

2413 9/28 ~~2020~~ ~~Notes~~ Summary of Differentiating

3 most important derivative rules:

$$(fg)' = f'g + fg' \quad [\text{Product}]$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad [\text{Quotient}]$$

$$\left[f(g(x))\right]' = f'(g(x))g'(x) \quad \left. \vphantom{\left[f(g(x))\right]}' \right\} [\text{Chain}]$$
$$\frac{dy}{dx} = \left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right)$$

Many other rules:

$$(f \pm g)' = f' \pm g' \quad [\text{Sum/Difference}]$$

$$(cf)' = cf' \quad [\text{Constant Multiple}]$$

$$(x^n)' = \cancel{n} x^{n-1} \quad [\text{Power}]$$

$$(f^n)' = n f^{n-1} f'$$

$$(1/f)' = -f'/f^2 \quad [\text{Reciprocal}]$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}; \quad \left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$|x|' = \begin{cases} 1 & : x > 0 \\ -1 & : x < 0 \end{cases} = \frac{|x|}{x} = \frac{x}{|x|}$$



2413 9/28

Many other rules:

$$x^r = r x^{r-1} \quad (r \text{ rational})$$

$$(f^r)' = r f^{r-1} f'$$

$$c' = 0 \quad (c \text{ constant}) \quad [\text{Constant}]$$

$$(a^x)' = a^x \ln a$$

$$(e^x)' = e^x$$

$$\left. \begin{aligned} (\log_a x)' &= \frac{1}{x \ln a} \\ (\ln x)' &= 1/x \end{aligned} \right\} \begin{array}{l} \text{only} \\ \text{for} \\ x > 0 \end{array}$$

$$(\sin \theta)' = \cos \theta$$

$$(\cos \theta)' = -\sin \theta$$

$$(\tan \theta)' = \sec^2 \theta$$

$$(\cot \theta)' = -\csc^2 \theta$$

$$(\sec \theta)' = \sec \theta \tan \theta$$

$$(\csc \theta)' = -\csc \theta \cot \theta$$

Trig. reminders:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = 1/\cos \theta$$

$$\csc \theta = 1/\sin \theta$$

$$\frac{dy}{dx} = \frac{1}{dx/dy} \quad [\text{Inverse Function}]$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$



2413 9/28 Practice with chain rule.  
(and other rules)

$$y = \tan(\ln^3 x - \csc \sqrt{x}); \quad \frac{dy}{dx} = ?$$

$$y = 2 \cos^5(x) 4^{x+3}; \quad \frac{dy}{dx} = ?$$

$$y = \frac{3 \sec(2x)}{x^2 + \log_7 \cot x}; \quad \frac{dy}{dx} = ?$$

$$y = 7e^{\left(\frac{x+1}{x-1}\right)} + \cos(e^x); \quad \frac{dy}{dx} = ?$$

$$y = \left[ \sin^3(x^4) e^{x^2} + 1 \right]^{\frac{8}{7}}; \quad \frac{dy}{dx} = ?$$

$$y = \frac{1}{\sqrt[3]{e^x + \ln(2 + (\sin x)/4)}}; \quad \frac{dy}{dx} = ?$$

$$y = \tan(\ln(\sin(\cos(x))))); \quad \frac{dy}{dx} = ?$$

$$y = \sqrt{1 + \sqrt[3]{1 + \sqrt[4]{x^2 - x}}}; \quad \frac{dy}{dx} = ?$$

$$y = e^{e^x}; \quad \frac{d^2y}{dx^2} = ?$$

$$y = 1/(\cos^2(2x) + \sin^2(3x)); \quad \frac{d^2y}{dx^2} = ?$$