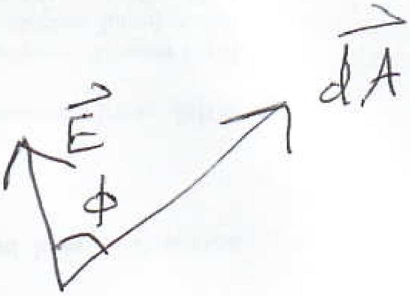
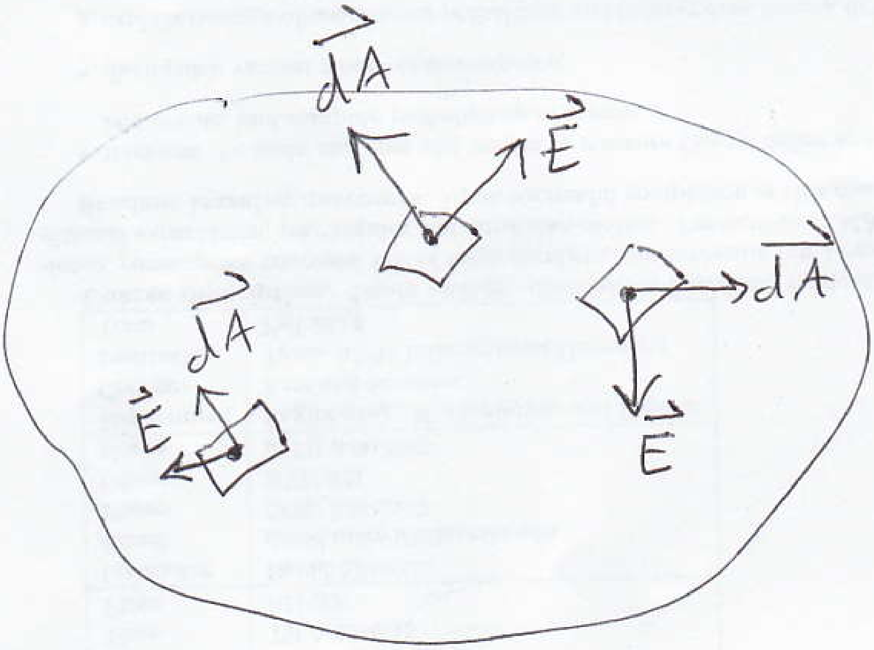
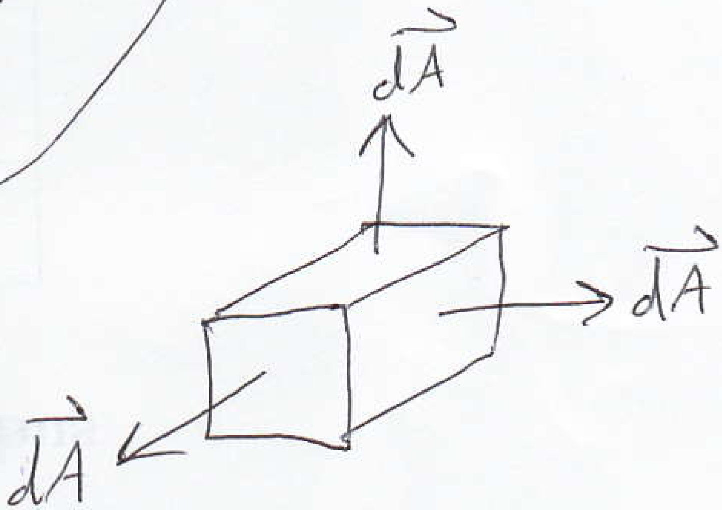
$$k = \frac{1}{4\pi \varepsilon_0} \qquad \Phi_E = \oint \frac{Q_{encl}}{\varepsilon_0}$$

$$\Phi_E = \underbrace{\oint}_{\text{closed}} \vec{E}\cdot d\vec{A} = \underbrace{\oiint}_{\text{closed}} \overbrace{\vec{E}\cdot d\vec{A}}^{E\,dA\cos\phi}$$

