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10/19 A

Last time: how to find the max & min values of continuous $f(x)$ over $[a,b]$, or $[a,b] \cup [c,d]$,

Today: how to find the max & min, if they exist, of continuous $f(x)$ over other ~~inter~~ kinds of intervals and unions of intervals.

⓪ Check that f is cts at all interior points of the intervals, cts from the right at all included left endpoints, and cts from the left at all included right endpoints

~~E.g. $\sqrt{x}$ is cts from right at 0~~

~~and cts~~

① Approximate each interval by a smaller hyperreal closed interval. (See other side.)

Same as last time

② Find all critical points in the interiors of these closed intervals.

③ List the values of $f(x)$ at all endpoints and critical points.

④ If the greatest value on the list is at a real, then it is the maximum value of f over the original interval(s).

(over please)

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If the greatest value is not at a real, then $f(x)$ has no maximum value.

If the least value is at a real, then it is the minimum of $f(x)$ over the original interval(s).

If the least value is not at a real, then $f(x)$ has no minimum over the original interval(s).

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This method works because of the Transfer Principle. Specifically, if a real function lacks a maximum/minimum at a real, ~~then it also lacks~~ over (a union of) real interval(s), then it lacks a hyperreal maximum/minimum at a hyperreal also.

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Approximating intervals: The approximation must include all the reals from the original intervals. In the examples below, $0 < \varepsilon \approx 0$, $0 < H$, and H is infinite.

$(3,4) \rightarrow [3+\varepsilon, 4-\varepsilon]$ $\quad$ $(3,\infty) \rightarrow [3+\varepsilon, H]$

$[3,4) \rightarrow [3, 4-\varepsilon]$ $\quad$ $[3,\infty) \rightarrow [3, H]$

$(3,4] \rightarrow [3+\varepsilon, 4]$ $\quad$ $(-\infty,3) \rightarrow [-H, 3-\varepsilon]$

$[3,4] \rightarrow [3, 4]$ $\quad$ $(-\infty,3] \rightarrow [-H, 3]$

$(-\infty,\infty) \rightarrow [-H, H]$