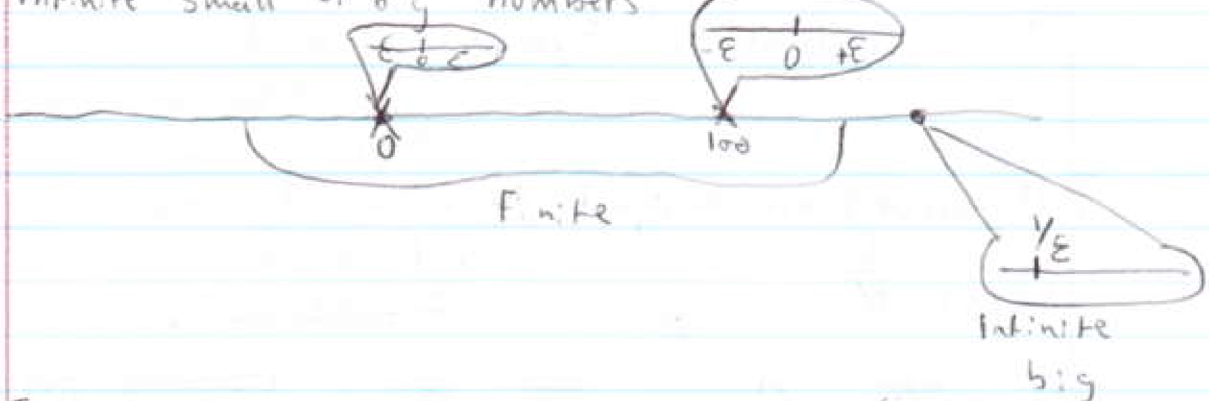
Joel A. Jurado 08/30/10

Hyperreals

Infinite small or big numbers



Three principles of Hyperreal numbers (Hyperreal number facts)

(1.5) Extension Principle
(1.5) Transfer Principle
(1.5) Standard Part Principle

<u>Extension Principle</u>

1.- The set of real numbers in the real line belongs as a subunit of the hyperreal line.

Also: • The real line is a subset of the hyperreal line. The < ordering <u>of reals</u> doesn't change when we look at hyperreals instead of reals.

E.g. $2 < 3$ is still true

• Functions (like cos, $\sqrt{\ }$, ...) and algebra operations (+, -, ·, /, powers), all are defined by hyperreals and they agree with the "old" real functions and operations.

E.g. $2+3=5$ is still **true**.

2.- <u>Transfer Principle</u>: "Real statements" like "$x + y = y + x$ is always true" or "If $0<x<y$ then $0 < \frac{1}{y} < \frac{1}{x}$" or "$\sqrt{x}$ is defined for all $x \geq 0$", etc. are true of the reals if and only if they are true of the hyperreals.

"Trickier transfers": Every real number is finite $\rightsquigarrow$ Every hyperreal number is hyperfinite.

* Hyperreals are designed to mimic the reals.

Suppose ε is a positive infinitesimal.

Is $\sqrt{\varepsilon} = \varepsilon$ or $\sqrt{\varepsilon} < \varepsilon$ or $\boxed{\sqrt{\varepsilon} > \varepsilon}$?

If $0 < x$ and x is a real, then $\sqrt{x} < x$

Transfer:

If $x > 1$ and x is a real, then $x > \sqrt{x}$

However, if $0 < x < 1$ and x is real, then $\sqrt{x} > x$.

Transfer: $0 < \varepsilon < 1$, so $\varepsilon < \sqrt{\varepsilon}$

Rules present in book pg 30 & 31

E.g. H infinite hyperreal > 0

Claim $\sqrt{H+1} - \sqrt{H-1}$ is infinitesmal.

$$\sqrt{1000001} - \sqrt{999,999} = .001$$