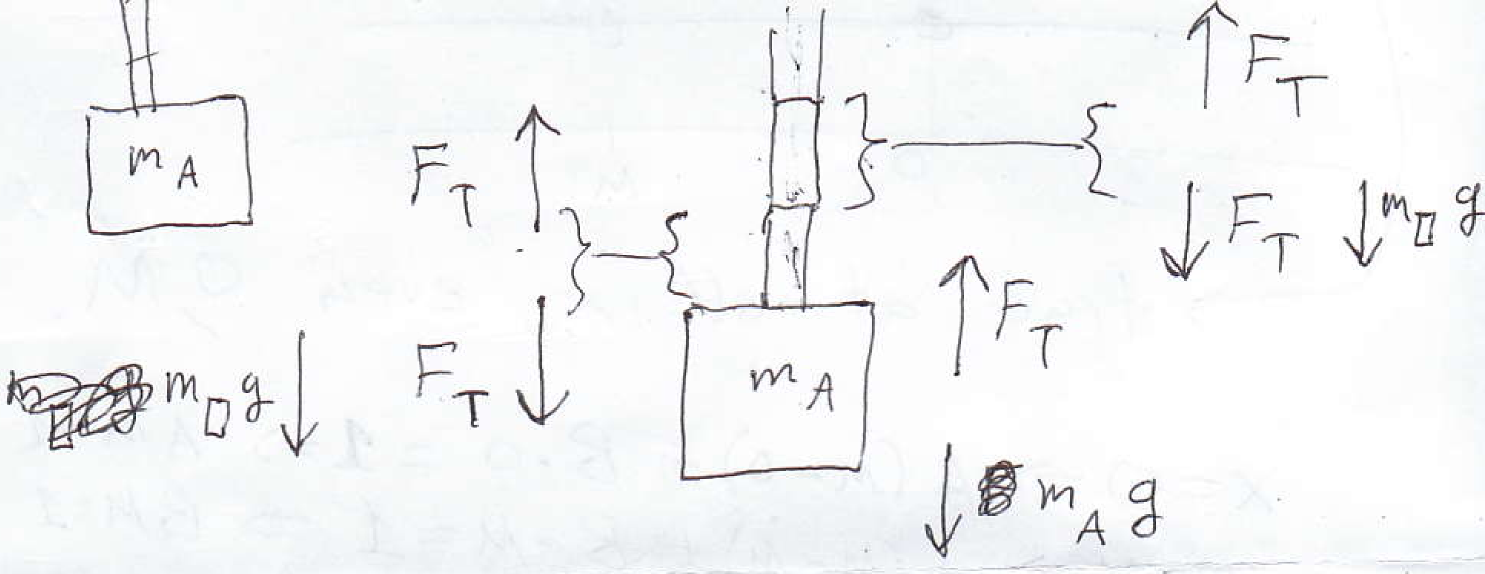
$$m_A a + m_B a = m_A g - F_T + F_T - m_B g$$
$$(m_A + m_B) a = (m_A - m_B) g$$

$$a = \frac{m_A - m_B}{m_A + m_B} g$$

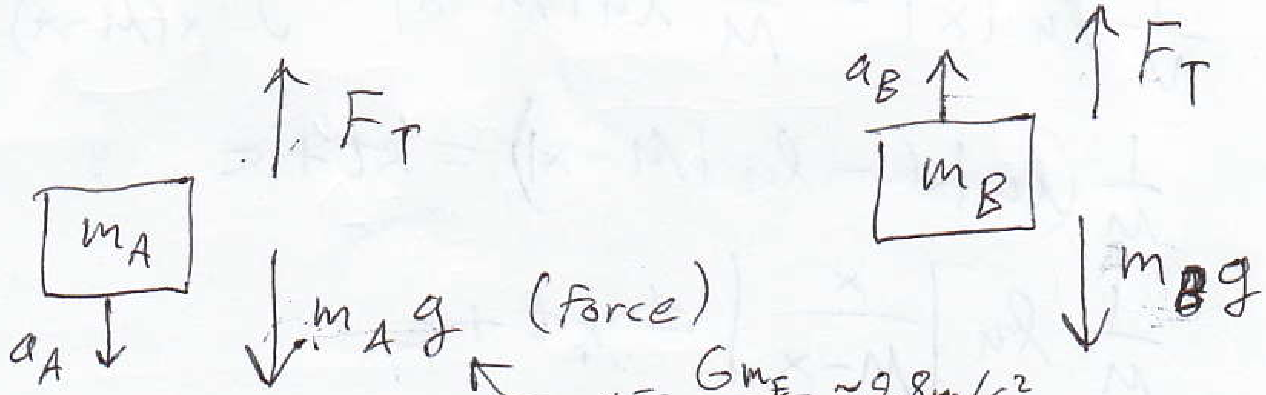


Each piece exerts a force on each of its neighbors, so that they stay connected.

