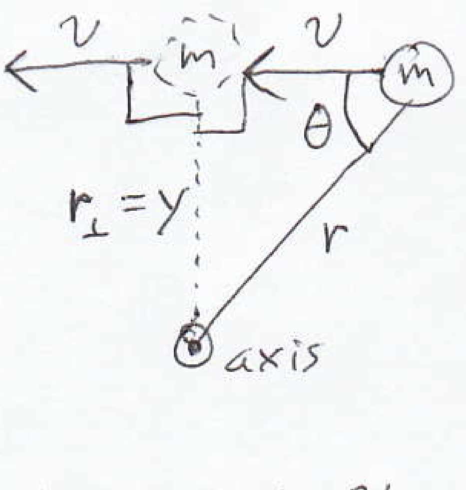
Like $F = \frac{dp}{dt}$, $\tau = \frac{dL}{dt}$.

Like $p = mv$, $|L| = m|v_\perp| r = mr|v_\perp|$

$|\tau| = |F_\perp| r = r|F_\perp|$

Today 4.1: angular momentum in the plane
(counter) clockwise rotation
about an axis perpendicular
to the plane

4.2: (Read for Thursday.)
Fully 3D angular momentum.



$\sin\theta = \frac{y}{r}$

$y = r\sin\theta$

$v = 2.0\,m/s$

$\theta = 60°$

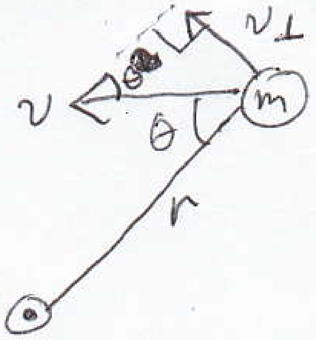
$r = 3.0\,m$

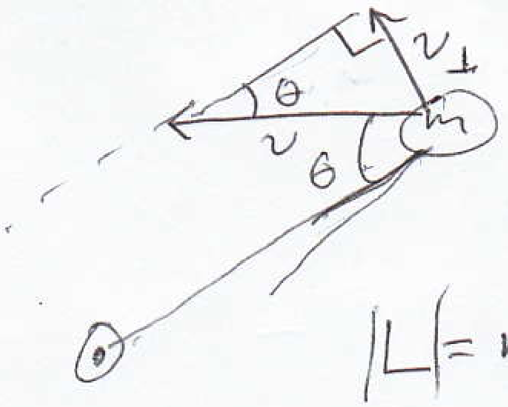
$m = 1.5\,kg$

$\underline{L = 7.8\,kg\,m^2/s}$

$L = myv = mr_\perp v$ also $L = mv_\perp r$

$$\boxed{m(r\sin\theta)v}$$



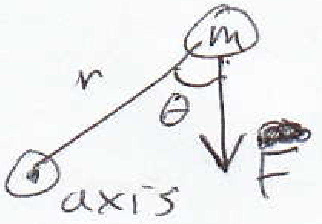


