Uniform circular motion

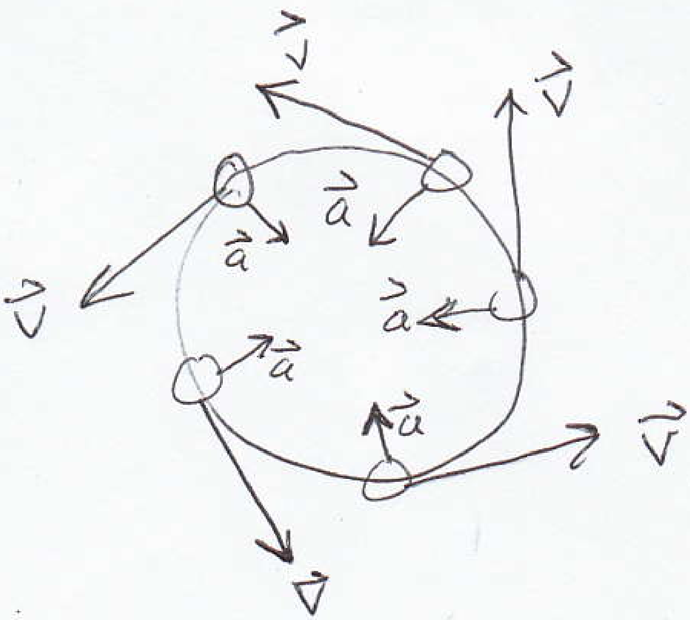
circular → not constant direction

Uniform → constant speed → not constant velocity

$$\vec{v} = \text{velocity} = (v_x, v_y, v_z)$$

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \text{speed}$$

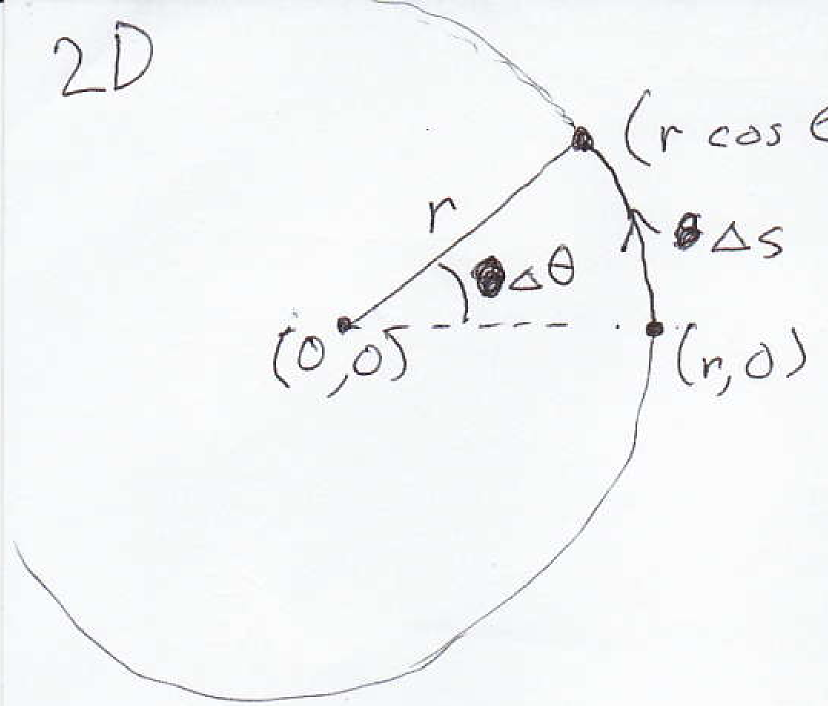
$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \text{direction} = \left(\frac{v_x}{\sqrt{v_x^2+v_y^2+v_z^2}}, \frac{v_y}{\sqrt{v_x^2+v_y^2+v_z^2}}, \frac{v_z}{\sqrt{v_x^2+v_y^2+v_z^2}} \right)$$



$$a = \frac{v^2}{r}$$

$$\frac{m}{s^2} = \frac{(m/s)^2}{m} \checkmark$$

2D



distance travelled from $(r,0)$ is arc length Δs

$\Delta s = r\Delta\theta$

$\uparrow$ θ in radians

If uniform circular motion, v is constant,

so v = average speed $= \dfrac{\Delta s}{\Delta t} = \dfrac{r\Delta\theta}{\Delta t}$

$$\Rightarrow \frac{\Delta\theta}{\Delta t} = \frac{v}{r} = \text{constant} \qquad \omega = \frac{d\theta}{dt} \text{ (notation)}$$

$$\Downarrow$$

$$\frac{v}{r} = \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega = \text{"angular velocity"}$$