Call this force F_T (T for tension)

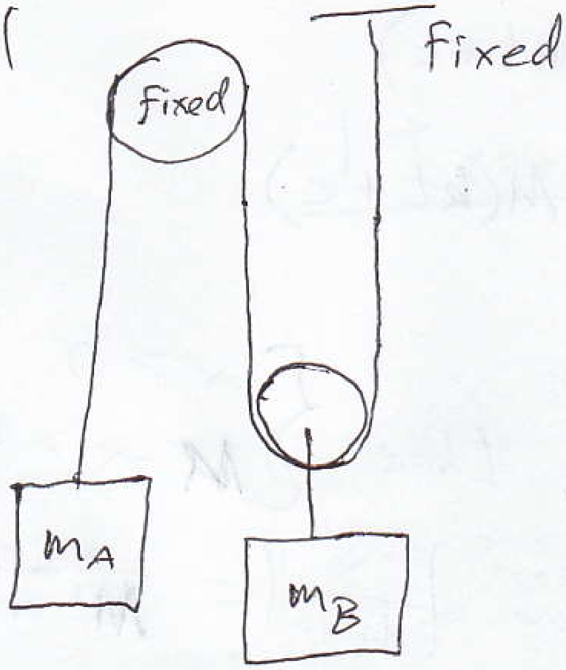


Ignore mass of rope



$g = \frac{G m_E}{r_E^2} \approx 9.8 m/s^2$

HW#1



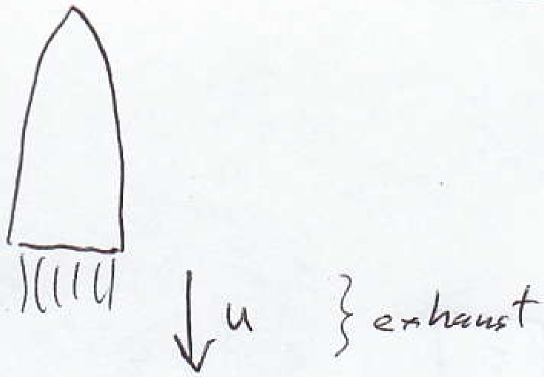
$m_A = 5.0\,kg$

$m_B = 3.0\,kg$

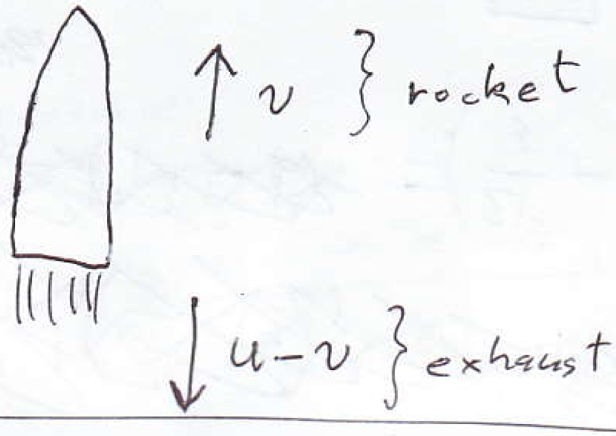
Find the acceleration of m_A and of m_B. What directions are they in?

