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Notes

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Ex

$$\frac{d\left(\sqrt{2w/3}\right)}{dw} = \left(\left(2w/3\right)^{1/2}\right)' = \left(2^{1/2}w^{1/2}/3^{1/2}\right)' = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(w^{1/2}\right)' =$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{1}{2} w \; \tfrac{1}{2}w = \boxed{\frac{\sqrt{2}}{2\sqrt{3}} w^{-1/2}}$$

Ex

Alternative $\quad y = \sqrt{2w/3} \quad y^2 = 2w/3 \quad 3y^2/2 = w$

$$\left(\sqrt{2w/3}\right)' = \frac{1}{3(2y)/2} = \frac{1}{3y} = \boxed{\frac{1}{3\sqrt{2w/3}}}$$

Ex $\quad z = x^{1/3} - 4x^{-3/5} \quad \frac{dz}{dx} = \boxed{\frac{1}{3}x^{\frac{1}{3}-1} - 4\left(-\frac{3}{5}\right)x^{-\frac{3}{5}-1}}$

2.5 part I: sin & cosine

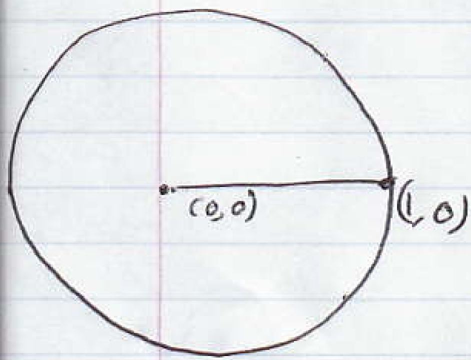
$(\sin x^\circ)' = \frac{\pi}{180}\cos x^\circ$

degrees

$\boxed{(\sin x)' = \cos x}$

radians

use this one



The unit circle
$x^2 + y^2 = 1$

radius = 1
counter = (0, 0)

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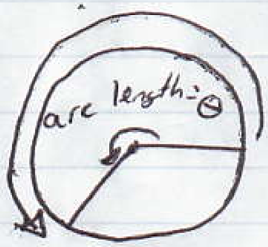
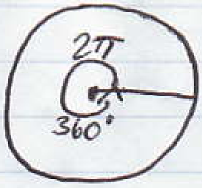
An angle of θ radians is an arc of length θ on the unit circle.



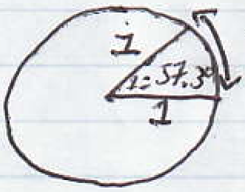
2π radians is the circumference 2π of the unit circle So, $2\pi = 360°$



$2\pi = 360° \Rightarrow \pi = 180° \Rightarrow 1 = 1 \text{ radian} = \left(\frac{180}{\pi}\right)$

$\Downarrow$

$\underline{57.3°}$



$1 = 57.3°$

$$\boxed{\approx \frac{\pi}{180} = 1° = 70° = 70\left(\frac{\pi}{180}\right) = 70\left(\frac{\pi}{18}\right) \text{ radians}}$$

If $\theta \geq 0$, then $\cos\theta$ & $\sin\theta$ are the x and y cordinates resulting from traveling distance θ from (1, 0) along the unit circle. counterclockwise

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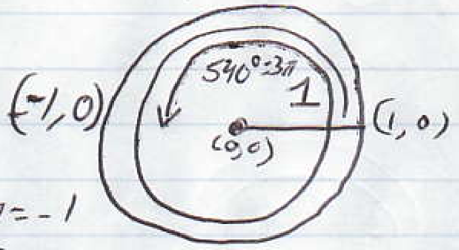
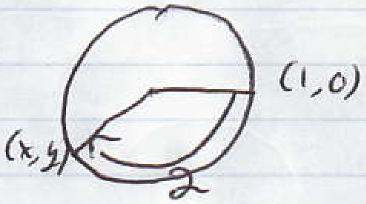