$\uparrow$ constant

If $\theta = 0$ when $t=0$, then $\Delta\theta = \omega\Delta t$

implies $\underline{\theta = \omega t.}$

$$\text{position} = \vec{r} = (r\cos\omega t, r\sin\omega t)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = (r\, d(\cos\omega t)/dt, \; r\, d(\sin\omega t)/dt)$$

$$\vec{v} = \left(r(-\sin\omega t)\, d(\omega t)/dt,\; r(\cos\omega t)\, \underbrace{d(\omega t)}_{\underbrace{\omega}_{\text{constant}}\, dt}/dt\right)$$

$$\vec{v} = \left(r(-\sin\omega t)\,\omega,\; r(\cos\omega t)\,\omega\right)$$

$$\frac{v}{r} = \omega \Rightarrow v = r\omega$$

$$\vec{v} = v(-\sin\omega t, \cos\omega t)$$

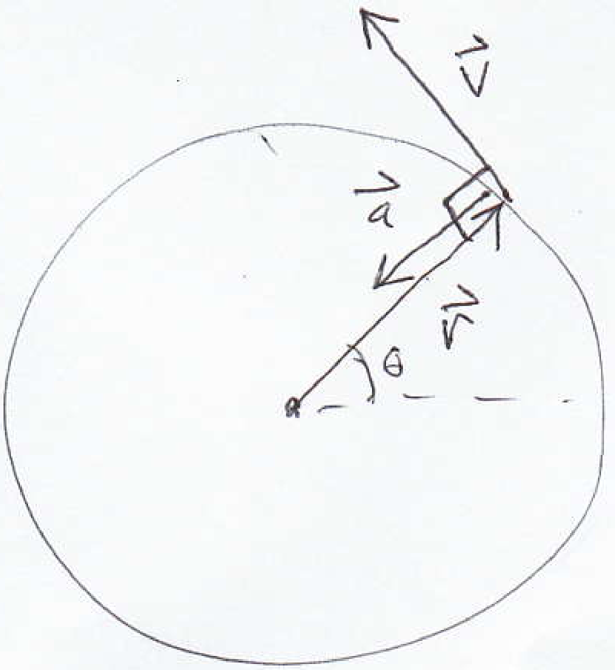
$$\vec{a} = \frac{d\vec{v}}{dt} = v\left((-\cos\omega t)\,\omega, (-\sin\omega t)\,\omega\right)$$

$$\vec{a} = v\omega(-\cos\omega t, -\sin\omega t)$$

$$\vec{a} = -\frac{v\omega}{r}(r\cos\omega t, r\sin\omega t) = -\frac{v\omega}{r}\vec{r}$$

$$\vec{a} = -\frac{v(v/r)}{r}\vec{r} = -\frac{v^2}{r^2}\vec{r}$$

$$a = |\vec{a}| = \frac{v^2}{r^2}r = \frac{v^2}{r}$$



$$a = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \frac{v}{r} \text{ if}$$
counterclockwise

$$\omega = \frac{-v}{r} \text{ if}$$
clockwise

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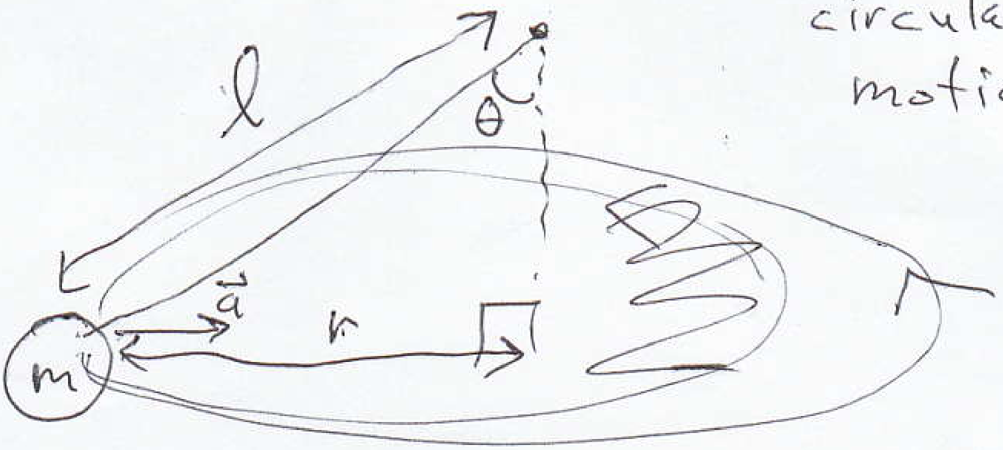
conical pendulum in <u>uniform</u> circular motion

