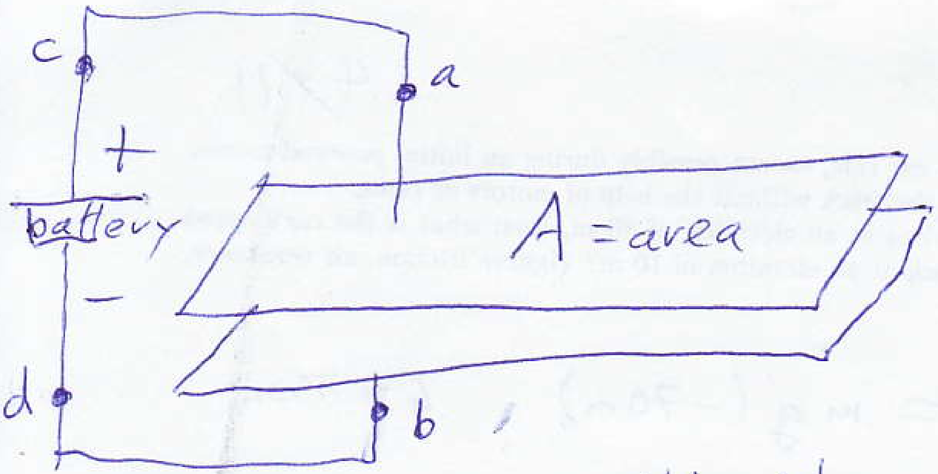


$$C = \varepsilon \frac{A}{d}$$

ε = permittivity of material inside gap

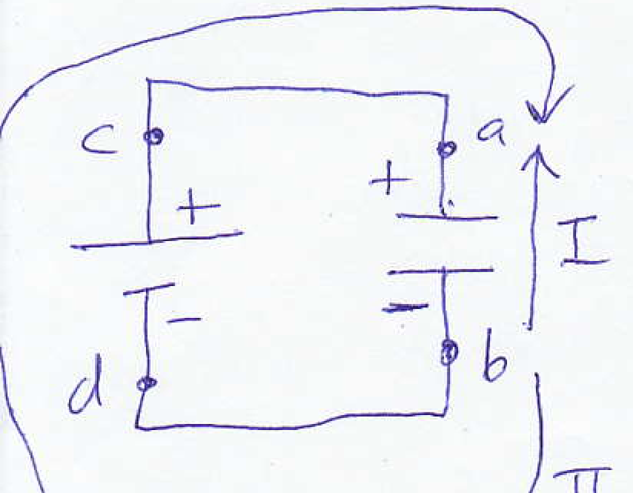
$$\varepsilon = K \varepsilon_0$$

↑ dielectric constant

$$V_{ac} = V_a - V_c = \int_c^a -\vec{E} \cdot \vec{dl} = 0$$

$\vec{E} = \vec{0}$ inside conductive wire

V_{cd} = voltage of battery = constant



$$V_{ab} = V_a - V_b = \int_b^a -\vec{E} \cdot \vec{dl} = ?$$

2 paths I & II

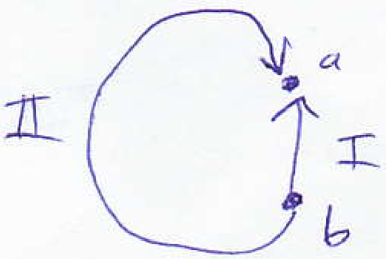
$$\text{II}: V_{ab} = V_{acdb}$$

$$= \underbrace{V_{ac}}_{0} + \underbrace{V_{cd}}_{\text{battery}} + \underbrace{V_{db}}_{0}$$

$V_{ab} = V_{cd}$ Key idea:

"conservativity of electrostatic field"

$$V_{ab} = \int_{I} -\vec{E} \cdot d\vec{l} = \int_{II} -\vec{E} \cdot d\vec{l}$$



Other key idea: conservation of (net) charge

Suppose we charge a capacitor with a battery. We can insert a dielectric, say, paper,

(1) with the battery connected, or

(2) without the battery connected.