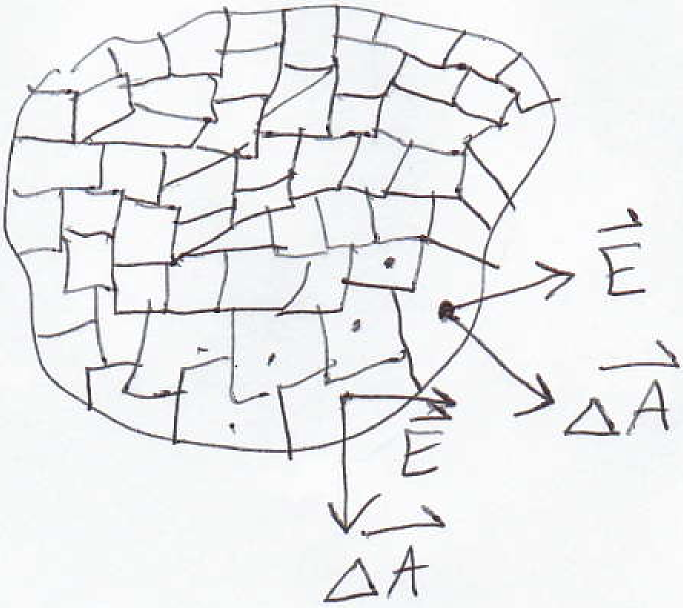


$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$





$$\oiint \vec{E} \cdot \vec{dA} \approx \sum_{\text{pieces}} \vec{E} \cdot \vec{\Delta A}$$



$|\vec{\Delta A}| = \Delta A =$ surface area of that piece

Symmetry helps!

$$\Phi_E = \oiint E \, dA \cos\phi$$

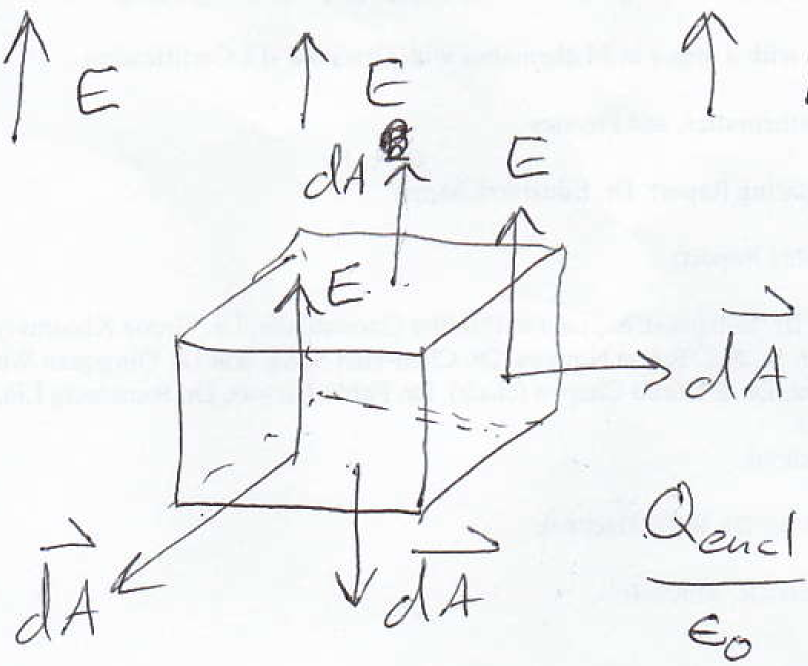
If ϕ & E are constant on the surface, then

$$\Phi_E = E \cos\phi \oiint dA = E(\cos\phi) A$$

- For parts of surface where $\cos\phi = 0$ (i.e., $\vec{E} \perp \vec{dA}$), $E \, dA \cos\phi = 0$, so ignore them.

Example

assuming $\vec{E}$ points up everywhere:



$$\frac{Q_{encl}}{\epsilon_0} = \Phi_E = \iint_{top} E\,dA - \iint_{bottom} E\,dA$$

