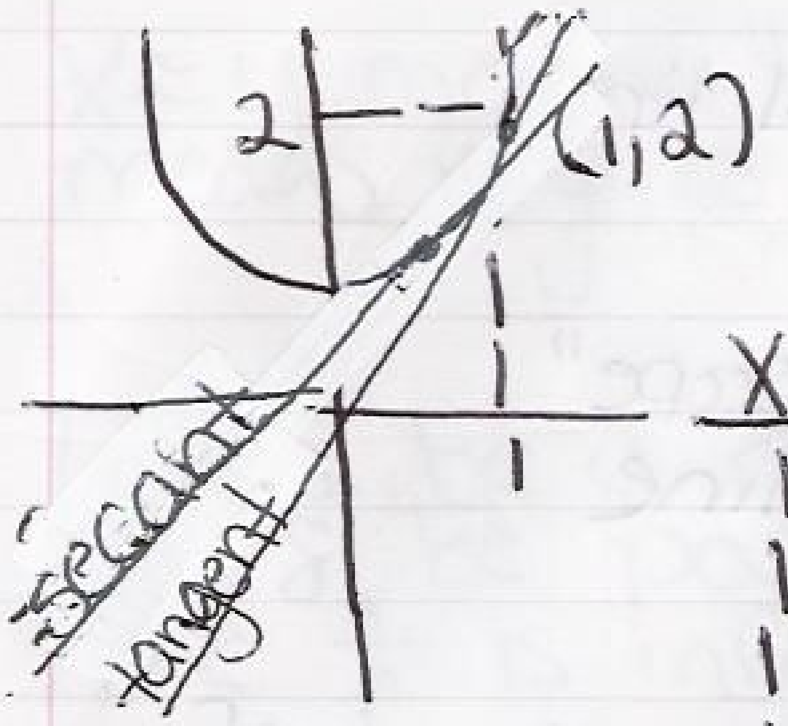
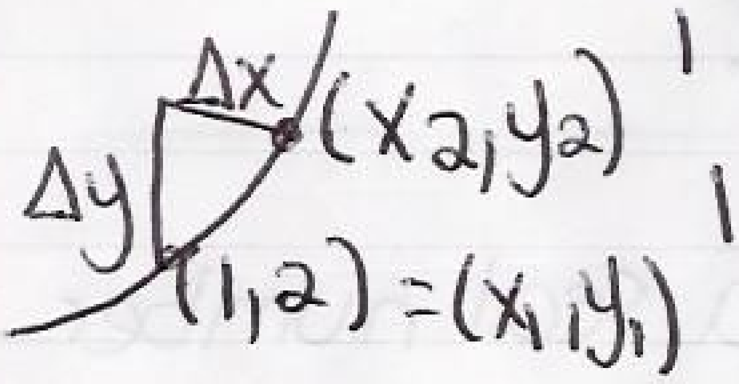
Janette Chavez
8/26

Notes



$y = x^2 + 1$

x_1	y_1	x_2	y_2	Δx	Δy	$\Delta y / \Delta x$
1	2	0.9	1.81	-0.1	-0.19	1.9
1	2	0.99	1.9801	-0.01	-0.0199	1.99
1	2	1	2	0	0	undefined
1	2	1.01	2.0201	0.01	0.0201	2.01
1	2	1.1	2.21	0.1	0.21	2.1
1	2	$1+\Delta x$	$2+2\Delta x+\Delta x^2$	Δx	$2\Delta x+\Delta x^2$	$2+\Delta x$