$$\sin\theta = \frac{v_\perp}{v}$$

$$v_\perp = v\sin\theta$$

$$|L| = mr|v_\perp| = \boxed{mrv\sin\theta}$$

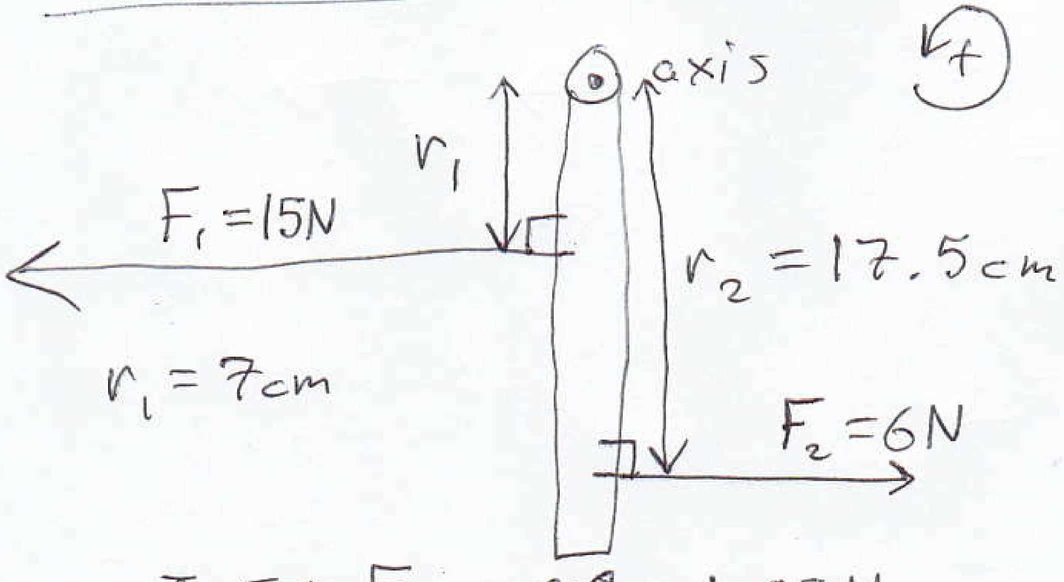
Similarly,



$$|\tau| = |F|r_\perp = |F_\perp|r = |F|r\sin\theta$$

sign of torque τ depends on which way is positive in your coordinate system.

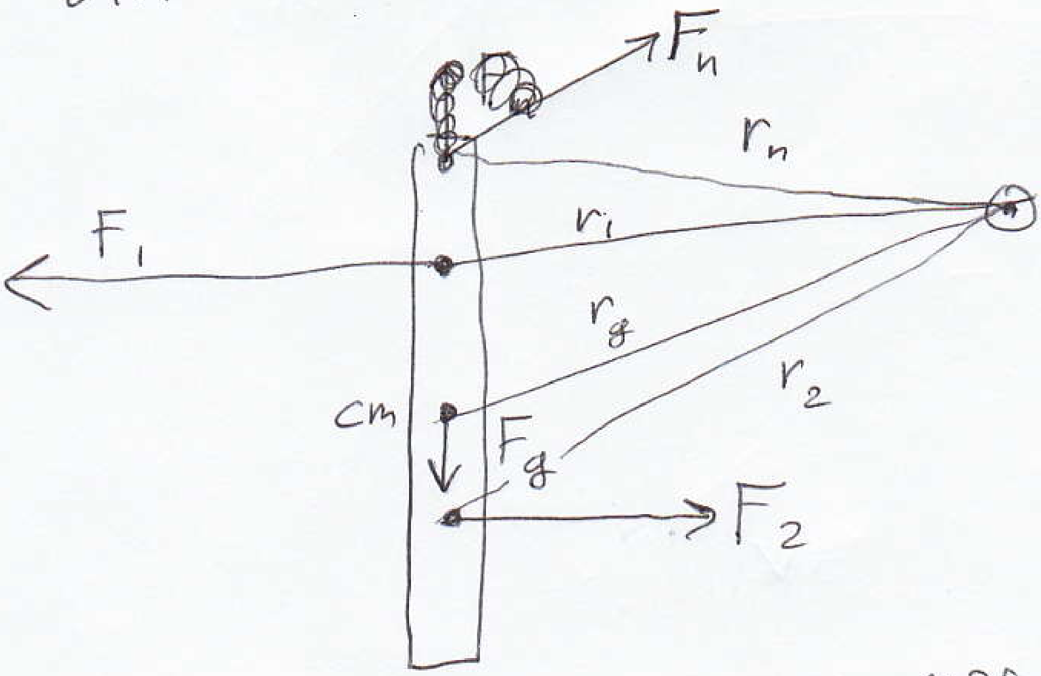
You can choose $\circlearrowleft +$ or $+\circlearrowright$.



$$\tau_2 = +r_2 F_2 \underbrace{\sin 90^\circ}_{1} = +1.05\,N\cdot m$$

$$\tau_1 = -r_1 F_1 \underbrace{\sin 90^\circ}_{1} = -1.05\,N\cdot m$$

Different choice of axis:



The torques would be different, but if the strip isn't accelerating its rotation, then they add to 0.

