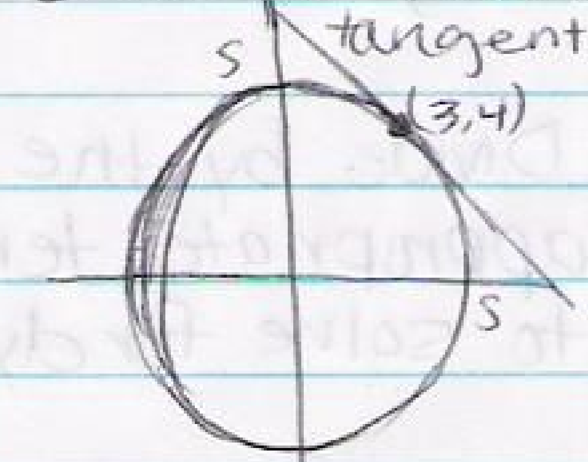
Veronica
Cardenas

Sept. 29, 2010

Today: Implicit Differentation (2.8)

Circle of radius 5 and center (0,0):



$x^2 + y^2 = 25$

$3^2 + 4^2 = 25$

$9 + 16 = 25$

What is the slope of tangent line at (3,4)?

1. Find dy/dx → Formula derivative of both sides of $x^2 + y^2 = 25$ using differentials:

$* \; d(f(x)) = f'(x)\,dx \; *$

$d(g(x)) = g'(y)\,dy$

$d(uv) = (du)v + u\,dv$

(Like $(fg)' = f'g + fg'$)

2. Plug in $x=3$, $y=4$.

$$\boxed{d\left(\frac{u}{v}\right) = \frac{(du)v - u\,dv}{v^2} \quad \text{like } \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

$d(u+v) = du + dv$

like $(f+g)' = f' + g'$

Back to $x^2 + y^2 = 25$

$d(x^2 + y^2) = d\underbrace{(25)}_{\text{constant}}$

$d(x^2) + d(y^2) = 0$

$2x\,dx + 2y\,dy = 0$

solve for $\frac{dy}{dx}$

1. Put all the dx's on one side of the equal sign, =, and all the dy's on the other:

$$\frac{2x\,dx}{-2y\,dx} = \frac{-\cancel{2}y\,dy}{-\cancel{2}x\,dx}$$

2. Divide by the appropriate terms to solve for dy/dx

$$= \boxed{-\frac{x}{y} = \frac{dy}{dx}}$$

3. Plug in (3,4).

$$\boxed{-\frac{3}{4} = \frac{dy}{dx} \text{ at } (3,4).}$$

$x^2y^3 = 6 + y + x$ Find $\frac{dy}{dx}$ at $(1,1)$

$$d(x^2y^3) = d(6 + y + x)$$

$$(d(x^2))y^3 + x^2 d(y^3) = 0 + dy + dx$$

$$(2x\,dx)y^3 + x^2(3y^2dy) = dy + dx$$

$$-dx - x^2(3y^2dy) \qquad -dx - x^2(3y^2dy)$$

use product Rule!

$d(uv) = (du)v + u\,dv$

$$(2x\,dx)y^3 - dx = dy - x^2(3y^2dy)$$

Now factor out the dx and dy.

$$(2xy^3 - 1)\frac{\cancel{dx}}{\cancel{dx}} = (1 - 3x^2y^2)\frac{dy}{dx}$$

Divide by dx

$$\frac{(2xy^3 - 1)}{(1 - 3x^2y^2)} = \frac{(1 - 3x^2y^2)}{(1 - 3x^2y^2)} \rightarrow \frac{2xy^3 - 1}{1 - 3x^2y^2} = \frac{dy}{dx}$$

→

$$\boxed{\text{Answer: } \frac{2(1)(1)^3-1}{1-3(1)^2(1)^2} = \frac{dy}{dx} \text{ at } (1,1)}$$

$\frac{5x}{y} + \sin^3(y^2x) = 2$ Find a formula for $\frac{dy}{dx}$.

$$d\Big(\underbrace{\frac{5x}{y}}_{\text{Part 1}} + \overbrace{\sin^3(y^2x)}^{\text{Part 2}}\Big) = \underbrace{d(2)}_{0}$$

$$d\left(\frac{5x}{y}\right) = 5d\left(\frac{x}{y}\right) = 5\left[\frac{(dx)y - xdy}{y^2}\right]$$

$$= \frac{(d(5x))y - 5xdy}{y^2}$$

$$d(5x) = 5dx$$

Now,

$$d(\sin^3(y^2x)) = ? \quad = d((\sin(y^2x))^3) = d(u^3)$$

$w = y^2x$

$u = \sin w = \sin(y^2x)$

$d(u^3) = 3u^2du$

$du = d(\sin w) = \cos w \, dw$

$$dw = d(y^2x) = \underbrace{(d(y^2))}_{2ydy}x + y^2dx$$

$dw = 2xydy + y^2dx$

$\rightarrow$

$du = \cos w \, dw$

$dw = (\cos(y^2x))(2xy\,dy + y^2\,dx)$

$d(u^3) = 3u^2\,du = 3(\sin(y^2x))^2(\cos(y^2x))(2xy\,dy + y^2\,dx)$

Shorter: $d(u^3) = 3u^2 \cos w \,(2xy\,dy + y^2\,dx)$

Now put everything together.

$$d\left(\frac{5x}{y} + \sin^3(y^2x)\right) = d(2)$$

$$5\left(\frac{(dx)\,y - x\,dy}{y^2}\right) + 3u^2\cos w\,(2xy\,dy + y^2\,dx) = 0$$

Move dy's over!

$$+\frac{5x\,dy}{y^2} \qquad\qquad +\frac{5x\,dy}{y^2}$$

$$-3u^2(\cos w)(2xy)\,dy \qquad\qquad -3u^2(\cos w)(2xy)\,dy$$

$$\frac{5(dx)\,y}{y^2} + 3u^2(\cos w)\,y^2\,dx = \frac{5x\,dy}{y^2} - 3u^2(\cos w)(2xy\,dy)$$

Factor out!

$$\frac{\left[\frac{5}{y} + 3u^2y^2\cos w\right]dx}{\left[\frac{5x}{y^2} - 6u^2xy\cos w\right]\cancel{dx}} = \frac{\left[\frac{5x}{y^2} - 6u^2xy\cos w\right]dy}{\left[\frac{5x}{y^2} - 6u^2xy\cos w\right]dx}$$

$$\frac{\frac{5}{y} + 3u^2y^2\cos w}{\frac{5x}{y^2} - 6u^2xy\cos w} = \frac{dy}{dx}$$

Final answer:

$$\frac{5/y + 3(\sin(y^2x))^2y^2\cos(y^2x)}{5x/y^2 - 6(\sin(y^2x))^2xy\cos(y^2x)} = \frac{dy}{dx}$$