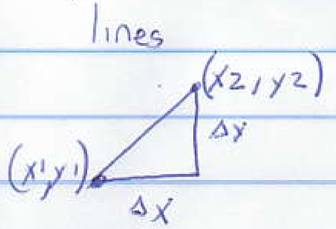
Review Notes

lines



$\Delta y = y^2 - y1$

$\Delta x = x^2 - x1$

slope $= m = \frac{\Delta y}{\Delta x}$

$y - y^1 = m(x - x_1)$

ε infinitesimal ⟺ ε infinitely close to 0

⟹ infinitely small ⟺ $st(\varepsilon) = 0$

b finite, non-infinitesimal ⟺ b infinitely close to some real

$\neq 0$

⟺ $st(b) \neq 0$

$st(a \pm b) = st(a) \pm st(b)$
$st(ab) = st(a)\,st(b)$ } if st(a) and st(b) exists

$st(a^n) = st(a)^n$
$st(\sqrt[n]{a}) = \sqrt[n]{st(a)}$ } if st(a) exists

$st(a/b) = \frac{st(a)}{st(b)}$

H, K > 0 and H, K are infinite
b, c > 0, b, c are finite then

infinitesimal		finite non-infinitesimal	"large" infinite		need more info
$\delta \pm \varepsilon$	ε / H "small"	$b+c$ "medium"	$H+K$	H/ε	$H-K$
$\delta \varepsilon$	$\sqrt[n]{\varepsilon}$	$b \pm \varepsilon$	$H \pm b$	H^n	$b-c$
$b \varepsilon$	ε^n	bc	$H \pm \varepsilon$	$\sqrt[n]{H}$	$H\varepsilon$
ε / b		b/c	Hb		H/K
$b/4$		$\sqrt[n]{b^n}$	H/b		ε/δ

Examples

$$5+(5-\varepsilon\delta) = 5-0\cdot 0=5$$

$$,A\ \delta,\varepsilon \approx 0$$

$$\frac{\overset{\text{infinite}}{\{}H^3-5}{\overset{\text{infinite}}{\{}4\boxed{H^2}+7} = \frac{\frac{H^3}{H^2}-\frac{5}{H^2}}{\frac{4H^2}{H^2}+\frac{7}{H^2}}$$

biggest

$$\frac{\sqrt{\varepsilon^2+7}-\sqrt{7}}{\varepsilon} = \frac{\varepsilon^2+7-7}{\varepsilon(\sqrt{\varepsilon^2+7}-\sqrt{7})}$$

multiply by up and bottom

$$\frac{\varepsilon^{\cancel{2}}}{\cancel{\varepsilon}(\sqrt{\varepsilon^2+7}+\sqrt{7})}$$

$$\frac{\varepsilon}{\sqrt{\varepsilon^2+7}+\sqrt{7}}$$

$$\frac{0}{\sqrt{0^2+7}+\sqrt{7}}=0$$

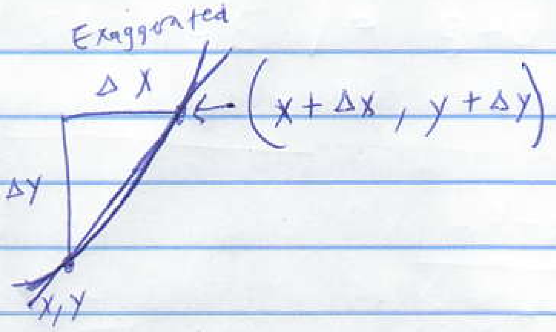
infinitesimal

$$\frac{H-\frac{5}{H^2}}{4+\frac{7}{H^2}}$$

$$= \frac{\text{big} - \text{small}}{4+\text{small}}$$

$$\frac{\text{big}}{\text{medium}} = \text{big}$$

$0 \neq \Delta x \simeq 0$



curve: $y = f(x)$

$\Delta y = f(x + \Delta x) - f(x)$

$\Delta y = f(x + \Delta x) - f(x)$

$f'(x) = st\left(\frac{\Delta y}{\Delta y}\right)$

$dx = \Delta x$

$dy = f'(x)\,dx$

$\frac{dy}{dx} = f'(x) = st\left(\frac{\Delta y}{\Delta x}\right)$

Increment: $\varepsilon \quad \underbrace{\frac{\Delta y}{\Delta x}}_{\text{slope of secant}} - \underbrace{\frac{dy}{dx}}_{\text{slope of tangent}}$