The forces also add to 0 if the strip isn't accelerating in the usual sense of motion of its cm.

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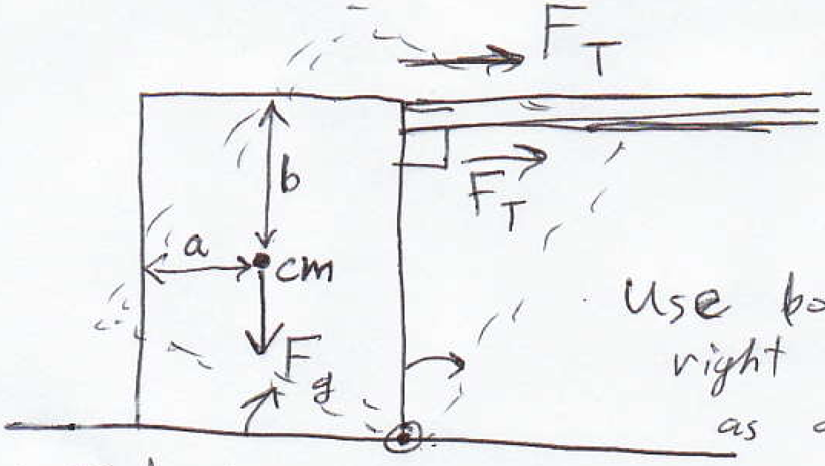
3 (more) homework problem (quick)

Ch. 4 #2, 6, 12

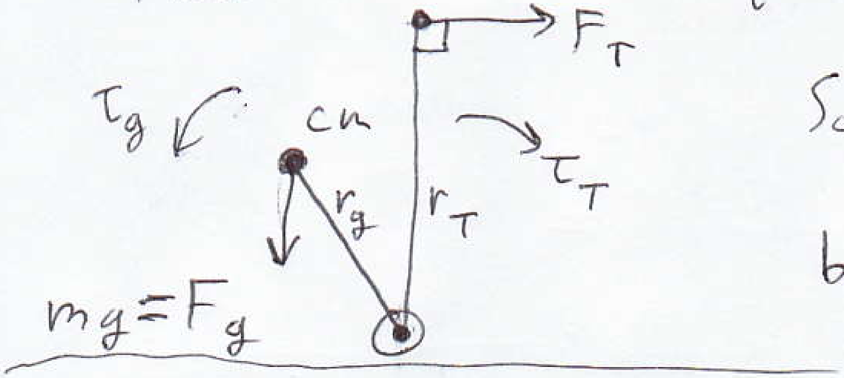
So, do #1, 2, 3 from Monday's email and Ch. 4 # 2, 6, 12



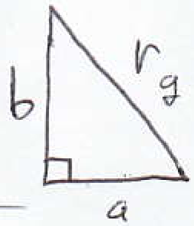
Use bottom right corner as axis.

m = box mass

How hard can we pull before it tips?