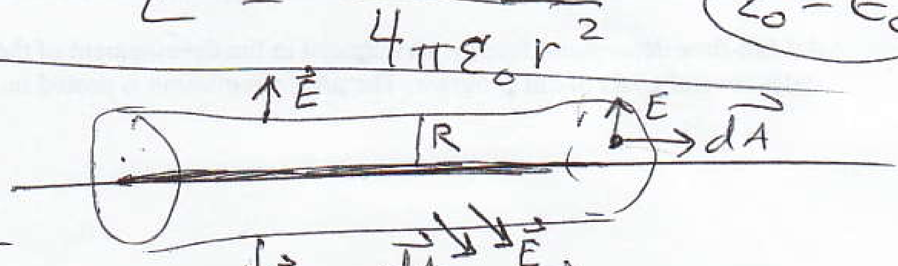
If E constant on top & bottom,

then $\frac{Q_{encl}}{\epsilon_0} = E_{top}A_{top} - E_{bottom}A_{bottom}$

---

point charge: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $\varepsilon_0 = \epsilon_0$

long line of ~~constant~~ uniform linear charge density λ:



$E = \frac{\lambda}{2\pi\varepsilon_0 R}$

length L & radius R cylinder:

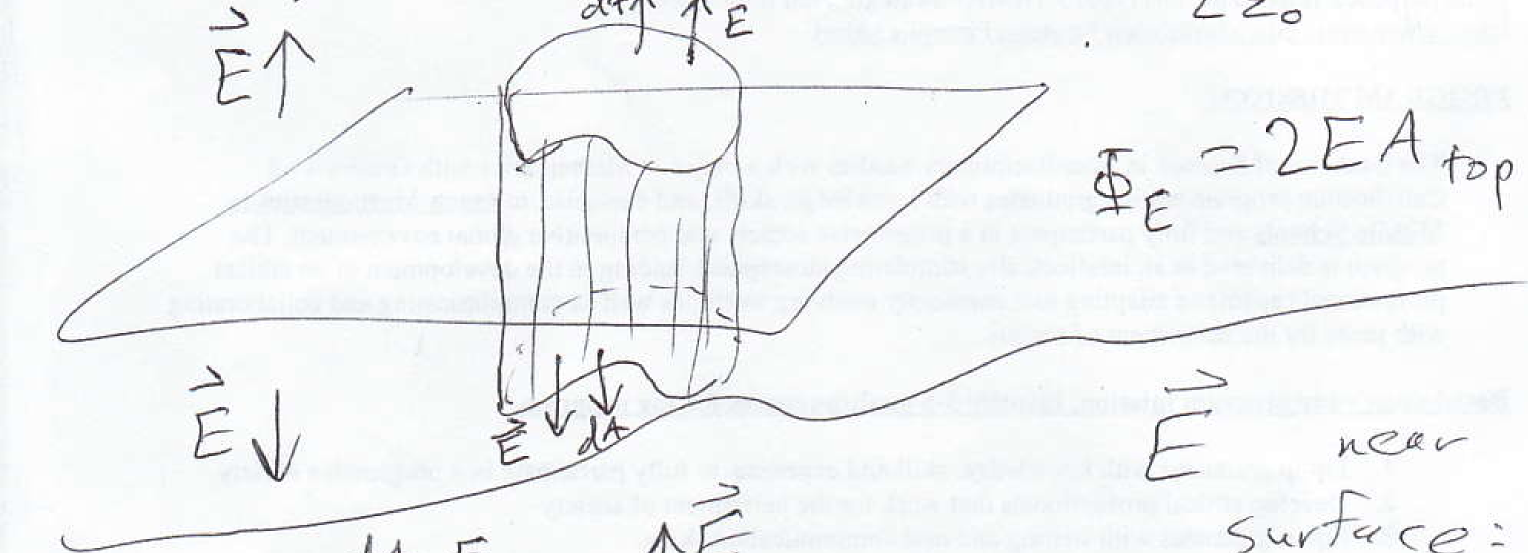
$$Q_{encl} = \lambda L \qquad \Phi_E = E A_{lateral}$$

$$\frac{\lambda L}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0} = \Phi_E = E\, 2\pi R L$$

