2.2 (Part I) (Notes)

Deferentials $\quad y=(x^4-7)$

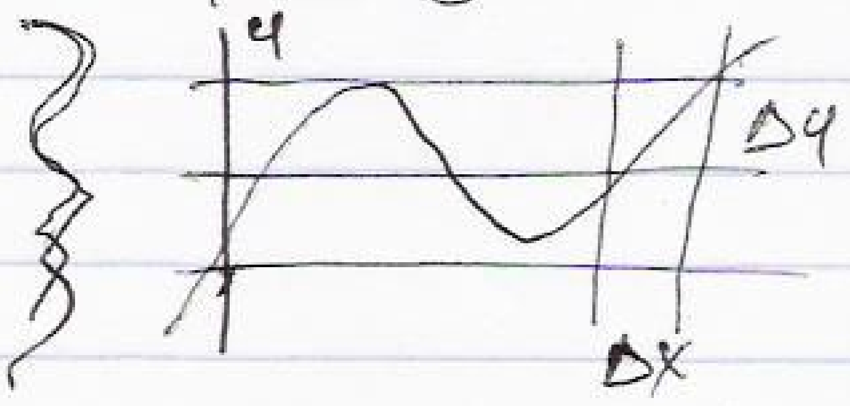
$\frac{dy}{dx} \quad y'=(x^4-7)'=4x^3$

$(x^n)'=nx^{n-1} \quad y'=0$

Recall Defenition: Assoming y' exsists:

$y'=st\left(\frac{\Delta y}{\Delta x}\right)$ for all $\Delta x \neq 0$, $\Delta x \approx 0$

where Δy is the change in y corresponding to a change Δx in x



$\Delta y=((x+\Delta x)^4-7)-(x^4-7)$

Definition

$dx=\Delta x$

$dy=y'\Delta x=y'dx$

$y' \approx \frac{\Delta y}{\Delta x} \quad \frac{1}{3}$

$\frac{dy}{dx}=\frac{y'dx}{dx}=y'$

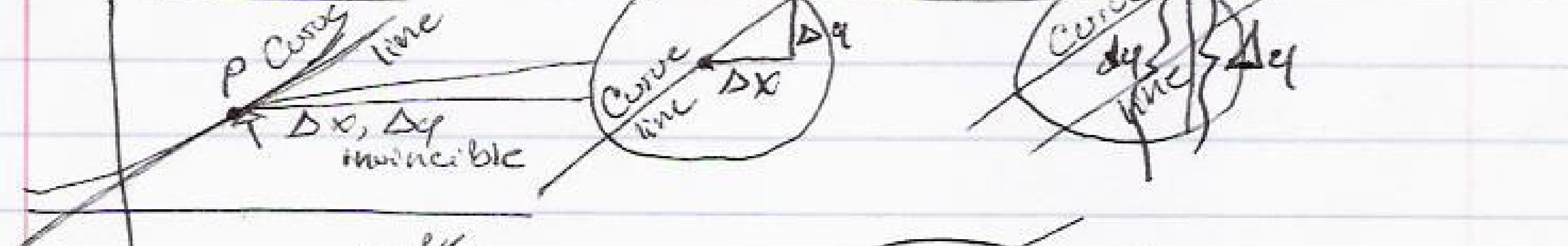
$\boxed{\frac{dy}{dx}=y' \approx \frac{\Delta y}{\Delta x}}$

