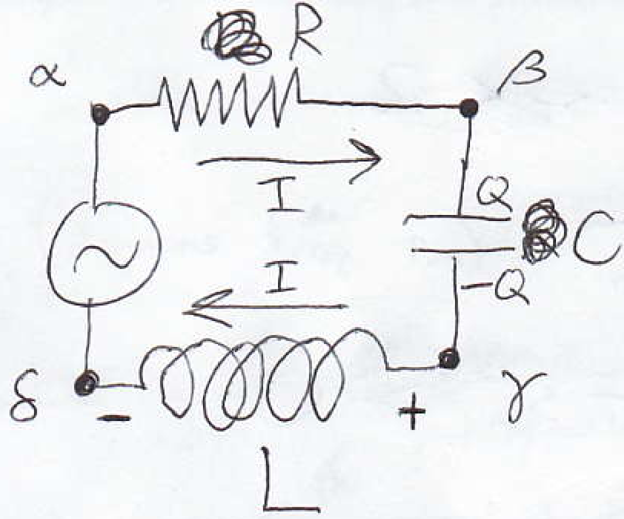



$V_{\alpha\delta}(t) = A_0 \sin(\omega t)$

Find non-transient current

$I(t) = A \sin(\omega t - \phi)$.

$dQ/dt = I$

$$V_{\alpha\delta} = V_{\alpha\beta\gamma\delta} = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\delta} = IR + \frac{Q}{C} + L\frac{dI}{dt}$$

$$A_0 \omega \cos(\omega t) = dV_{\alpha\delta}/dt = \frac{dI}{dt}R + \frac{I}{C} + L\frac{d^2 I}{dt^2}$$

$$A_0 \omega \cos(\omega t) = AR\omega \cos(\omega t - \phi) \quad \leftarrow \frac{dI}{dt}R$$

$$+ A C^{-1} \sin(\omega t - \phi) \quad \leftarrow I/C$$

$$- LA\omega^2 \sin(\omega t - \phi) \quad \leftarrow L d^2 I/dt^2$$

Let $X_L = L\omega$ & $X_C = \frac{1}{\omega C}$.

$$\Rightarrow A_0 \cos(\omega t) = A\left[R\cos(\omega t - \phi) + (X_C - X_L)\sin(\omega t - \phi)\right]$$

↖ (divided previous equation by $\omega$)

~~$A_0 \cos(\omega t) = A_0 \cos\left(\frac{1}{2}(\omega t + \phi) + \frac{1}{2}(\omega t - \phi)\right)$~~

$$A_0 \cos(\omega t) = A_0 \cos((\omega t - \phi) + \phi)$$

$$= A_0\left[\cos(\omega t - \phi)\cos\phi - \sin(\omega t - \phi)\sin\phi\right]$$

$$\Rightarrow \begin{cases} A_0 \cos\phi \cos(\omega t - \phi) = A R \cos(\omega t - \phi). \\ -A_0 \sin\phi \sin(\omega t - \phi) = A(X_C - X_L)\sin(\omega t - \phi) \end{cases}$$

because cosine & sine terms must equal

$$\Rightarrow \begin{cases} A_0 \cos\phi = A R \\ A_0 \sin\phi = A(X_L - X_C) \end{cases}$$

$$\Rightarrow \begin{cases} A_0^2 = A_0^2 \cos^2\phi + A_0^2 \sin^2\phi = A^2 R^2 + A^2 (X_L - X_C)^2 \\ \tan\phi = (A_0 \sin\phi)/(A_0 \cos\phi) = A(X_L - X_C)/(AR) \end{cases}$$

$$\Rightarrow \begin{cases} A_0 = A\sqrt{R^2 + (L\omega - \omega^{-1}C^{-1})^2} \\ \phi = \arctan((L\omega - \omega^{-1}C^{-1})/R) \end{cases}$$

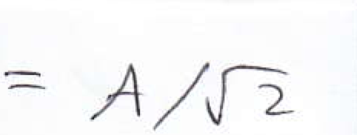
$$V_{\alpha\delta, rms} = \frac{A_0}{\sqrt{2}} \qquad I_{rms} = A/\sqrt{2}$$

$$Z = \text{impedance} = \frac{V_{\alpha\delta, rms}}{I_{rms}} = \frac{A_0}{A} = \sqrt{R^2 + \cdots}$$

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}$$

$\phi$ called "phase angle"

$\phi$ = angle by which current lags voltage.




E.g. $45^\circ = \frac{1}{8} 360^\circ = \frac{1}{8}$ cycle lag

$-30^\circ = -\frac{1}{12} 360^\circ = \frac{1}{12}$ cycle lead