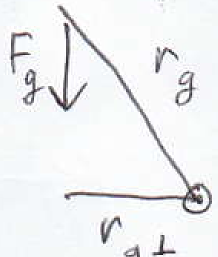


Solve for $\tau_T + \tau_g = 0$



$r_g = \sqrt{a^2+b^2}$

$r_T = 2b$



$r_{g\perp} = a$

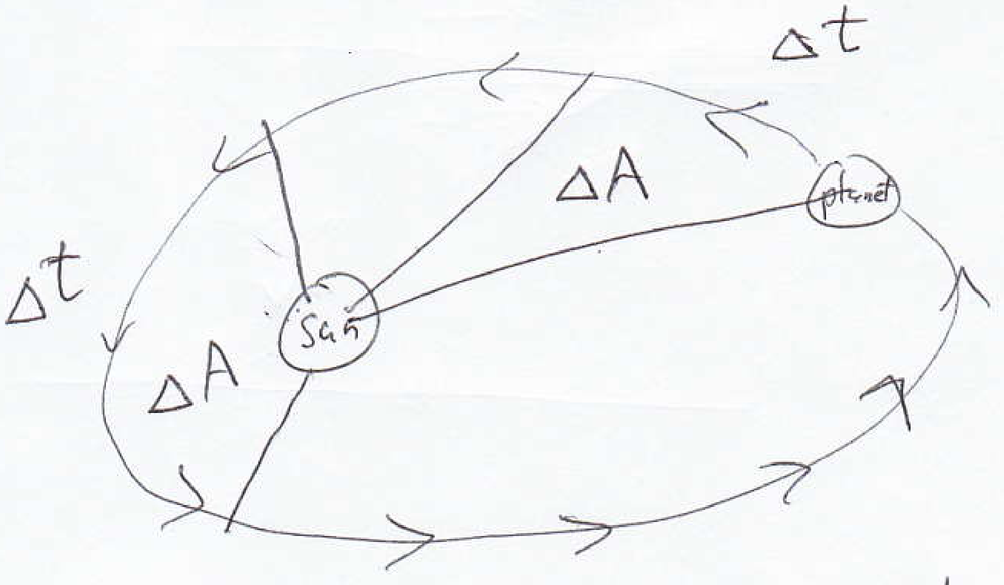
$\tau_g = r_{g\perp} F_g$

$\tau_g = amg$

$\cancel{r_T F_T = 2bF_T}$

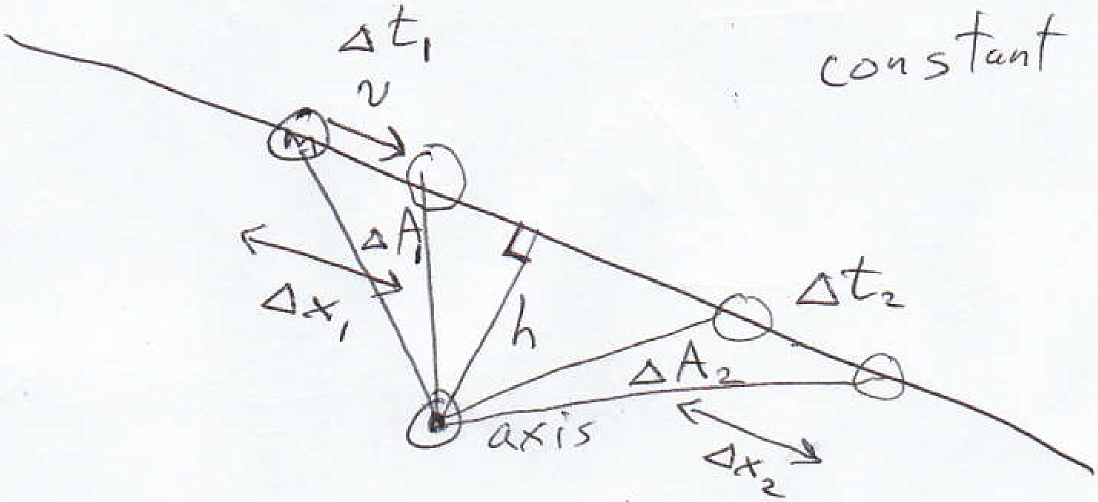
$\tau_T = -r_T F_T = -2bF_T$

$0 = \tau_T + \tau_g = -2bF_T + amg \Rightarrow \underbrace{F_T = \frac{amg}{2b}}_{\text{max force not tipping over box.}}$



Kepler's equal area law.

This is just conservation of angular momentum.



constant $\vec{v}$

If Δt is the same,

is ΔA the same?

If $\Delta t_1 = \Delta t_2$, does $\Delta A_1 = \Delta A_2$?

$$v = \frac{\Delta x_1}{\Delta t_1} = \frac{\Delta x_2}{\Delta t_2} \ \& \ \Delta t_1 = \Delta t_2 \Rightarrow \Delta x_1 = \Delta x_2$$

$$\Delta A_1 = \frac{1}{2} \Delta x_1 h \qquad \Delta A_2 = \frac{1}{2} \Delta x_2 h \Rightarrow \Delta A_1 = \Delta A_2$$