So, $E = \dfrac{\lambda}{2\pi\varepsilon_0 R}$

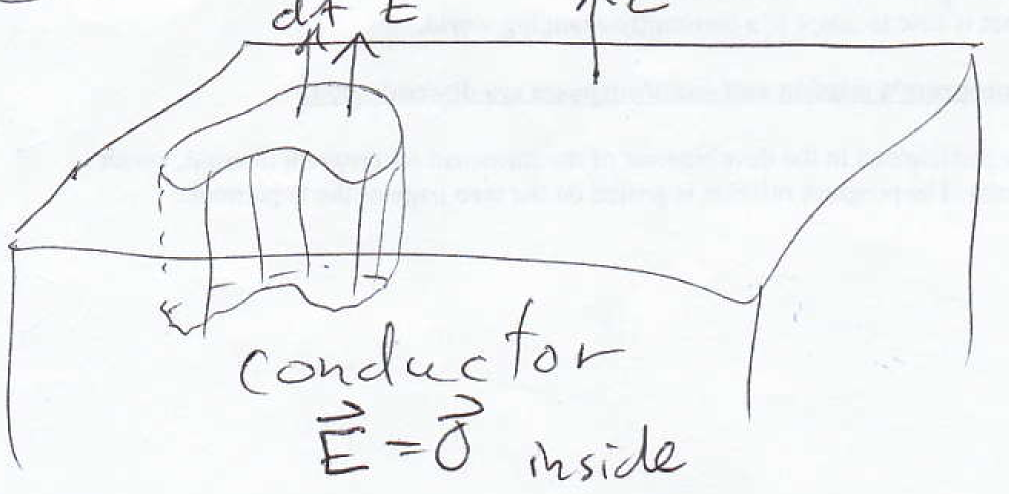
---

thin, infinite plane of uniform surface charge density σ: $E = \dfrac{\sigma}{2\varepsilon_0}$



$$\Phi_E = 2EA_{top}$$

$\vec{E}$ near surface:

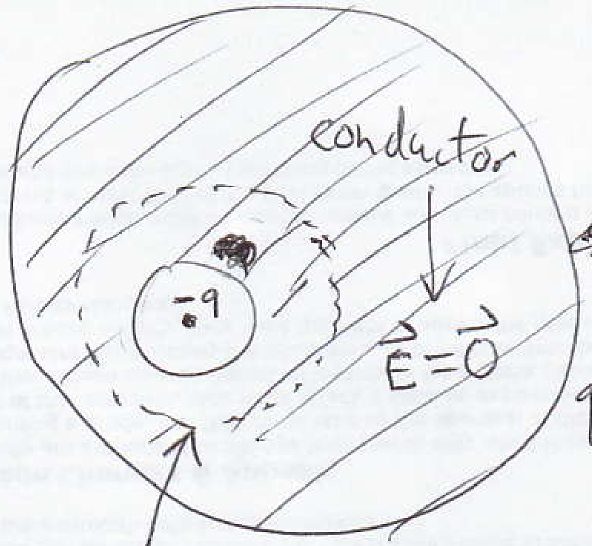


conductor

$\vec{E} = \vec{0}$ inside

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\Phi_E = EA_{top}$$



Q = net charge of conductor

$q_{outer\ surface}$

$$Q = \overbrace{q_{inner}}^{q} + q_{outer}$$

$$\Downarrow$$

$$q_{outer} = Q - q$$

(a)

Apply Gauss' Law

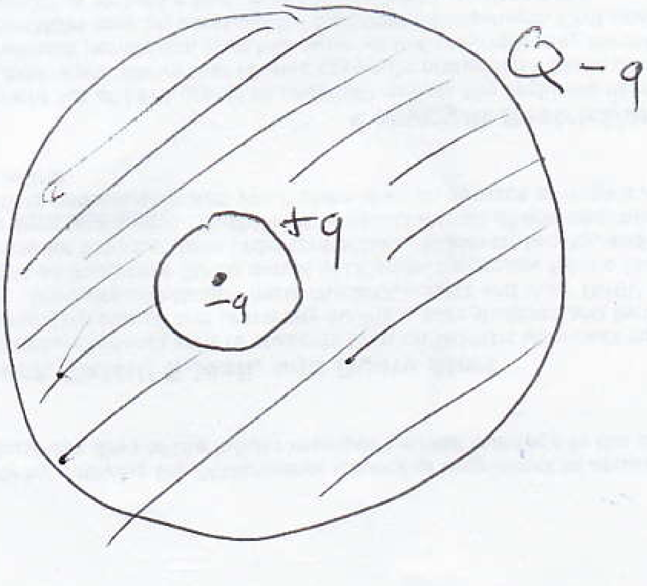
$$\frac{Q_{encl}}{\varepsilon_0} = \Phi_E = 0$$

$$\Downarrow$$

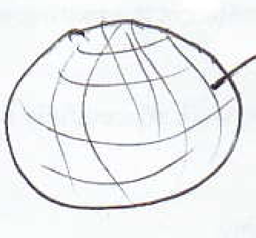
$$-q + q_{inner\ surface} = 0$$

(b) $q_{inner} = q$

[Q #12 Ch. 22]



Pr. # 62 (Ch. 22)



$\to E = 3\times10^{6}\ N/C$

$2R = 0.75\ cm = 7.5\times10^{-3}\ m$

Q = net charge of pea = ?