$\frac{Q_0}{V_0} = C_0 = \frac{A\varepsilon_0}{d}$ before paper $C_0 = \frac{Q_0}{V_0}$

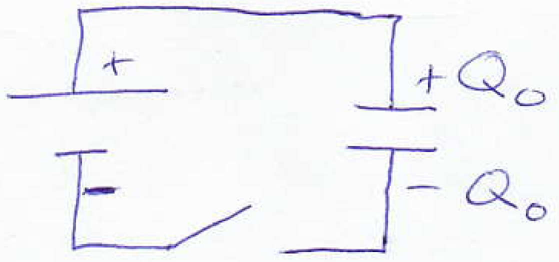
$\frac{Q}{V} = C = \frac{AK\varepsilon_0}{d}$ after paper $(K = 3.7)$

V_0 = battery voltage

Case (1): $V = V_0$, so

$$\frac{Q}{V} = \frac{Q}{V_0} = C = K C_0 = \frac{K Q_0}{V_0} \Rightarrow Q = K Q_0$$
$$\Rightarrow U = K U_0$$

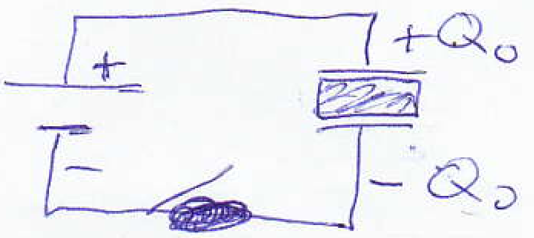
Case (2):



before paper

after paper

conservation of charge



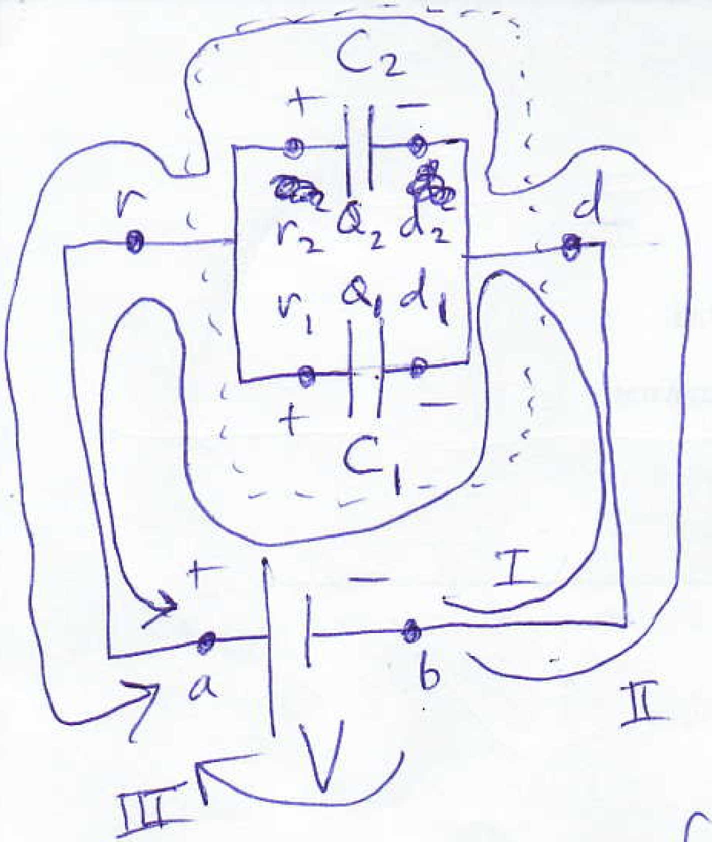
$Q = Q_0$

$$\frac{Q}{V} = \frac{Q_0}{V} = C = K C_0 = \frac{K Q_0}{V_0} \Rightarrow V = \frac{V_0}{K}$$
$$\Rightarrow U = U_0 / K$$