Increment Thm. f' exists $\Rightarrow \Delta y, \varepsilon \simeq 0$

Also $dy \simeq 0$

line

Tangent ∧ to $y = f(x)$ at $x = a$

Point-slope = $y - y_1 = m(x - x_1)$

$x_1 = a \quad y' = f(a) \quad m = f'(a)$

Shortcuts for finding $f'(x)$

$(mx+b)' = m$ $\qquad$ $(|x|)' = \begin{cases} 1 = x > 0 \\ -1 = x < 0 \end{cases}$

$(x^n)' = nx^{n-1}$ $\qquad$ $(f^n)' = nf^{n-1}f'$

$\nwarrow$ for $\nearrow$

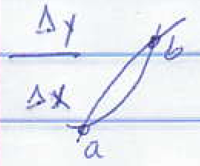
$(x^{-n})' = -nx^{-n-1}$ $\qquad$ $n = 1, 2, 3, 4, \ldots$

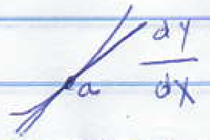
$(f^{-n})' = -nf^{-n-1}$

$(fg)' = f'g + fg'$ $\qquad$ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Average rate of change of y
from $x=a$ to $x=b$
$\Delta x = b-a$ $\qquad$ $\Delta y = f(b) - f(a)$

$\frac{\Delta y}{\Delta x}$ = avg. rate of change





Instantaneous rate of change
of y with respect
to x at $x=a$ is $f'(a)$

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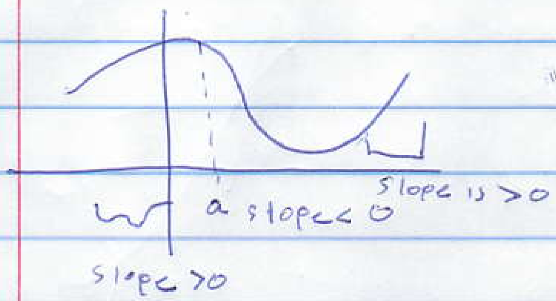
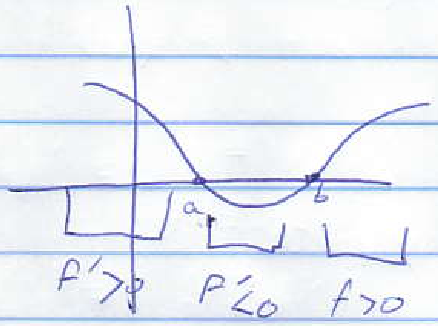
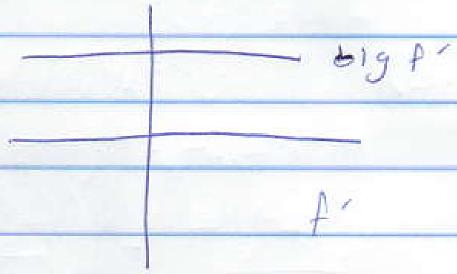
cost: $c = f(x)$ $\Rightarrow$ Marginal cost (Intantaneous rate of change)
$x=a$ is $c'(a)$ "marginal" = "derivatie"

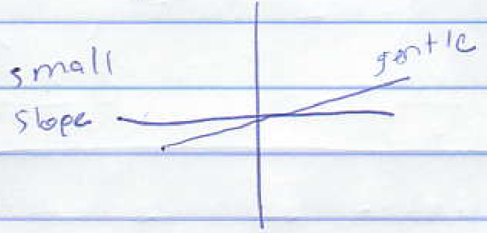
Questions?