$3\pi = (1 + \frac{1}{2})(\text{circumference})$

$\cos 3\pi = -1$

$\sin 3\pi = 0$

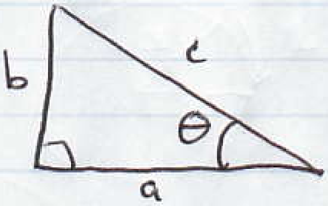
If $\theta < 0$, go clockwise



$x = \cos(-2)$

$y = \sin(-2)$

If $0^\circ = 0 < \theta < \pi/2 = 90^\circ$



then the $\left\{ \begin{array}{l} \cos \theta = a/c = \frac{adj}{hyp} \\ \sin \theta = \frac{b}{c} = \frac{opp}{hyp.} \end{array} \right\}$

$$\frac{d(\cos\theta)}{d\theta} = (\cos\theta)' = -\sin\theta$$

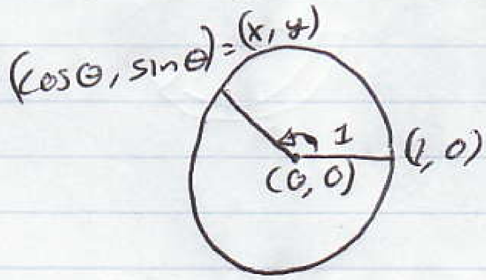
$$\frac{d(\sin\theta)}{} = (\sin\theta)' = \cos\theta$$

Example

$$(\sin^3\theta)' = 3\sin^2\theta \cdot (\sin\theta)' = 3\sin^2\theta\cos\theta$$

$$(f^n)' \; n f^{n-1} f'$$

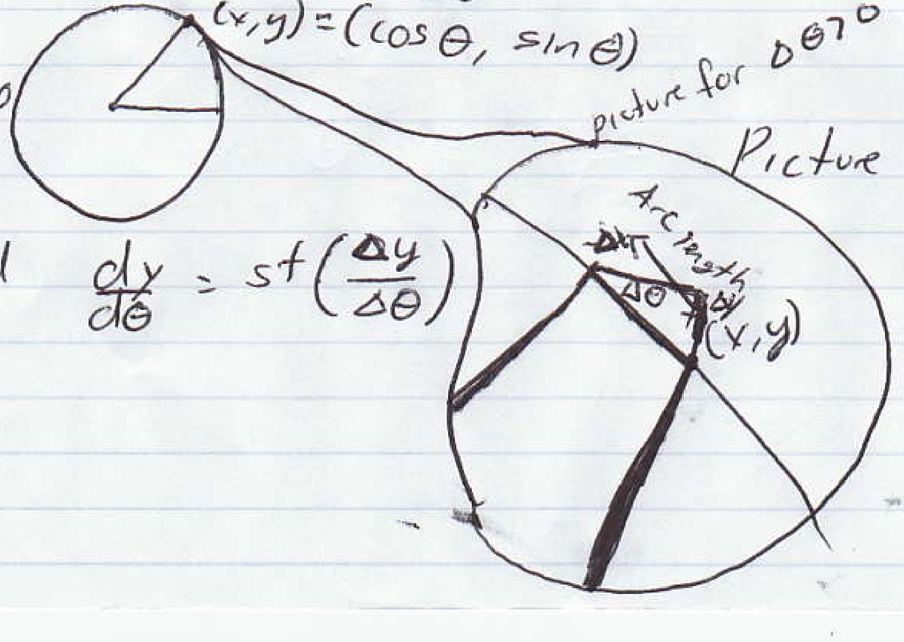
$$(\cos^2\theta + \sin^2\theta)' = 0$$



$$x^2 + y^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = \cos^2\theta + \sin^2\theta$$

$$\left.\begin{array}{l} x = \cos\theta \\ y = \sin\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{dx}{d\theta} = -\sin\theta \\ \frac{dy}{d\theta} = \cos\theta \end{array}\right.$$



if $0 \approx \Delta\theta \neq 0$

$\frac{dx}{d\theta} = st\left(\frac{\Delta x}{\Delta\theta}\right)$

and $\frac{dy}{d\theta} = st\left(\frac{\Delta y}{\Delta\theta}\right)$

Picture for $0 < \theta \frac{\pi}{2}$

That triangle is infinity close to a right triangle and its hypothenuse is infinely close to $\Delta\theta$; even on the scale here it looks like $h_{yp} = \Delta\theta$.

$$\cos\theta = \frac{adj}{hyp} = \frac{\Delta y}{\Delta\theta} \Rightarrow \cos\theta = \frac{dy}{d\theta}$$

$$\sin\theta = \frac{opp}{adj} \simeq \frac{-\Delta x}{\Delta\theta} \Rightarrow \cancel{\sin = -\frac{dx}{dy}} \quad \sin\theta = -\frac{dx}{d\theta}$$

$$\Rightarrow \boxed{(\sin\theta)' \cos\theta \;\&\; (\cos\theta)' = -\sin\theta}$$