$C = Q/V \rightarrow Q = CV$

$$U = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2}$$

↑ energy to charge capacitor from 0 to Q



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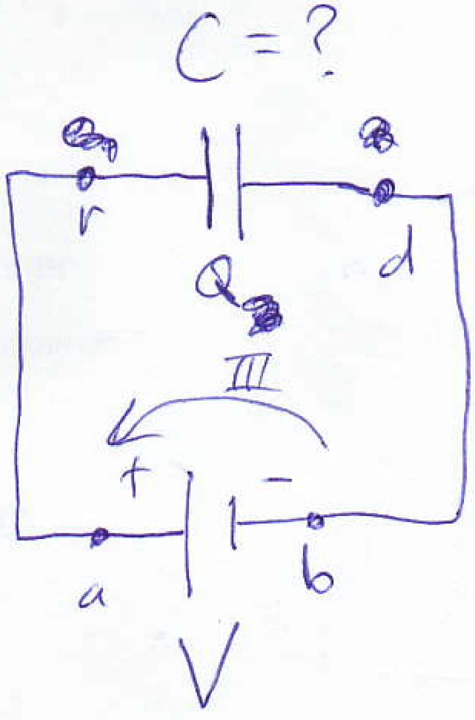


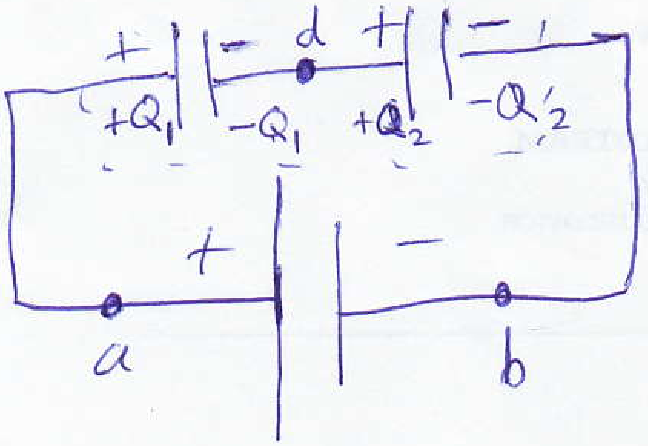
$$V = V_{ab} = V_a - V_b = \int_b^a -\vec{E}\cdot\vec{dl} = \int_{III} -\vec{E}\cdot\vec{dl}$$

$$V = V_{ab} = \underbrace{\int_{I} -\vec{E}\cdot\vec{dl}}_{V_{r_1 d_1}} = \underbrace{\int_{II} -\vec{E}\cdot dl}_{V_{r_2 d_2}} = V_{rd}$$

$$Q = Q_1 + Q_2 \qquad V = V_{r_1 d_1} = V_{r_2 d_2}$$

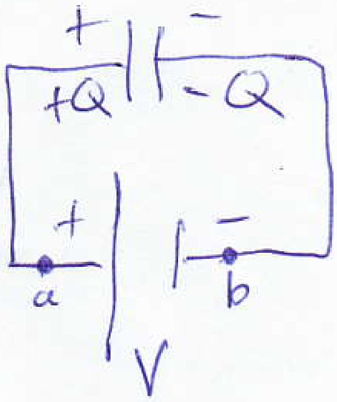
$$C = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$$

C_1 C_2:

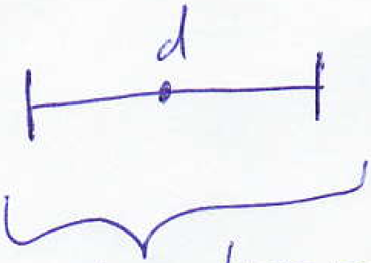


$\approx$

$C = ?$



$V = V_{ab}$ $\qquad V_{ab} = V_{adb} = \underbrace{V_{ad}}_{V_1} + \underbrace{V_{db}}_{V_2}$



net charge 0 is conserved

$$C = \frac{Q}{V}$$

$$\frac{1}{C} = \frac{V}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$0 = -Q_1 + Q_2 \Rightarrow Q_1 = Q_2 = Q$$

E.g. $C_1 = 470\mu F$
$C_2 = 300\mu F$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = 183\mu F$$