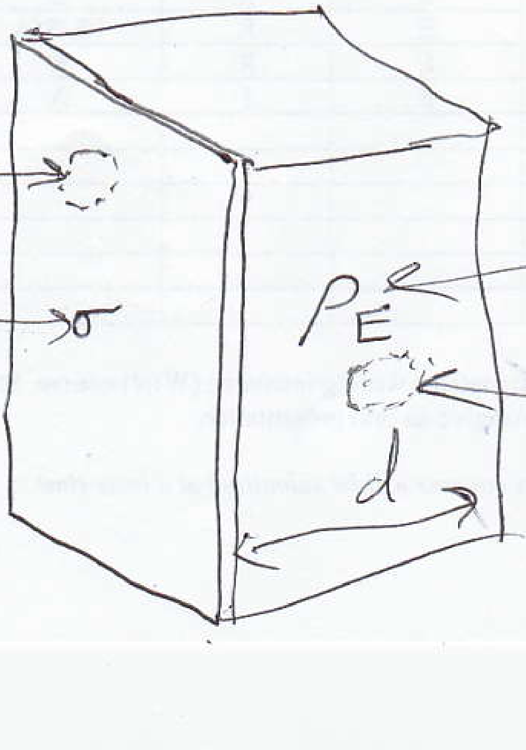
$4\pi R^2 = A = 4.7\times10^{-2}\ m^2$

symmetry: $\dfrac{Q}{\varepsilon_0} = EA \Rightarrow Q = \varepsilon_0 EA = 10^{-6}\ C$

$\varepsilon_0 = 8.85\cdot10^{-12}\ \dfrac{C^2}{N\cdot m^2}$

$\left(\dfrac{C^2}{N\cdot m^2}\right)\left(\dfrac{N}{C}\right)\left(m^2\right)$

Pr #60 (Ch. 22)



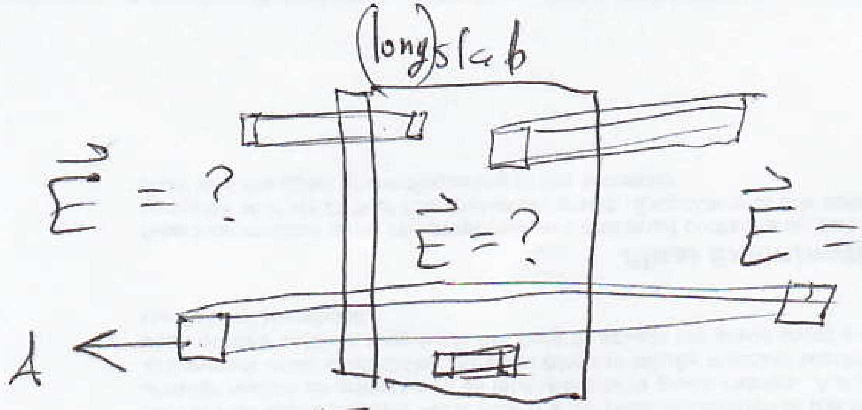
constant

$\dfrac{Q_{piece}}{A_{piece}} = \dfrac{\text{charge}}{\text{(surface area)}} \to \sigma$

