Rosario Garza

9-14-10

Last Time:

- $\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$ (Reciprocal Rule)

- $(f^{-n})' = -nf^{-n-1}f'$ (Negative Power Rule)

- $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ (Quotient Rule)

Today:

- Picture of Quotient Rule
- Inverse Function Rule

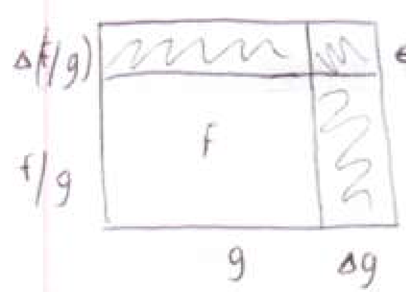
length: f/g

width: g

area: $(f/g)\cdot g = f$

Assume f, g depend on x; $g(x) \neq 0$; $f'(x)$ & $g'(x)$ exist; $0 \neq \Delta x \approx 0$

$f'(x) = st\left(\frac{\Delta f}{\Delta x}\right)$



$\leftarrow \Delta(\text{area}) = \Delta f$

$g'(x) = st\left(\frac{\Delta g}{\Delta x}\right)$

$$\Delta f = \Delta(f/g)\cdot g + \Delta(f/g)\cdot\Delta g + (f/g)\cdot\Delta g$$

$$\Delta f = \Delta(f/g)(g+\Delta y) + \frac{f}{g}\frac{\Delta g}{\Delta x}$$

$$st\left(\frac{\Delta f}{\Delta x}\right) = st\left(\frac{\Delta(f/g)}{\Delta x}\right)(g+\Delta g) + \frac{f}{g}st\left(\frac{\Delta g}{\Delta x}\right)$$

$$f' = st\left(\frac{\Delta(f/g)}{\Delta x}\right)(g+0) + \frac{f}{g}\cdot g'$$

$$\boxed{\text{increment theorem} \Rightarrow \Delta g \approx 0}$$

$$f' = st\left(\frac{\Delta(f/g)}{\Delta x}\right) g + \frac{f}{g}\cdot g'$$

$$f' - \frac{fg'}{g} = st\left(\frac{\Delta(f/g)}{\Delta x}\right) g$$

$$\underbrace{\frac{f'}{g} - \frac{fg'}{g^2}}_{\text{same for all } \Delta x} = st\left(\frac{\Delta(f/g)}{\Delta x}\right)$$

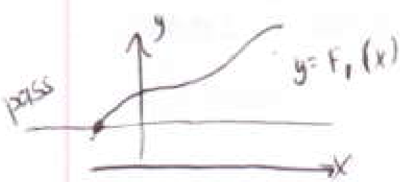
so, $(f/g)'$ exists & is $= \frac{f'g}{g^2} - \frac{fg'}{g^2} = \frac{f'g}{g^2} - \frac{fg'}{g^2} = \boxed{\frac{f'g - fg'}{g^2}}$

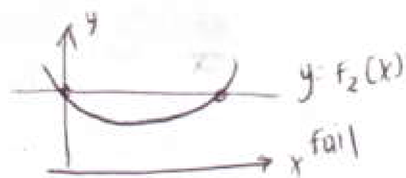
- <u>Inverse Functions</u>

If $f(x) = y$ & $g(y) = x$, then f & g are <u>inverses</u>

$$\begin{tikzcd} x \arrow[r, bend left, "f"] & y \arrow[l, bend left, "g"] \end{tikzcd}$$

