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9/13 A

Last time Product Rule $(fg)' = \boxed{f'g + fg'}$ Alright

$(f^2)' = (ff)' = \boxed{f'f + ff'} = 2ff'$

$(f^3)' \underset{2ff'}{=} (f^2 f)' = (f^2)'f + f^2 f'$

$= (2ff')f + f^2 f' = 3f^2 f'$

Improved Power Rule $(f^n)' = nf^{n-1}f' : n = 1, 2, 3, 4, \ldots$

Today: $\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$ (Reciprocal Rule)

$(f^{-n})' = -nf^{-n-1}f' : n = 1, 2, 3, 4, \ldots$

(negative power rule)

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ (Quotient Rule)

Special case: $\frac{d(x^{-n})}{dx} = -nx^{-n-1}$

Next: population example

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~~[illegible]~~

9/13B

At this ~~moment~~ instant in time, the population I of India [is about $1.180 \cdot 10^9$ and] growing at a rate of about 2085 people/hour. The world population W is about $6.868 \cdot 10^9$ [and growing at a rate of about 9181 per/hr]. What is the current rate of change of $F = I/W$, the fraction of humanity living in India?

$I(t)$, $W(t)$, $F(t) = \frac{I(t)}{W(t)}$

$\boxed{F(\text{now}) = 0.1718}$

Quotient Rule:

$$F'(t) = \frac{I'(t)W(t) - I(t)W'(t)}{W^2(t)}$$

$$F'(\text{now}) = \frac{I'(\text{now})W(\text{now}) - I(\text{now})W'(\text{now})}{W^2(\text{now})}$$

$$= \frac{2085 \cdot 6.868 \cdot 10^9 - 1.180 \cdot 10^9 \cdot 9181}{(6.868 \cdot 10^9)^2} \text{ per hour}$$

$$= 6.586 \cdot 10^{-8} \text{ per hour}$$

$$= 0.0005770 \text{ per year}$$

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Assuming $f'(x)$ exists & $f(x) \neq 0$

$$\left(\frac{1}{f(x)}\right)' = st\left(\frac{\frac{1}{f(x+\Delta x)} - \frac{1}{f(x)}}{\Delta x}\right)$$

where $0 \neq \Delta x \approx 0$

$$\Delta f = f(x+\Delta x) - f(x)$$

$$f(x) + \Delta f = f(x+\Delta x)$$

$$\Rightarrow \left(\frac{1}{f(x)}\right)' = st\left(\frac{\frac{1}{f(x)+\Delta f} - \frac{1}{f(x)}}{\Delta x}\right)$$

$$= st\left(\frac{1}{(\Delta x)(f(x)+\Delta f)} - \frac{1}{(\Delta x) f(x)}\right)$$

$$= st\left(\frac{1 \cdot f(x)}{(\Delta x)(f(x)+\Delta f) f(x)} - \frac{1 \cdot (f(x)+\Delta f)}{(\Delta x) f(x)(f(x)+\Delta f)}\right)$$

$$= st\left(\frac{f(x) - (f(x)+\Delta f)}{(\Delta x) f(x)(f(x)+\Delta f)}\right) = st\left(\frac{-\Delta f}{(\Delta x) f(x)(f(x)+\Delta f)}\right)$$

$$= -st\left(\frac{\Delta f}{\Delta x}\right) \cdot \frac{1}{f(x)(f(x)+st(\Delta f))}$$

$\Delta f \approx 0$ by Increment Theorem

$$= -f'(x) \cdot \frac{1}{f(x)(f(x)+0)} = \frac{-f'(x)}{f^2(x)}$$

$$\Rightarrow \left(\frac{1}{f}\right)' = -\frac{f'}{f^2} \quad \text{(Reciprocal Rule)}$$

$$\left(\frac{1}{x^2+1}\right)' = \frac{-(x^2+1)'}{(x^2+1)^2} = \frac{-(2x+0)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$$

9/13D ~~n=1,2,3,...~~ $n=1,2,3,\ldots$:

$$(f^{-n})' = \left(\frac{1}{f^n}\right)' = -\frac{(f^n)'}{(f^n)^2}$$

$$= -\frac{nf^{n-1}f'}{f^{2n}} = -nf^{n-1-2n}f'$$

$$= \boxed{-nf^{-n-1}f'}$$

$$((5-\sqrt{x})^{-3})' = -3(5-\sqrt{x})^{-4}(5-\sqrt{x})'$$

$$= -3(5-\sqrt{x})^{-4}\left(0-\frac{1}{2\sqrt{x}}\right) = \frac{3}{2\sqrt{x}(5-\sqrt{x})}$$

$$(x^{-n})' = -nx^{-n-1}x' = \boxed{-nx^{-n}}$$

$(x^{-5})' = -5x^{-6}$; $(x^{-1})' = -1x^{-}$

agrees with $(1/x)' = -1/x^2$.

$$\left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right)' = f' \cdot \frac{1}{g} + f \cdot \left(\frac{1}{g}\right)'$$

$$= \frac{f'}{g} + f \cdot \left(-\frac{g'}{g^2}\right) = \frac{f'g}{g^2} - \frac{fg'}{g^2}$$

$$= \frac{f'g - fg'}{g^2}$$

$$\left(\frac{x^3}{x^2+6}\right)' = \frac{(x^3)'(x^2+6) - x^3(x^2+6)'}{(x^2+6)^2} = \frac{3x^2(x^2+6) - x^3(2x}{(x^2+6)^2}$$

~~Next: differential form of product, quot, ...~~