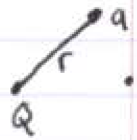
Notes 8/30

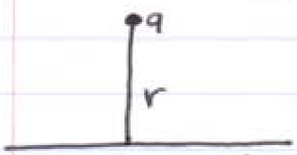
Brief mention of 21, and starting 22

Last week



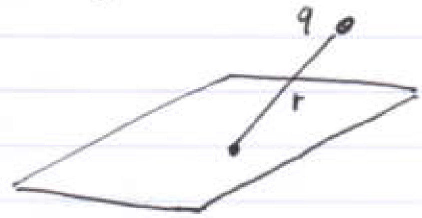
• Point charge $\Rightarrow E \propto \frac{1}{r^2}$ $\left(E = \frac{KQ}{r^2} = \frac{Q}{4\pi\epsilon_0 r^2}\right)$

Long thin line of charge $\Big\} \Rightarrow E \propto \frac{1}{r}$ $\frac{F}{q} = E = \frac{2K\lambda}{r} = \frac{\lambda}{2\pi\epsilon_0 r}$



$\lambda = $ charge/length

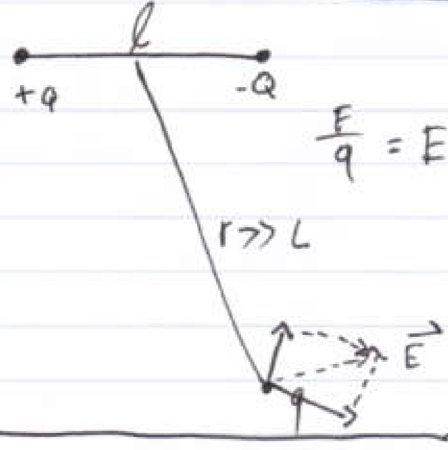
Big (Flat) plane of charge



$\frac{F}{q} = E = 2\pi k$
$= \frac{\sigma}{2\epsilon_0}$

σ = charge per area

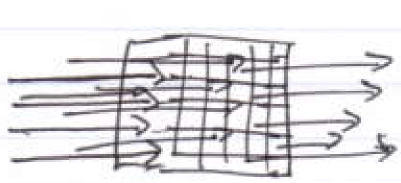
Dipoles

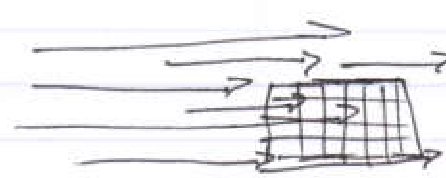


$\frac{F}{q} = E$ approximately $\propto \frac{1}{r^3}$

Gauss' Law : Chapter 22

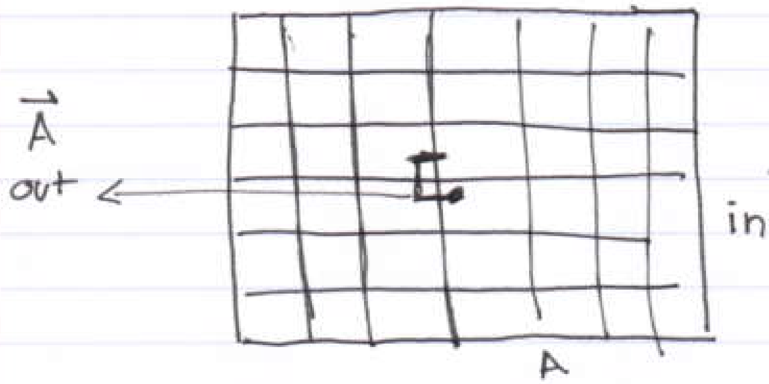
Flux : how much "goes through"





vector $\vec{V}$

Plane A



- Vector $\vec{V}$
- Plane A
- choose on "out" side

$\vec{A}$ is the "out ward normal" area vector
$\vec{A}$ has magnitude = area of Plane
something like $3m^2\hat{i} - 4m^2\hat{j} = \vec{A}$

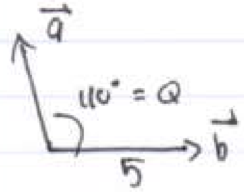
$$|\vec{A}| = \sqrt{(3m^2)^2 + (-4m^2)^2} = 5m^2$$

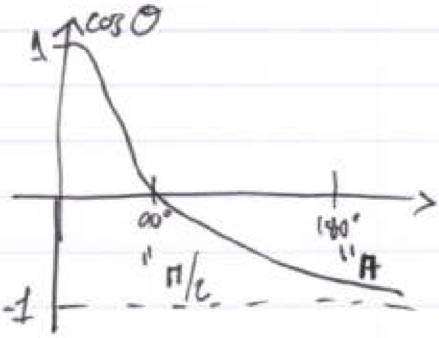
The flux of of $\vec{V}$ through $\vec{A}$
is $\vec{V} \cdot \vec{A}$.