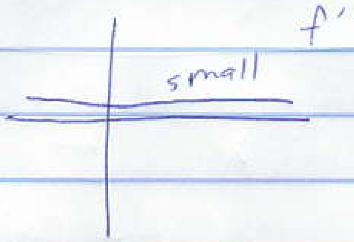

slope >0
a
slope< 0
slope is >0


a
b
f' >0
f' <0
f >0

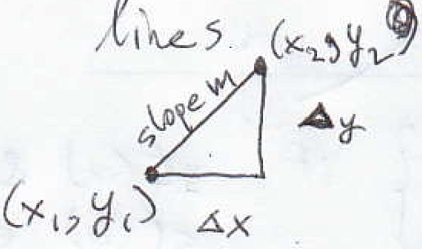

big slope
steep


big f'
f'


small slope
gentle


f'
small

lines.



$\Delta y = y_2 - y_1$

$\Delta x = x_2 - x_1$

[redacted] slope $= m = \frac{\Delta y}{\Delta x}$

Point-slope: $y - y_1 = m(x - x_1)$
Formula for line

~~When $dx = \Delta x$ is nonzero infinitesimal~~

~~$f'(a) = m$ means~~ [redacted]

Hyperreals:

[redacted] ε infinitesimal $\iff$ ε infinitely close to 0 $\iff$ infinitely small $\iff$ $st(\varepsilon) = 0$

b [redacted] finite, non-infinitesimal $\iff$ [redacted] b infinitely close to some real $r \neq 0$ $\iff$ $st(b) \neq 0$.

H infinite $\iff$ H infinitely big $\iff$ H infinitely far from all reals $\iff$ st(H) does not exist.

Name: ______________________

§1.6

$$\left.\begin{aligned} st(a \pm b) &= st(a) \pm st(b) \\ st(ab) &= st(a)st(b) \end{aligned}\right\}$$ if st(a) & st(b) exist

$$\left.\begin{aligned} st(a^n) &= st(a)^n \\ st(\sqrt[n]{a}) &= \sqrt[n]{st(a)} \end{aligned}\right\}$$ if st(a) exists

$$st(a/b) = st(a)/st(b)$$ if st(a) & st(b) exist & $st(b) \neq 0$

§1.5

If δ, ε are infinitesimal, $\delta, \varepsilon > 0$, $b, c > 0$, b, c are finite non-infitesimal, $H, K > 0$, and H, K are infinite, then:

infinitesimal: "small"	Finite non-infitesimal "medium"	infinite "large"	need more information
$\delta \pm \varepsilon$	$b + c$	$H + K$	$H - K$
$\delta\varepsilon$	$b \pm \varepsilon$	$H \pm b$	$b - c$
$b\varepsilon$	bc	$H \pm \varepsilon$	$H\varepsilon$
ε/b	b/c	$Hb, b/\varepsilon$	H/K
b/H		H/b	ε/δ
ε/H	$\sqrt[n]{b}$	H/ε	
$\sqrt[n]{\varepsilon}$	b^n	H^n, HK	
ε^n		$\sqrt[n]{H}$	

Date: Tuesday, September 21, 2010.