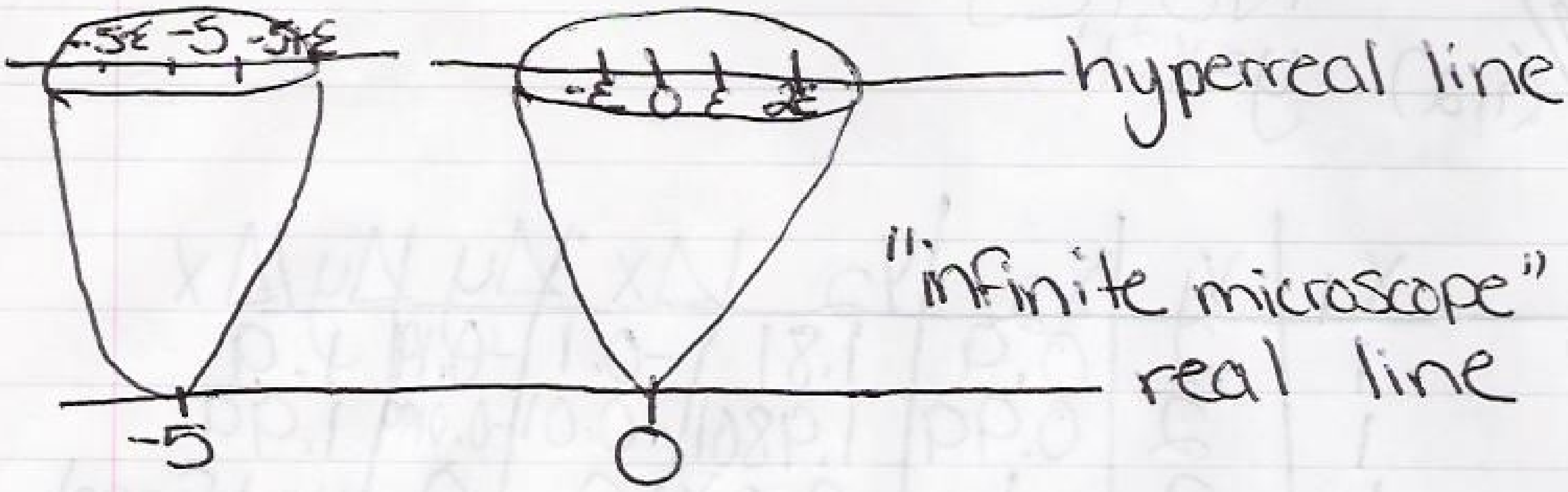


$\Delta y = y_2 - y_1$ $\quad$ Slope $= \dfrac{\Delta y}{\Delta x}$
$\Delta x = x_2 - x_1$

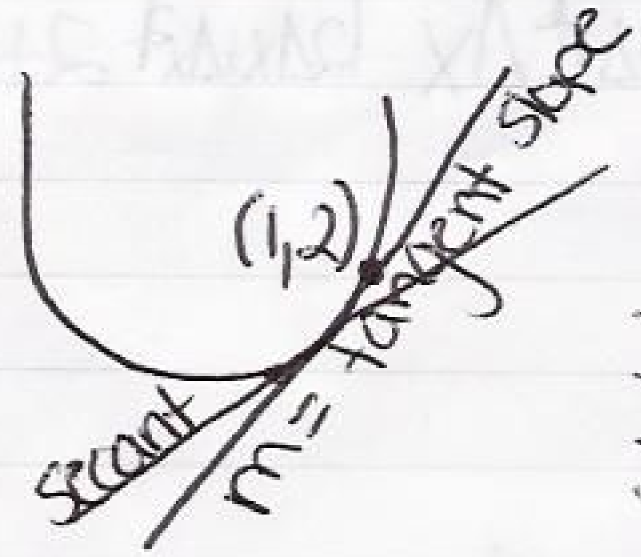
Idea: Let Δx be infinitely small, but not 0 (whatever that means). Then the secant line will have slope $2+\Delta x$, which is infinitely close to 2. Also, the secant line will be infinitely close to the tangent line, so the tangent's slope must be two.

There are infinitely small, nonzero "hyperreal" numbers.

Let ε be an infinitely small, positive number, for example.



Every real number has a "halo" or cloud of hyperreals infinitely close to it.



$y=x^2+1$ m is a real number

If we make Δx infinitely small, then the secant's slope, which is $2+\Delta x$, is infinitely close to m and to 2.

