The book uses the magnitude of impedance $|Z|$ and phase angle of impedance ϕ.

A better way is to have Z be a complex # that encodes the magnitude & phase angle via $Z = |Z|e^{i\phi}$.

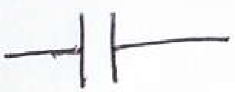
<u>When using complex impedances:</u>

impedances add in series

impedances' reciprocals add in parallel

$Z = R$

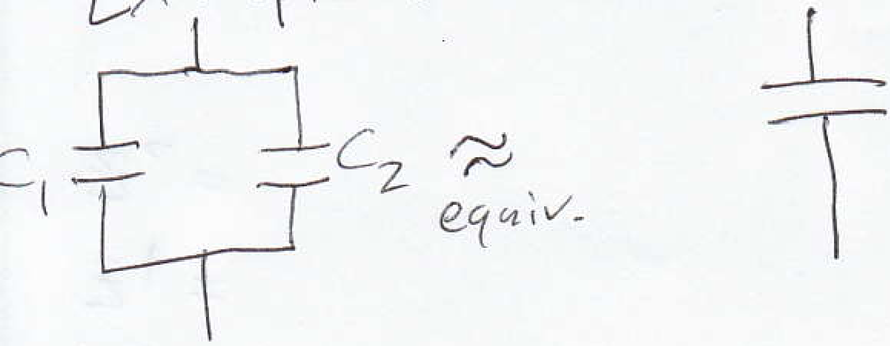
$Z = -\frac{i}{\omega C} = \frac{1}{i\omega C}$



$Z = i\omega L$

Note : $\frac{1}{i} = -i$ because $i^2 = -1$

Example 1:

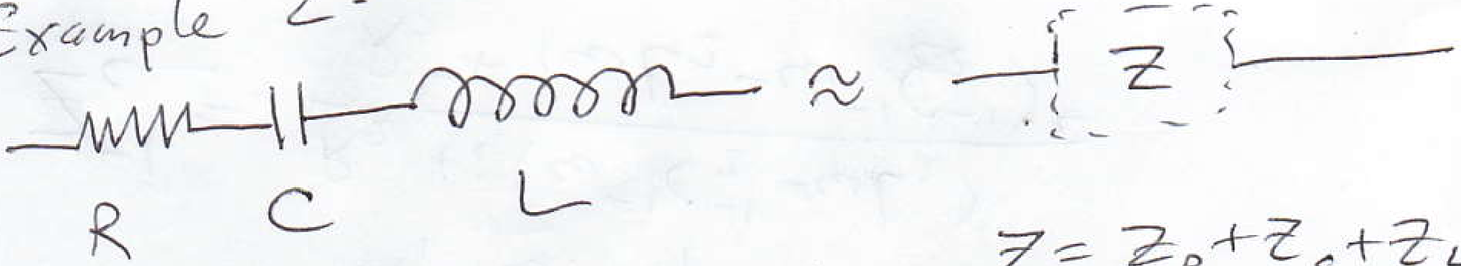


$C_1 + C_2 = C$

$$Z_1 = \frac{1}{i\omega C_1} \quad Z_2 = \frac{1}{i\omega C_2} \Rightarrow Z = (Z_1^{-1} + Z_2^{-1})^{-1}$$

$$\Rightarrow Z = \frac{1}{Z_1^{-1} + Z_2^{-1}} = \frac{1}{i\omega C_1 + i\omega C_2} = \frac{1}{i\omega C} \checkmark$$

Example 2:



$$Z = Z_R + Z_c + Z_L$$

$$Z_R = R \quad Z_c = \frac{1}{i\omega C} \quad Z_L = i\omega L$$

$$Z = R + i\left(\frac{-1}{\omega C} + \omega L\right)$$

$$X_c = \frac{1}{\omega C} \;\&\; X_L = \omega L \Rightarrow Z = R + i(X_L - X_c)$$

magnitude of impedance:

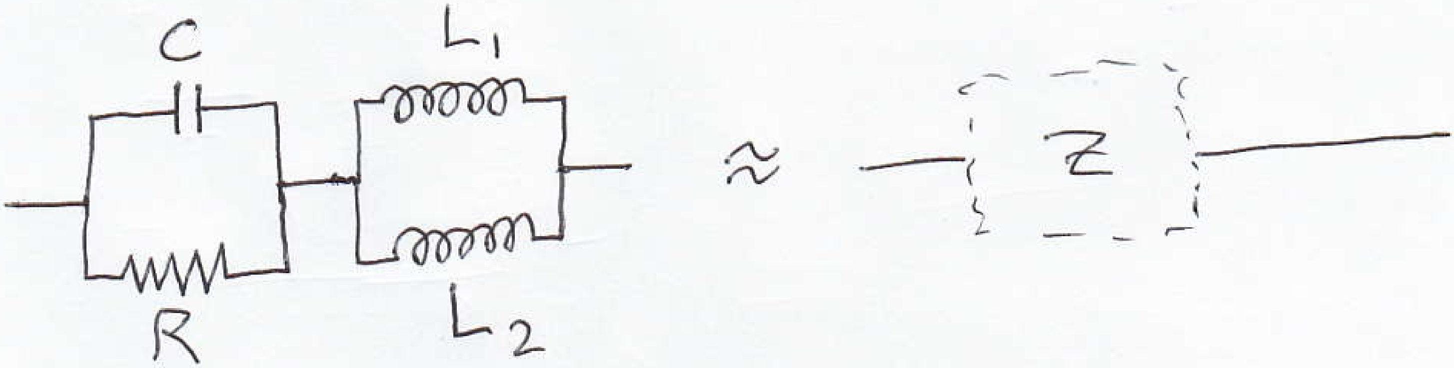
$$|a+bi| = \sqrt{a^2+b^2}$$

$$|Z| = \sqrt{R^2 + (X_L - X_c)^2} \quad \boxed{\text{just as in book}}$$

phase angle of $a+bi = \arctan\left(\frac{b}{a}\right) \Rightarrow \phi = \arctan\left(\frac{X_L - X_c}{R}\right)$

(For $a \geq 0$)

Example 3:



$$Z = \left(Z_R^{-1} + Z_C^{-1}\right)^{-1} + \left(Z_{L_1}^{-1} + Z_{L_2}^{-1}\right)^{-1}$$

$$Z = \left(\frac{1}{R} + i\omega C\right)^{-1} + \left(\frac{1}{i\omega L_1} + \frac{1}{i\omega L_2}\right)^{-1}$$

$$(a+bi)^{-1} = \frac{a-bi}{a^2+b^2} = \frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$$

$$Z = \frac{R^{-1} - i\omega C}{R^{-2} + \omega^2 C^2} + \frac{1}{i\omega}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}$$

$$Z = \frac{R^{-1}}{R^{-2} + \omega^2 C^2} - i\left[\frac{\omega C}{R^{-2} + \omega^2 C^2} + \frac{1}{\omega}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}\right]$$

$$|Z| = \sqrt{\left(\frac{R^{-1}}{R^{-2} + \omega^2 C^2}\right)^2 + \left(\frac{\omega C}{R^{-2} + \omega^2 C^2} + \frac{L_1 L_2}{\omega(L_1 + L_2)}\right)^2}$$

$$\left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1} = \left(\frac{L_2 + L_1}{L_1 L_2}\right)^{-1} = \frac{L_1 L_2}{L_1 + L_2}$$