$$st(5-\varepsilon\delta) = 5 - 0\cdot 0 = 5$$

if $\delta, \varepsilon \approx 0$

$\Longrightarrow$ $5 - \varepsilon\delta$ is finite, non-infintesimal

$$\frac{\text{infinite}\{\; H^3-5}{\text{infinite}\{\; 4\underset{\underset{\text{biggest}}{\uparrow}}{\boxed{H^2}}+7} = \frac{(H^3-5)/H^2}{(4H^2+7)/H^2}$$

$$= \frac{H - 5/H^2}{4 + 7/H^2}$$

$$= \frac{\text{big} - \text{small}}{4 + \text{small}}$$

$$= \frac{\text{big}}{\text{medium}} = \text{big}$$

$\Longrightarrow$ infinite

$$\frac{\sqrt{\varepsilon^2+7}-\sqrt{7}}{\varepsilon} = \frac{\varepsilon^2+7-7}{\varepsilon\left(\sqrt{\varepsilon^2+7}+\sqrt{7}\right)}$$

$$(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a-b$$

$$\frac{\sqrt{a}-\sqrt{b}}{c} = \frac{\sqrt{a}-\sqrt{b}}{c}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{a-b}{c(\sqrt{a}+\sqrt{b})}$$

$$= \frac{\varepsilon^{\cancel{2}}}{\cancel{\varepsilon}\left(\sqrt{\varepsilon^2+7}+\sqrt{7}\right)} = \frac{\varepsilon}{\sqrt{\varepsilon^2+7}+\sqrt{7}}$$

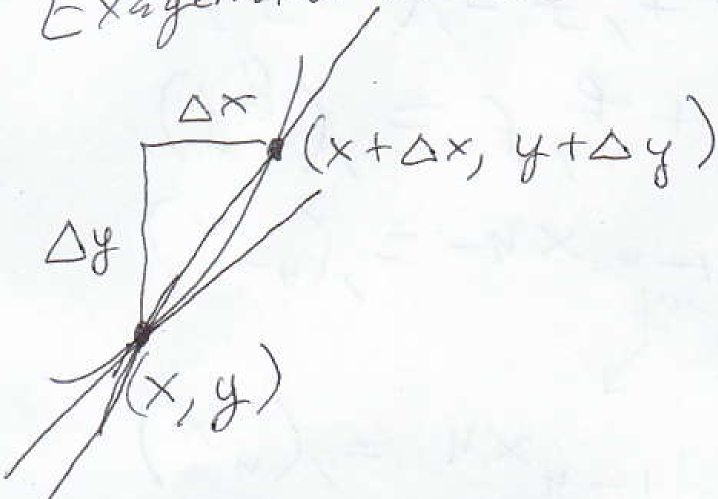
standard part is : $\frac{0}{\sqrt{0^2+7}+\sqrt{7}} = 0$

$\Rightarrow$ infinitesimal

Give a precise description of each E-equivalence class.

$0 \neq \Delta x \approx 0$ curve: $y = f(x)$

Exagerated Picture



$$\Delta y = f(x+\Delta x) - f(x$$

$$f'(x) = st\left(\frac{\Delta y}{\Delta x}\right)$$

$$dx = \Delta x$$

$$dy = f'(x)\,dx$$

$$\frac{dy}{dx} = f'(x) = st\left(\frac{\Delta y}{\Delta x}\right)$$

Increment: $\varepsilon = \underbrace{\frac{\Delta y}{\Delta x}}_{\text{slope of secant}} - \underbrace{\frac{dy}{dx}}_{\text{slope of tangent}}$

Increment Thm.: f' exists $\Rightarrow \Delta y, \varepsilon \approx 0$

Also $dy \approx 0$.

Tangent line to $y = f(x)$ at $x = a$

Point-slope: $y - y_1 = m(x - x_1)$

$x_1 = a$ $\quad$ $y_1 = f(a)$ $\quad$ $m = f'(a)$

Shortcuts for finding $f'(x)$

$(mx + b)' = m$ $\quad$ $(|x|)' = \begin{cases} 1 : x > 0 \\ -1 : x < 0 \end{cases}$

constant

$(x^n)' = nx^{n-1}$ $\quad$ $(f^n)' = nf^{n-1}f'$

for $n = 1, 2, 3, 4, \ldots$

$(x^{-n})' = -nx^{-n-1}$ $\quad$ $(f^{-n})' = -nf^{-n-1}f'$

$(fg)' = f'g + fg'$ $\quad$ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$(f \pm g)' = f' \pm g'$