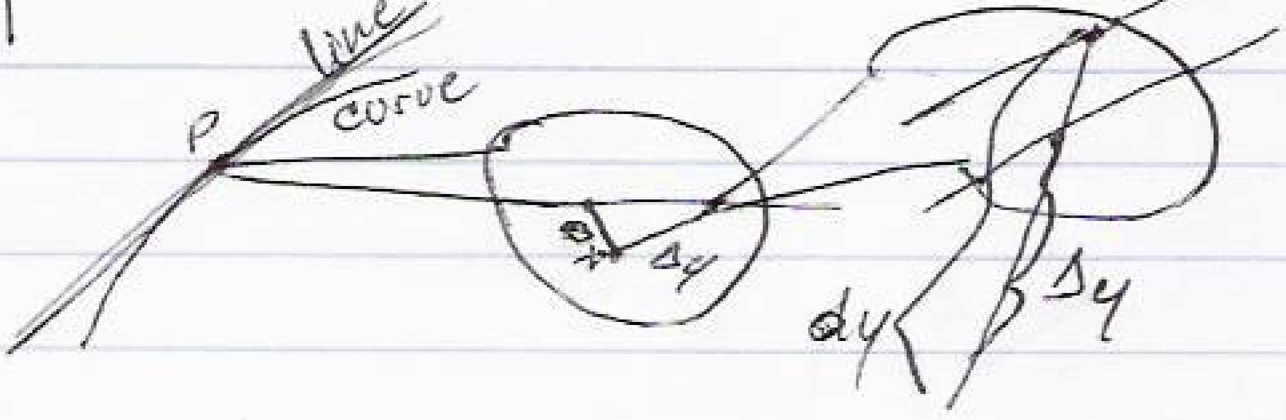
$dx=\Delta x=$ change in x

$\Delta y=$ corresponding change along the curve in y

$dy=$ corresponding change along the tangent line in y

Picture





Increment Theorem

- $\Delta y \approx 0$
- There is some $\varepsilon \approx 0$ such that

$\Delta y = y' \cdot \Delta x + \varepsilon \cdot \underset{0 \approx}{\Delta x} = dy + \varepsilon \cdot dx$

$= (y' + \varepsilon)\Delta x$

$= (y' + \varepsilon)dx$

(Assuming y' exsists)

Remember

$dy = y' \cdot dx$; $dx = \Delta x$ by def.

$y' = \frac{dy}{dx}$

$dy = y' \cdot dx =$ (finite)(infinitesimal) $=$ infinitesimal

$y = x^3$ $\qquad y' = 3x^2$

$\Delta y = (x + \Delta x)^3 - x^3$

$\Delta y = 3x^2 \Delta x + 3x\Delta x^2 + \Delta x^3$

$= y'dx + 3x\Delta x^2 + dx^3$

$= dy + \underbrace{(3xdx + dx^2)}_{\varepsilon}dx$

ε — called "increment"

Always choose

$0 \neq \Delta x \approx 0$ ($= dx$)

$\varepsilon = 3xdx + dx^2$

(finite)(infinitesimal) + infinitesimal^2

= infinitesimal + infinitesimal

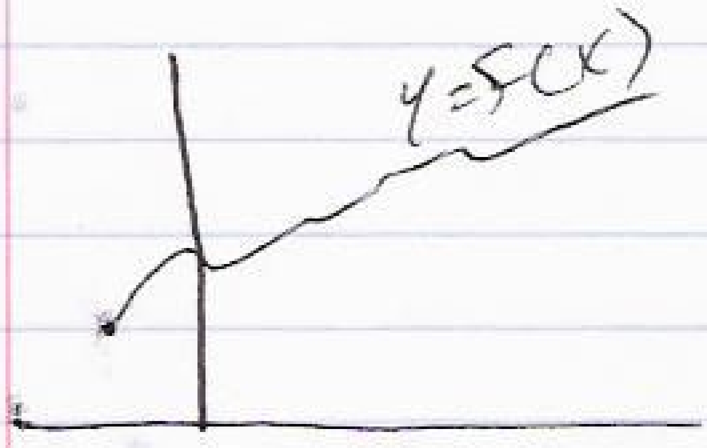
= infinitesimal

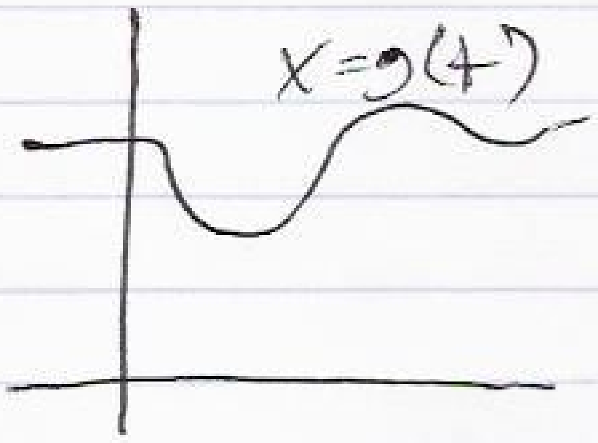
Summary:

$0 \approx \Delta y = dy + \varepsilon dx$

for some $\varepsilon \approx 0$.

$(x^n)' = nx^{n-1}$	$\frac{d(x^n)}{dx} = nx^{n-1}$	$\cancel{nx^{n-1}}$ $d(x^n) = nx^{n-1}$
$(f+g)' = f' + g'$	$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$	$d(f+g) = df + dg$
$(cf)' = c(f')$	$d(cf) = c \cdot \frac{df}{dx}$	$d(cf) = c \cdot df$
$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$	$\frac{d(1/x)}{dx} = -1/x^2$	$d(1/x) = -dx/x^2$
$\sqrt{x}\,' = \frac{1}{2\sqrt{x}}$	$\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$	$d(\sqrt{x}) = dx/2\sqrt{x}$
$mx+b' = m$	$\frac{d(mx+b)}{dx} = m$	$d(mx+b) = mdx$





x indipendent variable

y dependent variable

t independent variable

x dependent variable

In the context of $g(t+\Delta t)-g(t)$

$$dx = g'(t)\cdot dt$$