$\rho_E \leftarrow \dfrac{\text{charge}}{\text{volume}}$ constant

$\dfrac{Q_{piece}}{V_{piece}} = \rho_E$

d



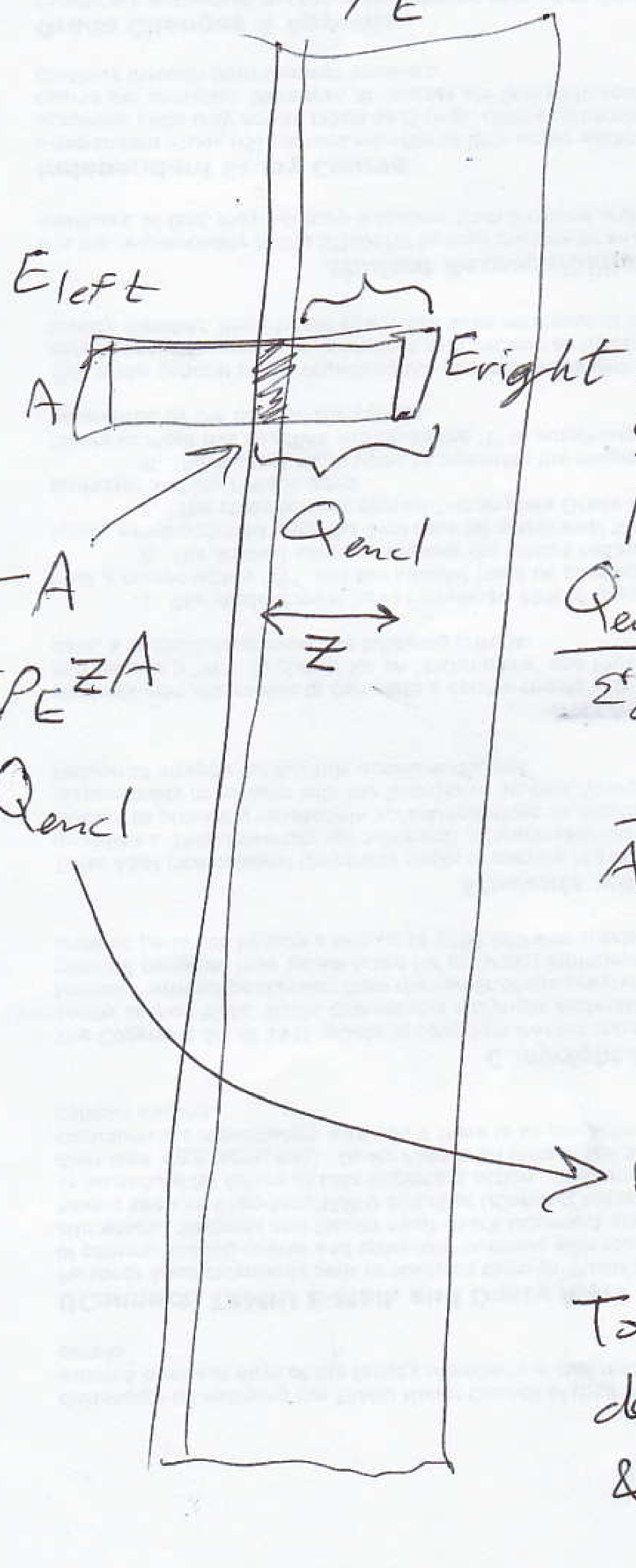
$\vec{E}$ always points $\rightarrow$ or $\leftarrow$

σ

ρ_E

$$\Phi_E = \vec{E}_{left} \cdot \vec{A}_{left} + \vec{E}_{right} \cdot \vec{A}_{right}$$

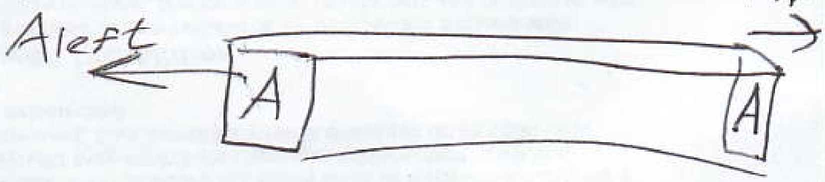
A = area of left base of cylinder



$\sigma A + \rho_E z A = Q_{encl}$

$$\underbrace{\Phi_E}_{\frac{Q_{encl}}{\varepsilon_0}} = \underbrace{\vec{E}_{left} \cdot \vec{A}_{left}}_{\pm E_{left} A} + \underbrace{\vec{E}_{right} \cdot \vec{A}_{right}}_{\pm E_{right} A}$$