$(cf)' = cf'$

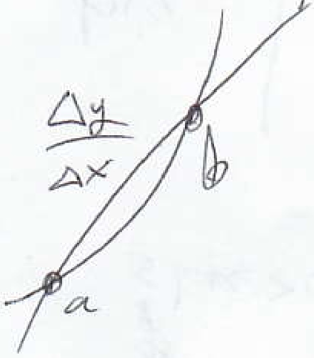
constant

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$

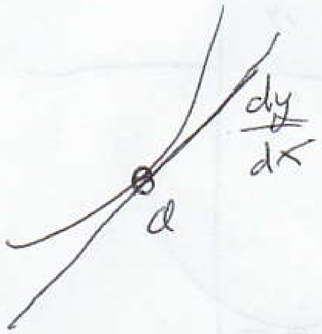
$y = f(x)$

Average rate of change of y from $x = a$ to $x = b$



$\Delta x = b - a \qquad \Delta y = f(b) - f(a)$

$\frac{\Delta y}{\Delta x}$ = avg. rate of change

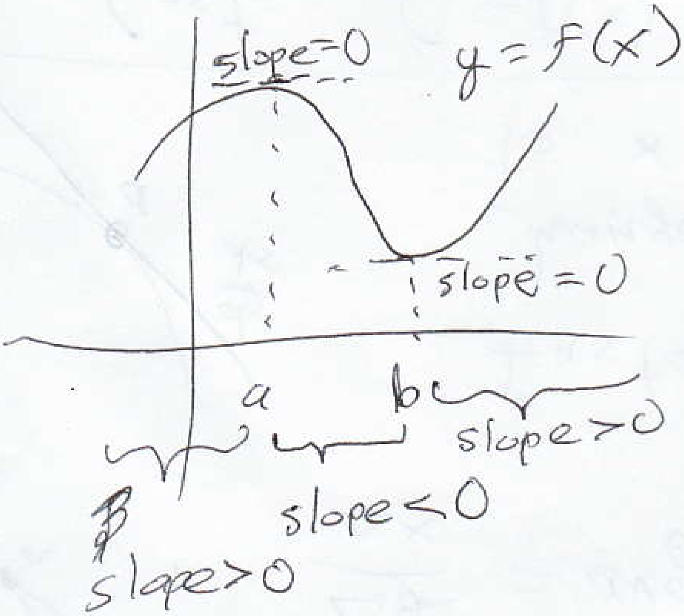


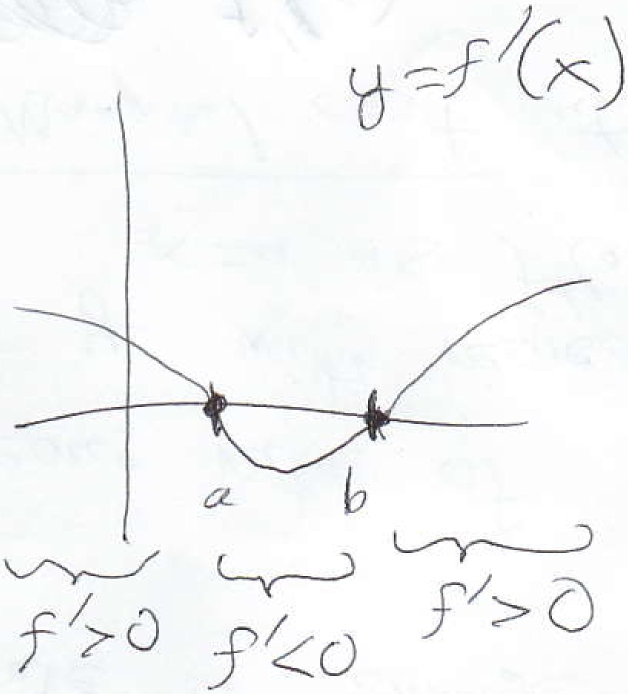
Instantaneous rate of change of y with respect to x at $x = a$ is $f'(a)$.