Write $m \simeq 2+\Delta x \simeq 2$.
If two real numbers are infinitely close, then they're equal, so $m=2$

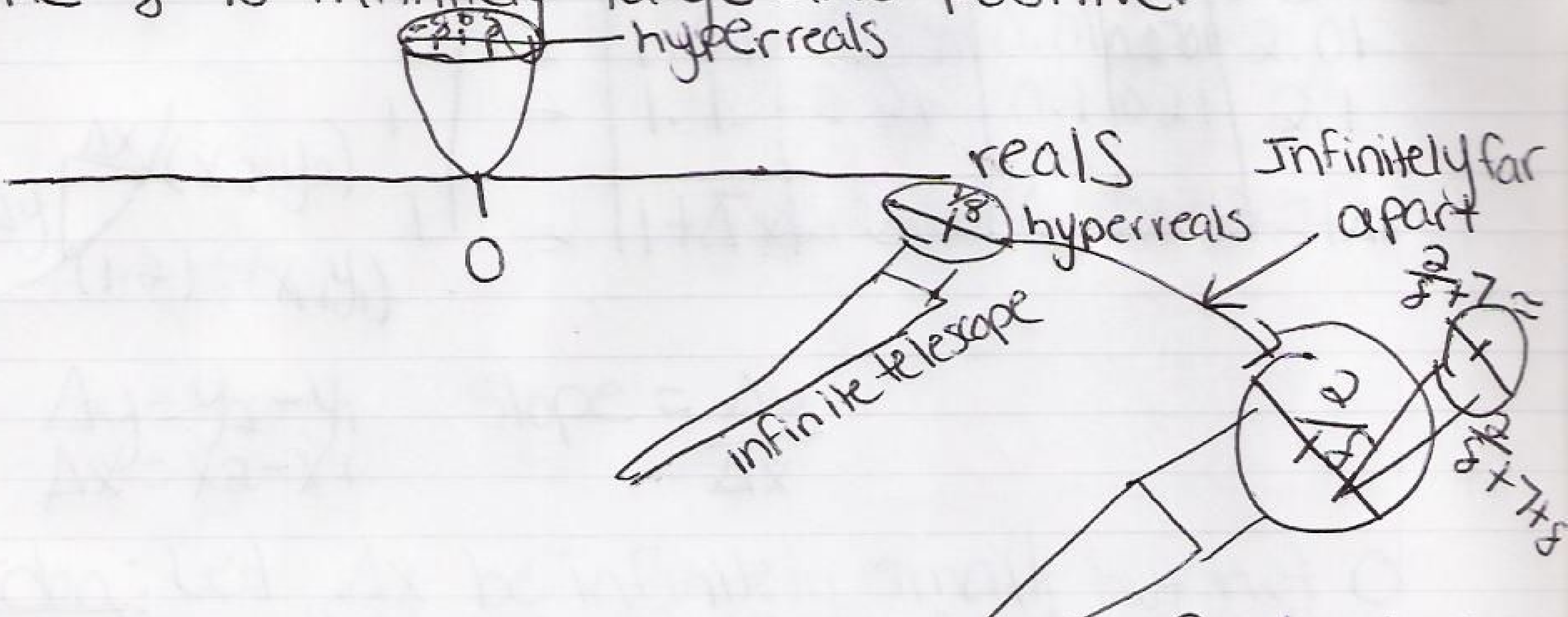
"infinitesimal" = "infinitely small"

Formally, infinitely small means this: ε is infinitely small if for every positive real a,

$-a<\varepsilon<a$

$x \approx y$ means "x is infinitely close to y"
means $x-y$ is infinitely small.

Let δ be a nonzero infinitesimal;
let δ be positive.
The $\frac{1}{\delta}$ is infinitely large and positive.



$\frac{2}{\delta} - \frac{1}{\delta} = \frac{1}{\delta}$ is infinite