A function f has an inverse if it passes the horizontal line test

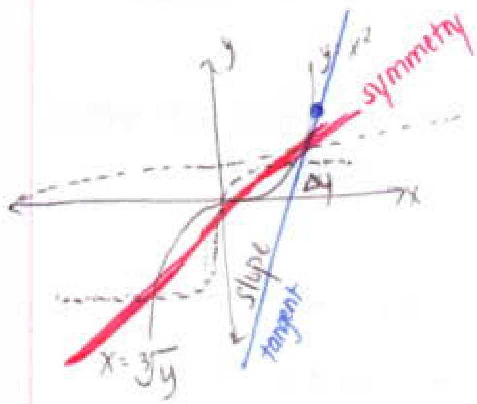


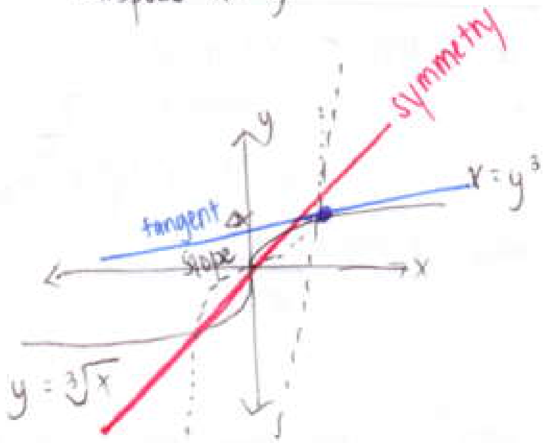


2 or more points on a horizontal line

$\underbrace{y = x^3 \qquad \& \qquad x = \sqrt[3]{y}}_{\text{inverse}}$

Transpose x & y





$m = \frac{\Delta y}{\Delta x}$

$\frac{1}{m} = \frac{\Delta x}{\Delta x}$

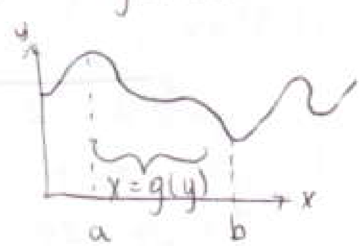
$\frac{\Delta y}{\Delta x} = \frac{\text{old } \Delta x}{\text{old } \Delta y} = \frac{1}{m}$

Idea : slope of inverse function should be $\frac{1}{\text{slope of original function}}$

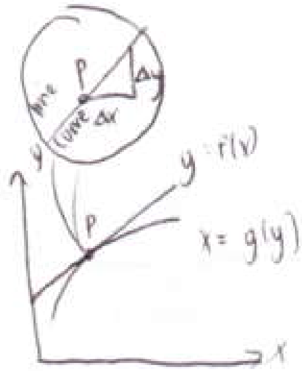
Inverse Function Theorem :

If $f'(x)$ exists & $f'(x) \neq 0$ whenever $a < x < b$, then there is a function g such that $x = g(y)$ & $y = f(x)$

$g'(y)$ exists and $g'(y) \neq 0$ whenever $y = f(x)$ and $a < x < b$



Inverse Function Rule :



If $y = f(x)$ + $x = g(y)$
& $f'(x)$ exists & $f'(x) \neq 0$
& $g'(y)$ exists & $g'(y) \neq 0$
then $f'(x) = \dfrac{1}{g'(y)}$

tangent line exists not verticals
not horizontal

Why? Let there be $0 \approx \Delta x \neq 0$. By the I.T, $\Delta y \approx 0$
Since f passes the horizontal line test, $\Delta y \neq 0$

Next : $f'(x) = st\left(\dfrac{\Delta y}{\Delta x}\right)$ & $g'(y) = st\left(\dfrac{\Delta x}{\Delta y}\right)$

because $0 \approx \Delta x \neq 0$ & $y = f(x)$

because $0 \approx \Delta y \neq 0$ + $x = g(y)$

$$f'(x)\, g'(y) = st\left(\frac{\Delta y}{\Delta x}\right) st\left(\frac{\Delta x}{\Delta y}\right) = st\left(\frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta y}\right) = st(1) = 1$$

$$f'(x)\, g'(y) = 1$$

$$\boxed{f'(x) = \frac{1}{g'(y)}}$$

$\left(\sqrt[3]{4x-2}\right)' = ?$ $\quad \dfrac{1}{\left(\frac{y^3}{4} + \frac{1}{2}\right)'} = \dfrac{1}{\frac{1}{4} \cdot 3y^2 + 0}$

$y = \sqrt[3]{4x-2}$

$y^3 = 4x - 2$

$y^3 + 2 = 4x$

$\dfrac{y^3}{4} + \dfrac{1}{2} = x$

$$\boxed{\frac{4}{3\left(\sqrt[3]{4x-2}\right)^2}} = \frac{4}{3y^2}$$

Answer with same variable you started with

$$\left(\frac{y^3}{4}\right)' = \Big(\underbrace{\frac{1}{4}}_{\text{constant}} \cdot y^3\Big)' = \frac{1}{4}\left(y^3\right)' = \frac{1}{4} \cdot 3y^{3-1} = \frac{1}{4} \cdot 3y^2$$

$\frac{1}{4} \cdot 3y^2 = \left(\frac{1}{2}\right)^2 3y^2 = 3\left(\frac{y}{2}\right)^2 = \frac{3y^2}{4} = \cdots$ are all equally good!

$$\frac{1}{\frac{1}{4} \cdot 3y^2} = \frac{1}{\frac{3y^2}{4}} = \frac{4}{3y^2}$$