Example: $\vec{V} = (-2\hat{j} + 1\hat{k})\frac{N}{C}$

$$\vec{V} \cdot \vec{A} = (3\hat{j} - 4\hat{j}) \cdot (-2\hat{j} + \hat{k}) \frac{N \cdot m^2}{C}$$
$$= (-4\hat{j})(-2\hat{j}) \frac{N \cdot m^2}{C} = 8 \frac{N m^2}{C}$$

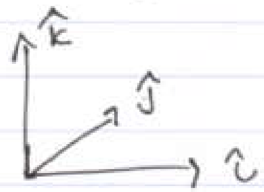
Ex. $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \underbrace{\theta}_{\text{angle between vectors}} = 3.5 \cdot \cos 110° \approx -5.1$





In reverse: $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}| + |\vec{b}|} = \cos\theta$ $\qquad \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}\right) = \theta$

$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0 = 1\cdot 1\cdot 1 = 1$

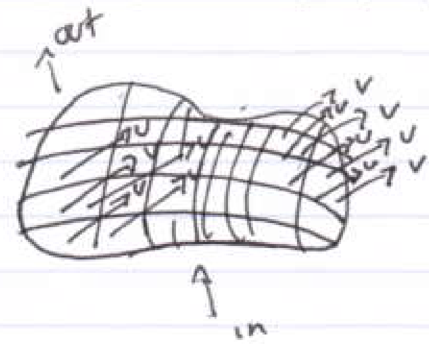


$\hat{i}\cdot\hat{j} = |\hat{i}|\cdot|\hat{j}|\cos 90° = 1\cdot 1\cdot 0 = 0$

$(a\hat{i} + b\hat{j} + c\hat{k})\cdot(u\hat{i} + v\hat{j} + w\hat{k}) = au + bv + cw$

$(x_1, y_1, z_1)\cdot(x_2, y_2, z_2) = x_1x_2 + y_1y_2 + z_1z_2$

Flux for curve surfaces : Break up into little pieces that are almost flat.



compute flux for each ~~each~~ piece and add up:

$\vec{V}\cdot A_1 + v_1\cdot A_2 + \dots\dots$

Limiting process of smaller & smaller pieces

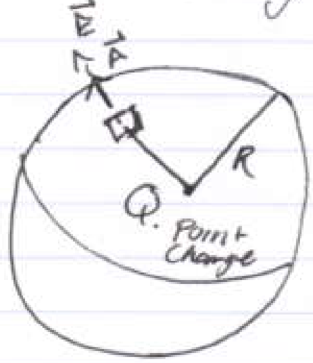
$\int \vec{V}\cdot d\vec{A}$

Flux for vector field : $\vec{V}$ is different for each little piece of the surface.

same formula : $\int \vec{V} \cdot d\vec{A}$, but $\vec{V}$ is not constant

Example :

$\vec{E} \cdot \vec{A} = EA = |\vec{E}||\vec{A}|$



Q charge inside a sphere of radius R
Q is at the center

Total flux

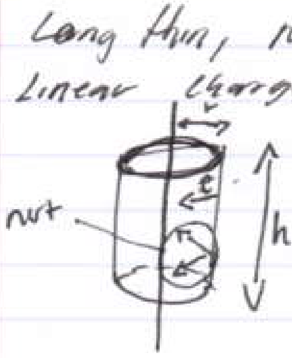
$$= \int_{\text{sphere}} \vec{E} \cdot d\vec{A} = \int_{\text{sphere}} \underbrace{E}_{\text{constant} = \frac{kQ}{R^2}} dA$$

$$= \frac{kQ}{R^2} \int_{\text{sphere}} dA = \frac{kQ}{R^2} 4\pi R^2$$

$$= 4\pi k Q = \frac{Q}{\epsilon_0}$$

For any closed surface, $\int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

Long thin, negatively charged wire with uniform linear charge density $-\lambda$



Electric field always pointing straight toward the wire.

$$\frac{Q_{\text{enclosed}}}{\epsilon_0} = \text{Flux}$$

$$\frac{h(-\lambda)}{\epsilon_0} = \int \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A}$$

$$\rightarrow 0 + 0 + \int_{\text{side}} (\ldots) dA$$

$$= \boxed{E \cdot 2\pi r h}$$

$$\frac{h\lambda}{\epsilon_0} = E \cdot 2\pi r h$$

$$\frac{\lambda}{2\pi r \epsilon_0} = E$$

$$\parallel$$

$$\frac{2k\lambda}{r}$$