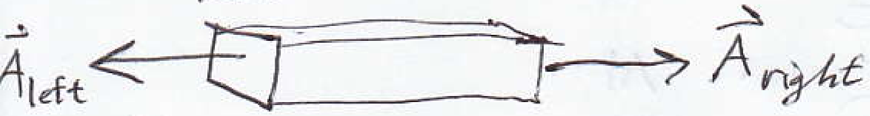
area of base



$$\sigma + \rho_E z = \pm E_{left} \pm E_{right}$$

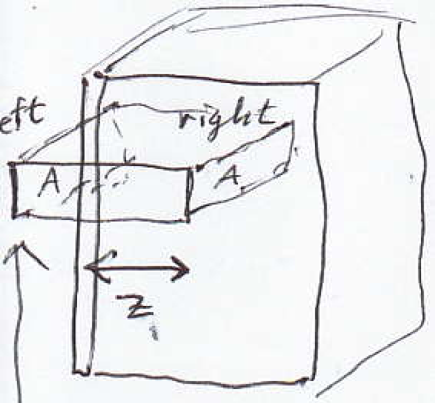
To banish the $\pm$'s, declare $\rightarrow \hat{z}$ direction & ~~compute~~ work with $E_{left,z}$ & $E_{right,z}$

$\vec{A}_{left} = -A\hat{k} \quad \vec{A}_{right} = A\hat{k}$



$\xrightarrow{1} \hat{k}$

components in z-direction



$$\Phi_E = \boxed{E_{left,z}}\,\hat{k} \cdot (-A\hat{k}) + \boxed{E_{right,z}}\,\hat{k} \cdot A\hat{k}$$

$$\Phi_E = (-E_{left,z} + E_{right,z})A$$

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_0} = \frac{\sigma A + \rho_E z A}{\varepsilon_0}$$

$$\Downarrow$$

here $\vec{E} = \underbrace{E_{left,z}}_{\text{may be a positive or negative quantity}}\hat{k}$

$$(\star)\; E_{right,z} - E_{left,z} = (\sigma + \rho_E z)/\varepsilon_0$$

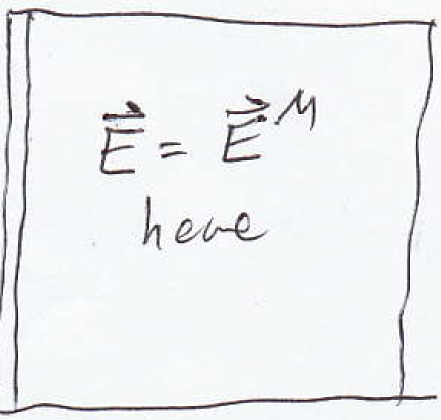
2 unknowns

We need more equations. So, try more cylinders with Gauss' Law.

Let's add some notation:

L, M, R : left, middle, right

$\vec{E} = \vec{E}^L$ here



$\vec{E} = \vec{E}^R$ here

In this notation, (☆) becomes

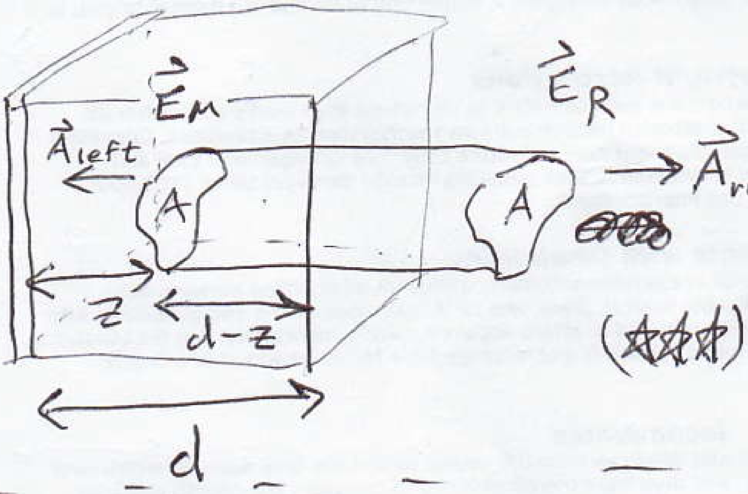
$$E_z^M - E_z^L = \frac{\sigma + \rho_E z}{\varepsilon_0}. \qquad (☆☆)$$