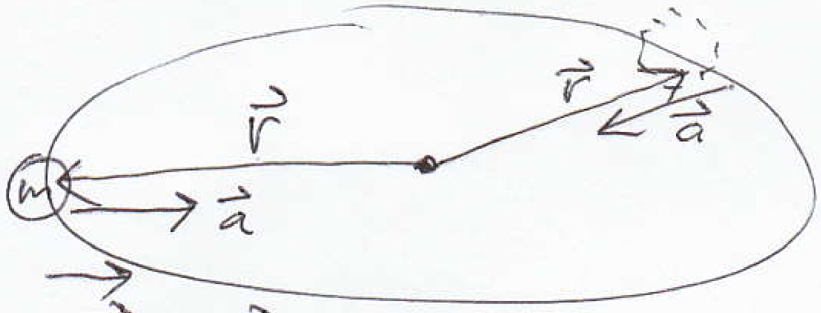


$$r = \ell \sin\theta \Leftarrow \sin\theta = \frac{opp}{hyp} = \frac{r}{\ell}$$

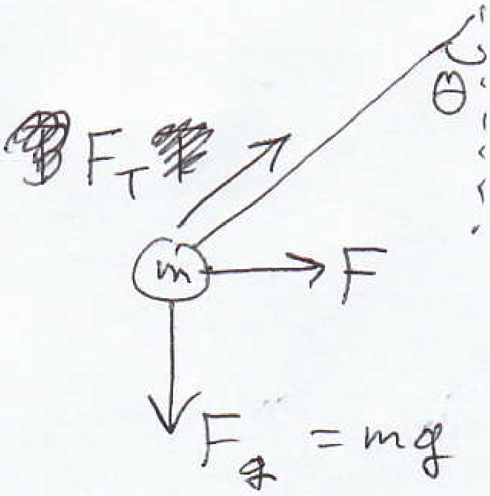
T = period = time for one revolution

$$v = \frac{\text{circumference}}{T} = \frac{2\pi r}{T} = \frac{2\pi \ell \sin\theta}{T}$$

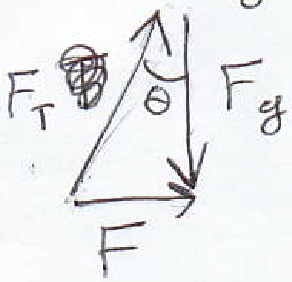
$$a = v^2/r \qquad \vec{F} = m\vec{a}$$

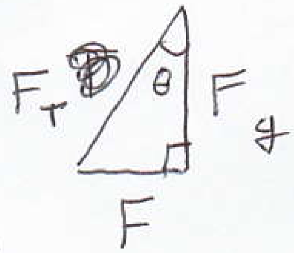


$\vec{F} = \dfrac{\vec{a}}{m}$ $\quad$ $\vec{F}$ & $\vec{a}$ are horizontal



$\vec{F} = \vec{F}_T + \vec{F}_g$





$\sin\theta = \dfrac{F}{F_T}$

$\cos\theta = \dfrac{F_g}{F_T}$

$$mg\tan\theta = F = F_g \tan\theta$$

$$F = F_T \sin\theta$$

$$F = \frac{F_g \sin\theta}{\cos\theta} \quad F_T = \frac{F_g}{\cos\theta} = \frac{mg}{\cos\theta}$$

$$m\frac{v^2}{r} = ma = F = mg\tan\theta \Rightarrow \frac{v^2}{r} = g\tan\theta$$

$$v = \frac{2\pi r}{T} \qquad \frac{(2\pi r/T)^2}{r} = g\tan\theta$$

here T is period, not tension.

$$\frac{4\pi^2 r}{T^2} = \frac{4\pi^2 r^2/T^2}{r} = g\tan\theta$$

$$\theta = \underbrace{\arctan}_{\tan^{-1}}\left(\frac{4\pi^2 r}{gT^2}\right)$$

$$\frac{4\pi^2 \ell \cancel{\sin\theta}}{T^2} = g\,\frac{\cancel{\sin\theta}}{\cos\theta}$$

$$\frac{T^2}{4\pi^2 \ell} = \frac{\cos\theta}{g}$$

$$\theta = \arccos\left(\frac{gT^2}{4\pi^2 \ell}\right)$$

$$\theta = \arcsin\left(\frac{r}{\ell}\right) \text{ because } \sin\theta = \frac{r}{\ell}$$

You can measure r, ℓ, T and check that both formulas equal the same angle

HW #1 If a mass of 340g travels around a circle with radius 40cm with frequency of 8 revolutions per minute, then Find:

a) acceleration in m/s^2
b) speed in m/s
c) angular velocity in rad/s (= 1/s)
d) period in s
e) kinetic energy in J

(Assume speed is constant.)

HW #2 If a string breaks at tension 1300N, then what is the maximum mass that we could swing from the string with constant speed and period 2.0s? The length of the string is 85cm.

#3 Working with 2D vector notation, if $\vec{r} = (3.0\,\text{m} \cos \omega t,\ 3.0\,\text{m} \sin \omega t)$

and $\omega = \text{constant} = 5.2\ \text{rad/s}$,

find $\vec{v}$, v, $\hat{v}$, $\vec{a}$, a, $\hat{a}$

when $t = 0.40\,\text{s}$.

Sketch the vectors $\vec{v}$, $\hat{v}$, $\vec{a}$, $\hat{a}$, $\vec{r}$.

Read 4.1.