Sep 6 2010

Notes

$(x^n)' = n \cdot x^{n-1}$

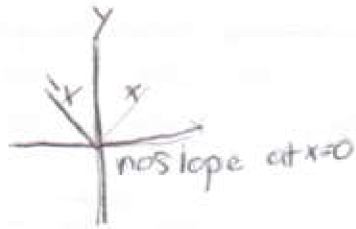
$(mx+b)' = m$

m, b constant

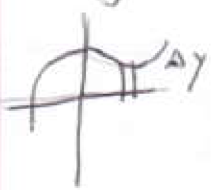
$(b)' = 0$

$|x|' = \begin{cases} -1 : x<0 \\ 1 : x>0 \\ \text{undefined } x=0 \end{cases}$

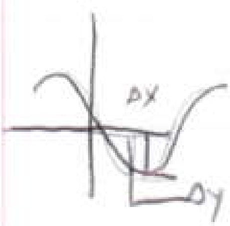
$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$



$(f+g)' = f'+g'$ Eg $(x^3+x^2)' = 3x^2+2x$



$f'(x) = st(\Delta f/\Delta x)$ for $0 \neq \Delta x \approx 0$



$g'(x) = st\left(\frac{\Delta y}{\Delta x}\right)$

for $0 \neq \Delta x \approx 0$

$(f+g)' = st\left(\frac{\Delta(f+g)}{\Delta x}\right)$

$st\left(\frac{\Delta f + \Delta g}{\Delta x}\right)$

$= st\left(\frac{\Delta f}{\Delta x}\right) + st\left(\frac{\Delta g}{\Delta x}\right)$

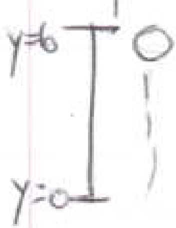
$f'(x) + g'(x)$

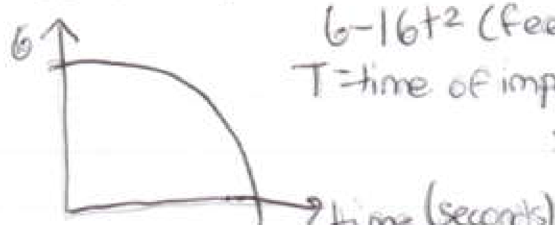
$(cf)' = c \cdot f'$

constant Ex $(5x^2) = 5 \cdot (x^2)$

$(cf)' = st\left(\frac{\Delta(cf)}{\Delta x}\right) = st\left(\frac{c\Delta f}{\Delta x}\right) = c\,st\left(\frac{\Delta f}{\Delta x}\right) = cf'(x)$

Drop rock from 6 ft heig:





$6 - 16t^2$ (feet) speed $= \underline{d} \div t$

T = time of impact Δt = negative infinitesimal

speed of impact $= (6-16t^2)'$

$= -32t$

$-32T$ at $T = t$ $\qquad T^2 = \frac{6}{18} = \frac{3}{5} = T = \sqrt{\frac{3}{8}}$

$0 = y = 6 - 6T^2$

$16T^2 = 6$

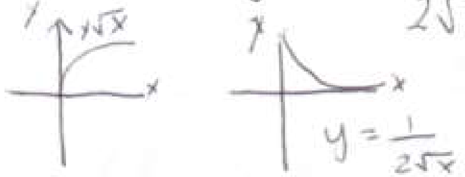
$32\sqrt{3/8}$ feet / second ≈ 19.6 ft/sec

instantaneous rate of change > Rates of changes are $\overbrace{\text{slopes}}^{\text{slopes graph with } \angle \text{ st}}$ are derivatives

Eg.: velocity is rate of change of position

At impact our rock had speed $32\sqrt{3/8} \approx 19.6$ (ft/sec) Over its entire descent, the average speed was $\frac{6 \text{ ft}}{\sqrt{\frac{3}{8}} \text{ seconds}} \approx 9.8$ ft/sec

Another rule: $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$



$(\sqrt{x})' = st\left(\frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}\right) = st\left(\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\Delta x(\sqrt{x} + \sqrt{x})}\right)$

$= st\left(\frac{x + \Delta x - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}\right)$

where $0 \neq \Delta x \approx 0$

$st\left(\frac{x + \Delta x - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}\right) = st\left(\frac{\Delta x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}\right) = \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}}$

It costs a factory $10000 + 3\sqrt{x}$ dollars/month to make x pencils per moth

Estimate the cost of making 1 more pencil per moth (called marginal

Exact $\quad (10000 + 3\sqrt{x+1}) - (10000 + 3\sqrt{x}) = .0014\,99999\ldots$

Estimate $\quad (10000 + 3\sqrt{x})' = \frac{3}{2\sqrt{x}} = .0015$

If $x = 10^6 = 1000\,000$