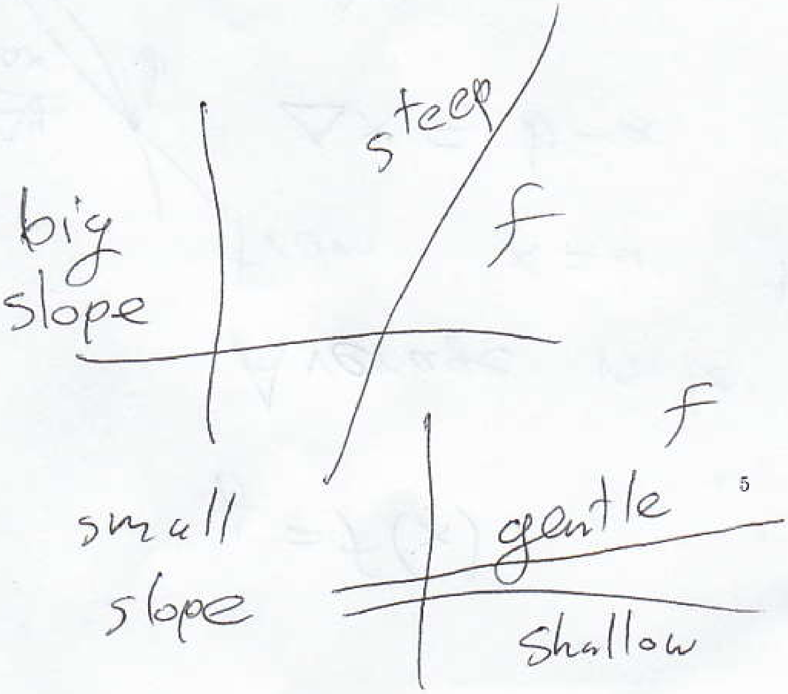
Cost: $C = f(x) \Rightarrow$ Marginal cost at $x = a$ is $f'(a)$.

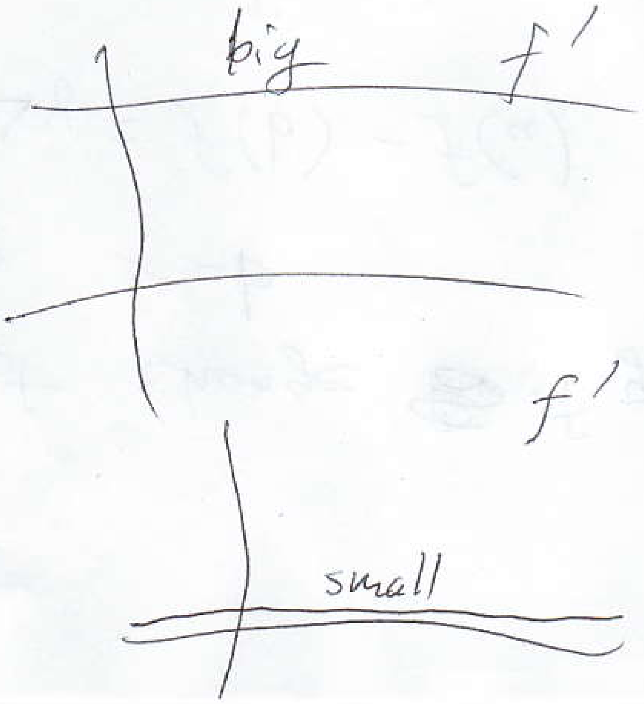
"Marginal" = "derivative"

Questions?


slope=0
y=f(x)
slope=0
a
b
slope>0
slope<0
slope>0


y=f'(x)
a
b
f'>0
f'<0
f'>0


steep
f
big
slope
f
small
slope
gentle
shallow


big
f'
f'
small