Another cylinder: $\Phi_E = E_z^R A - E_z^M A$

$\Phi_E = Q_{encl}/\varepsilon_0$

$Q_{encl} = \rho_E A(d-z)$

$\Downarrow$



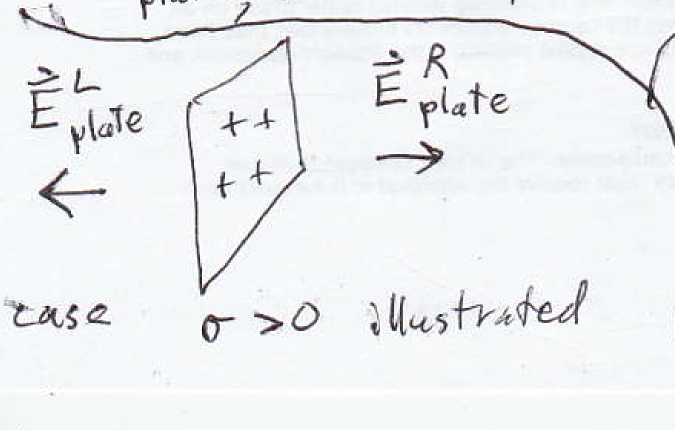
$$(☆☆☆) \quad E_z^R - E_z^M = \frac{\rho_E (d-z)}{\varepsilon_0}$$

~~Another cylinder~~

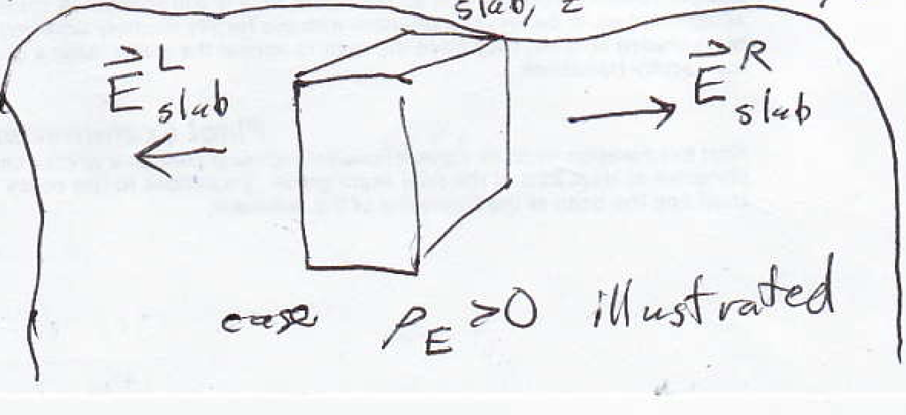
Combine (☆☆) & (☆☆☆) by ~~adding~~ subtracting the equations to get (☆4):

$$E_z^R - E_z^L = (E_z^R - E_z^M) + (E_z^M - E_z^L) = \frac{\rho_E (d-z)}{\varepsilon_0} + \frac{\sigma + \rho_E z}{\varepsilon_0} = \frac{\sigma + \rho_E d}{\varepsilon_0}$$

$\cancel{2E_z^M = (E_z^M - E_z^L)}$ By symmetry, the field created by the plate, E_{plate}, satisfies $E^L_{plate,z} = -E^R_{plate,z}$, & likewise $E^L_{slab,z} = -E^R_{slab,z}$



case $\sigma > 0$ illustrated



case $\rho_E > 0$ illustrated

Hence, $E_z^L = E^L_{plate,\,z} + E^L_{slab,\,z}$

$$E_z^L = -E^R_{plate,\,z} - E^R_{slab,\,z}$$

$$E_z^L = -E_z^R$$

Combine this with ($\star$4):

$$-2E_z^L = 2E_z^R = \frac{\sigma + \rho_E d}{\varepsilon_0}$$

Hence, $\boxed{E_z^L = -\frac{\sigma + \rho_E d}{2\varepsilon_0}}$ & $\boxed{E_z^R = \frac{\sigma + \rho_E d}{2\varepsilon_0}}$.

Combine this with ($\star\star$):

$$E_z^M = E_z^L + \frac{\sigma + \rho_E z}{\varepsilon_0} = -\frac{\sigma + \rho_E d}{2\varepsilon_0} + \frac{2\sigma + 2\rho_E z}{2\varepsilon_0}$$

$$\Rightarrow \boxed{E_z^M = \frac{\sigma + \rho_E (2z - d)}{2\varepsilon_0}}$$ as shown below:

(where z is distance from plate)