---

rocket frame:



Earth frame:



Assume initial rocket mass $m_0 = 1.2 \times 10^5$ kg.

u = exhaust velocity downward relative to rocket

Assume $u = 2.4 \times 10^3$ m/s is constant

Earth's frame

$$\frac{dp_{rocket}}{dt} = \underbrace{\underbrace{-m_{rocket}\, g}_{\text{Force of gravity on rocket}}}_{\text{rate upward momentum transferred to rocket due to gravity}} + \underbrace{\underbrace{\underbrace{\left(-\frac{dm_{rocket}}{dt}\right)}_{\frac{dm_{exhaust}}{dt}}\underbrace{(u - v_{rocket})}_{v_{exhaust}}}_{\text{rate downward momentum transferred to exhaust gas due to burning}}}_{\text{rate upward momentum transferred to rocket due to burning}}$$

upward momentum's rate of change

downward

("−" sign because gravity pulls down)

$$g = \frac{G m_E}{r^2}$$

r = distance from rocket to earth's center

For simplicity, we'll just study the rocket's flight when r is not too different from r_E, that is, when the rocket's altitude is small compared to $r_E = 6.4 \times 10^6$ m.

So, $g \approx \dfrac{Gm_E}{r_E^2} = 9.8\,\text{m/s}^2$

Assume initial speed v_0 is 0.

Assume $\dfrac{dm_{rocket}}{dt} = -1.0 \times 10^3$ kg/s is constant

Call $w = 1.0 \times 10^3$ kg/s $= -dm_{rocket}/dt$

$$\frac{dp_{rocket}}{dt} = m_{rocket}\, g + w(u - v)$$

$$\cancel{m_{rocket} - m_0 = \int_{m_0}^{m_{rocket}} dm_{rocket} = \int_{t=0}^{t=t} -w\,dt = -w(t-0)}$$

Because w is constant,

$$\frac{m - m_0}{t - 0} = \frac{\Delta m}{\Delta t} = -w, \text{ so } m - m_0 = -w(t - 0),$$

so $m = m_0 - wt$.

(To save space, I've stopped using "rocket".)

$$\frac{dp}{dt} = -mg + w(u-v) = \boxed{-(m_0 - wt)g + w(u-v)}$$

$$\frac{dp}{dt} = \frac{d(mv)}{dt} = \frac{dm \cdot v + m \cdot dv}{dt}$$

$$\Rightarrow \frac{dp}{dt} = \frac{dm}{dt}v + m\frac{dv}{dt} = -wv + m\frac{dv}{dt}$$

$$\Rightarrow \frac{dp}{dt} = \boxed{-wv + (m_0 - wt)\frac{dv}{dt}}$$

$$-wv + (m_0 - wt)\frac{dv}{dt} = (wt - m_0)g + wu - wv$$

$$\frac{dv}{dt} = \frac{(wt - m_0)g}{m_0 - wt} + \frac{wu}{m_0 - wt}$$

$$\frac{dv}{dt} = -g + \frac{wu}{m_0 - wt}$$

$$dv = -g\,dt + \frac{wu\,dt}{m_0 - wt}$$

$$v = v - 0 = v - v_0 = \int_{v_0}^{v} dv = \int_{t_0}^{t} -g\,dt + \int_{t_0}^{t} \frac{wu\,dt}{m_0 - wt}$$

$t_0 = 0$

$$v = -g(t - t_0) + \int_0^t \frac{wu\,dt}{m_0 - wt} = -gt + wu\int_0^t \frac{dt}{m_0 - wt}$$

$$v = -gt + wu \underbrace{\int_{t_0}^{t} \frac{dt}{m_0 - wt}}$$

$m = m_0 - wt$

$dm = 0 - w\,dt$

$-dm/w = dt$

$$= \int_{m_0}^{m} \frac{-dm/w}{m} = \frac{-1}{w}\int_{m_0}^{m} \frac{dm}{m}$$

$$= -\frac{1}{w} \ln \frac{m}{m_0}$$

$$v = -gt + wu\left(-\frac{1}{w}\ln\frac{m}{m_0}\right)$$

In general, if $0 < a < b$, then $\int_a^b \frac{dx}{x} = \ln\frac{b}{a}$.

$$v = -gt - u \ln\frac{m}{m_0}$$

$$v = -gt + u \ln\frac{m_0}{m}$$

$$\boxed{v = -gt + u \ln\left(\frac{m_0}{m_0 - wt}\right)}$$

For example, if $t = 90\text{s}$, then

$v = 2.4 \times 10^3 \text{ m/s}$.

If $t = 100\text{s}$, $v = 3.3 \times 10^3 \text{ m/s}$.

($0 \le t < 120\text{s}$ because in 120s, all the mass of the rocket would be gone, burned into exhaust gas, so before then we run out of fuel to burn.

$120\text{s} = \frac{m_0}{w}$ is how I found 120s.)