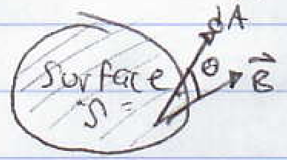
10/18/10

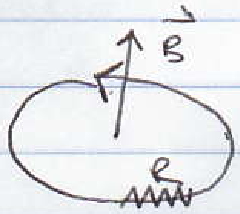
Faraday's Law

induced $\mathcal{E} = \dfrac{-d\Phi_B}{dt}$



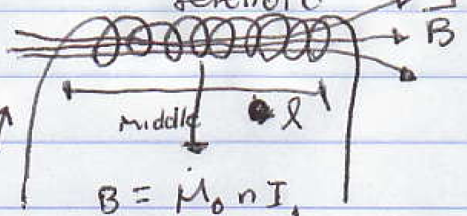
$\Phi_B = \int_S \underbrace{\vec{B} \cdot d\vec{A}}_{B \cos\theta \, dA}$

dA, B uniform $\Rightarrow \Phi_B = B \underbrace{A}_{\text{Area of } S} \cos\theta$



direction of positive induced current

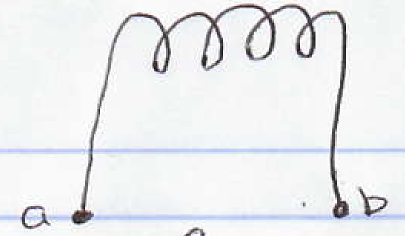
$I = \dfrac{\mathcal{E}}{R} = -\dfrac{1}{R}\dfrac{d\Phi_B}{dt}$



$B = \mu_0 n I_1$

$n = \dfrac{N}{L} = \dfrac{\#\text{ Loops}}{\text{lenght}}$

If I_1 changes, then B changes, which causes the total magnetic flux for the second coil to change which indeces an emf on the second coil (which results in an induced current)



N_2 loops for second coil

$\mathcal{E}_2$

$$\mathcal{E}_2 = \frac{-d\Phi_2}{dt} = -N_2 \frac{dB}{dt} A$$

$$\mathcal{E}_2 = -N_2 \frac{d}{dt}(\mu_0 n_1 I_1) A = \underbrace{-N_2 \mu_0 n_1 A}_{M} \frac{dI_1}{dt}$$

mutual inductance

$$M = N_2 \mu_0 n_1 A$$

$$M = N_2 \mu_0 \frac{N_1}{L} A = \frac{N_1 N_2 \mu_0 A}{L}$$

M doesn't change if switch 1 & 2

$$\mathcal{E}_1 = -N_1 \mu_0 n_2 A \frac{dI_2}{dt}$$

induced

$$\mathcal{E}_2 = -M \frac{dI_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{dI_2}{dt}$$

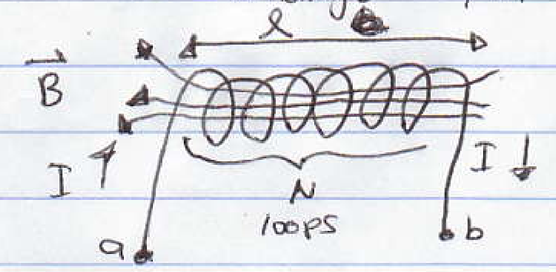
The unit of inductance is the henry:

$$1\,H = 1 \frac{T \cdot m^2}{A} = \frac{1\,V}{A/s} = \frac{1\,V \cdot s}{A} = 1\,\Omega \cdot s$$

$$\boxed{M = \frac{-\mathcal{E}_2}{dI_1/dt}}$$

→ Defines mutual inductance between any two circuits, not just two selenoids.

if I changes, it induces an emf.



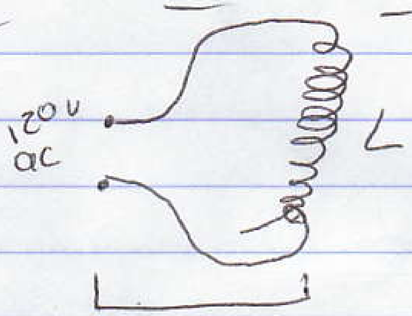
$$\mathcal{E} = -\frac{d\Phi}{dt} = -N\frac{dB}{dt}A$$

$$n = N/\ell$$

$$= -N\frac{d}{dt}\left(\mu_0 n I\right)A = -\underbrace{\frac{N^2\mu_0 A}{\ell}}_{L}\frac{dI}{dt}$$

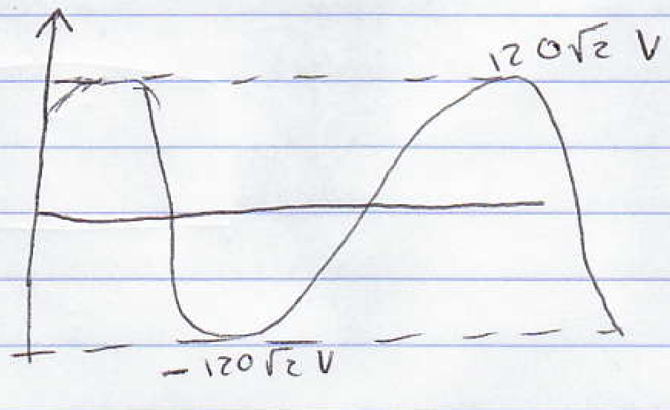
self-inductance

$$L = \frac{-\mathcal{E}}{dI/dt} \qquad \Rightarrow \mathcal{E} = -L\frac{dI}{dt}$$



$$V_{peak} = \sqrt{2}\, V_{rms}$$

$$V = (120\text{ V})\sqrt{2}\,\sin(\omega t + \underbrace{\phi}_{\text{constant}})$$



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

$$\underset{\text{for ac}}{=} \frac{V_{peak}}{\sqrt{2}}$$

$$V_p = 120\sqrt{2}\ V$$

$$V = V_p \sin(\omega t + \phi)$$

$$I = \frac{V_p}{R} \sin(\omega t + \phi)$$

$$\frac{dI}{dt} = \frac{V_p}{R} \cos(\omega t + \phi) \frac{d}{dt}(\omega t + \phi)$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$\mathcal{E} = -\frac{V_p}{R} \omega \cos(\omega t + \phi)$$